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# PAPERS, REPORTS, DISCUSSIONS, AND MEMOIRS

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AMERICAN SOCIETY OF CIVIL ENGINEERS
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# PAPERS AND DISCUSSIONS

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## EXPERIMENTS ON SPILLWAY DISCHARGE OF THE KEOKUK DAM

By Floyd A. Nagler* and Albion Davis,† Members, Am. Soc. C. E.

### Synopsis and Conclusions

This paper describes rather unusual discharge measurements made on the crest of the spillways of the Keokuk Dam, in Iowa. These studies were supplemented by tests made on a laboratory model, in order to determine a more accurate spillway rating than had formerly been available and in order to make possible a final and precise revision of the daily flow records of the Mississippi River at this point. Volumes of flow averaging about 4 000 sec-ft. were measured in the 30 ft. of spillway width, at heads slightly less than 11 ft.

Unless one has attempted to measure water running 24 ft. per sec. he cannot appreciate all the difficulties attendant on an accurate spillway crest gauging under a head of 11 ft. Special equipment was required to handle properly the stay-wires supporting the meters. By this means it was possible to make about 150 velocity observations in each gauging, fully exploring the entire cross-section up to within 0.4 ft. of the spillway edge. It was also possible to measure all the vertical and horizontal angles of flow and to rate the meters so as to obtain results that were accurate within 2 per cent.

It was found that opening the gates adjacent to the spillway where the water was being measured, had a very considerable effect on the discharge coefficients. In the formula, $Q = C L H^{\frac{3}{2}}$, the coefficient, $C$, for the normal operating head‡ of 10.8 ft., varied from 3.71, when a single spillway was discharging, up to 3.90, when many spillways were discharging. The gaugings also showed clearly that with a crest unobstructed by piers, $C$, for a 10.8-ft. head, increased to 4.00.

Note.—Written discussion on this paper will be closed in May, 1929.

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‡ Throughout this paper, $H$ in this formula denotes the depth of the pond water above the highest part of the concrete crest.

The coefficients developed by these measurements are in rather striking contrast to the conservative coefficient of 3.33 that was used in providing spillway capacity in the Keokuk Dam, notwithstanding the fact that in designing the dam it was realized that the actual coefficients would probably be considerably greater than 3.33. The use of the new spillway rating yields flood flows that are in some cases 10% in excess of the former determination.

The field measurements were followed by a study of the behavior of a model of three of the spillways built to a scale of one-eleventh the size of the Keokuk Dam in the Hydraulic Laboratory of the State University of Iowa.

These experiments duplicated the results observed at the dam itself under normal conditions with remarkable reliability, and studies were made to determine the probable behavior of the Keokuk spillways when discharging under abnormal pond levels.

Experiments were performed on the model when its surface was both smooth and rough, and with and without a fill of sediment up stream from the dam. It was demonstrated that in experimenting with such a large model, the relative amount of friction caused by the varying surface conditions of the model was of small significance.

Several pier shapes differing from those used at Keokuk were constructed on the model. Extending the pier nose up stream, or giving it a tapered form so that it attained its full thickness a short distance down stream from the highest part of the spillway crest, produced a dam with a greater discharge than that obtained with the Keokuk design.

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## History of the Calibration of the Spillways

Both prior and subsequent to the actual construction of the water power development, the engineers of the Mississippi River Power Company have kept continuous records of daily flow at Keokuk, as well as at other points on the Upper Mississippi River.

The flow record at Keokuk subsequent to the construction of the dam has been computed with considerable accuracy by calculating the discharge through the water turbines, and the quantity of surplus water discharged over the spillways. Field measurements of the hydraulic efficiency of the various turbine units were available. These served as a basis for accurate computations of turbine discharge.

Pond elevations taken every hour and a record of gate operations permit computation of the flow over the spillways by the use of a weir formula. The coefficients for this formula to compensate properly for the nappe contractions over the crest and around the piers, however, must be determined definitely by measurements if an accurate record of discharge over the dam is to be obtained.

Previous to 1924, coefficients had been used which were based partly on some early measurements made on the discharge through single spillways at times when the pond level was maintained at an elevation of less than 5.28 ft. above the crest, and also on a rating curve for these spillways prepared by John R. Freeman, Past-President, Am. Soc. C. E., and derived from experi-

ments on models of the Croton Dam. The normal operating head on the Keokuk spillway is 11 ft., a head much in excess of that existing in any of these tests, and much higher than had existed in any measurements known to the writers.

For some time it was feared that the coefficients obtained in this manner were too small for use under heads as great as 11 ft., perhaps to the extent of 10%, but no change was made from the original rating so that all records of daily flow would be comparable. From the daily computations of flow over a period of several years, and from current-meter measurements of flood discharge above and below the dam, several fairly definite conclusions were developed. It was noted that there was a tendency for the computed flow at Keokuk to be lower on Sundays than during week days when there had been no apparent change in natural flow entering the pond. The only difference in Sunday operation was that more water passed over the spillways and less through the water turbines than on contiguous week days. Flood discharges computed from the several measurements in the open river invariably indicated a higher flow than those computed from the records of plant and spillway operation. It became evident that a new and more nearly correct rating of the spillways must be obtained if the Keokuk records were to represent the true discharge at that point. The tests described herein were undertaken as the first step toward making the flow records available for publication for the benefit of the profession.

The co-operation of the engineers of the Hydraulic Laboratory at the State University of Iowa, was enlisted in obtaining the actual measurements of flow through the Keokuk spillways. The Laboratory Staff was interested in undertaking this work as a research problem not only because of the value to the Engineering Profession of the actual field measurements of discharge over the Keokuk spillway, but also because of the opportunity afforded thereby to make comparisons between the large-scale results and the performance of models.

Eight measurements of the discharge over the spillways were made by operating Ott current meters in the cross-section of the nappe above the crest during the period from April 29 to May 8, 1924.

A model of three sections of the spillway one-eleventh of the full size was constructed in the testing canal of the University of Iowa Hydraulic Laboratory, and studies were made during the period from September 16 to October 29, 1925.

Before removing the model from the testing flume, an investigation was made to compare the contraction effects of certain other forms of piers with those of the Keokuk design. These experiments were performed during the period from November 19, 1925, to May 5, 1926.

## Description of Spillway

The actual dam consists of 119 spillway sections extending from the Illinois shore to the up-stream corner of the power house. A view of the spillway is shown in Fig. 1. Each spillway section is 30 ft. wide with a crest about 30 ft. above the former natural stream bed. The spillways are separated by

piers which are 6 ft. thick, with a cross-section at the up-stream end in the form of a semi-circle (see Fig. 2).

The typical cross-section of the spillway shown in Fig. 3 gives the details and dimensions of the form of the crest. Soundings made up stream from the spillways selected for the tests indicated that the bed of the stream approaching the dam is much higher than the natural stream bed at these points. This is due in some cases to coffer-dam material left in place and, in other cases, to natural sedimentation. In the twelve years since the construction of the dam, there has been considerable silting in front of those gates that are operated very infrequently and where "dead water" exists nearly all the time. In front of the gates that are operated more frequently, however, there is practically no sedimentation.

Fig. 2, which is a view of the up-stream face of the dam, shows the well-rounded approach to the crest of the spillway. Extending from the concrete crest upward there is a slot 21 in. wide and 14 in. deep in the face of each pier, in which steel gates, 11 ft. high and 32 ft. wide, are operated. The water level behind the dam is maintained at all times close to the top of the gate which is about 11 ft. above the crest of the dam. The steel gates, which are of the plain sliding variety without rollers, are finished on the bottom with a timber sill which makes the water seal against the carefully finished concrete crest of the dam. The gates are raised by electrically operated traveling cranes, and are always lifted clear of the water surface, so that a spillway discharges to its full capacity. What little leakage there is when a gate is lowered, is cut off effectively by a few shovelfuls of cinders, when the leakage becomes a factor in the operation of the plant.

The spillways are numbered in order from the Illinois shore toward the power house. The discharge measurements were made on Spillways Nos. 68 and 103. Spillway No. 68 was chosen as representative of those near the center of the dam, and No. 103 was selected from among those nearer the power house where the pond is deeper.

## Current Meter Discharge Measurements at Keokuk Dam, April 29 to May 8, 1924

After a careful study had been made of the various possible methods of accurately measuring the discharge through the spillway, it was decided to operate current meters in the nappe as it flowed over the crest of the dam. It was appreciated, however, that stream velocities would be nearly 25 ft. per sec., which is greatly in excess of speeds ordinarily encountered in current meter practice; and that, in parts of the nappe, stream filaments would approach a rigidly supported meter at a considerable angle with the meter axis. Non-rigid suspension of metering equipment was entirely out of the question.

*Development of Method of Measurement.*—The Ott type of current meter seemed admirably adapted for these measurements, not only because it measures the normal component of angular currents more accurately than other types, but also because the bearing at the end of its horizontal axis is capable of taking the strong horizontal thrust caused by these high velocities.

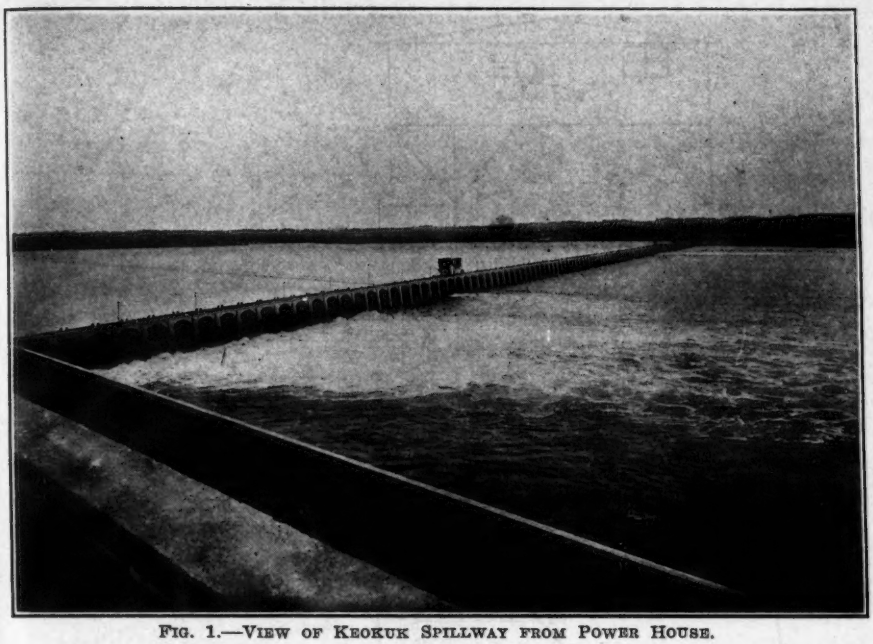


FIG. 1.—VIEW OF KEOKUK SPILLWAY FROM POWER HOUSE.

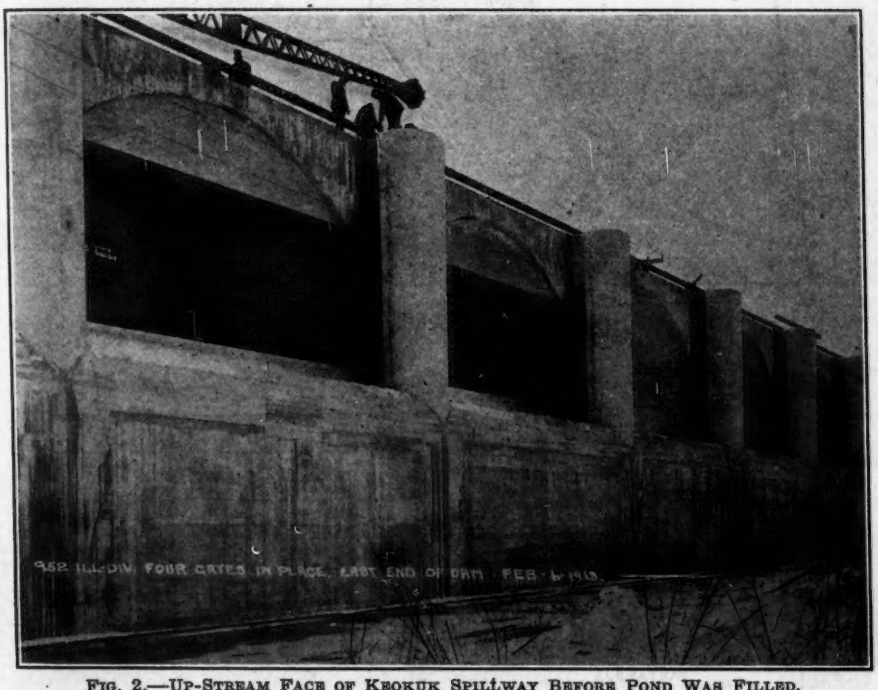



FIG. 2.—UP-STREAM FACE OF KEOKUK SPILLWAY BEFORE POND WAS FILLED, FEBRUARY 6, 1913.

FIG. 1.—VIEW OF KROEBUK SPILLWAY FROM POWER HOUSE.

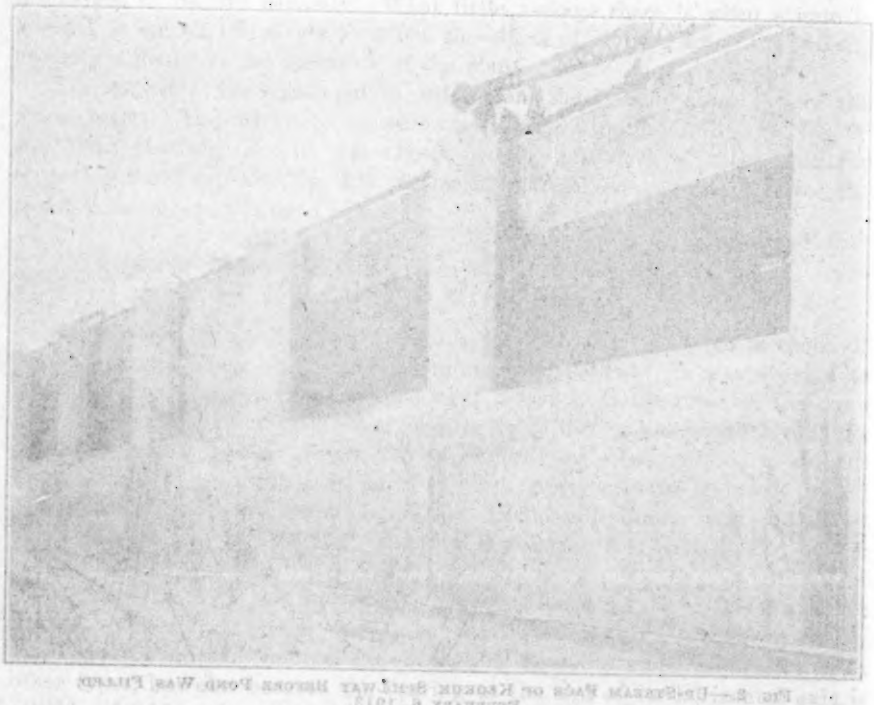


FIG. 2.—DOWNSTREAM FACE OF KROEBUK SPILLWAY BEFORE TOPS WAS PLACED. FEBRUARY 6, 1913.

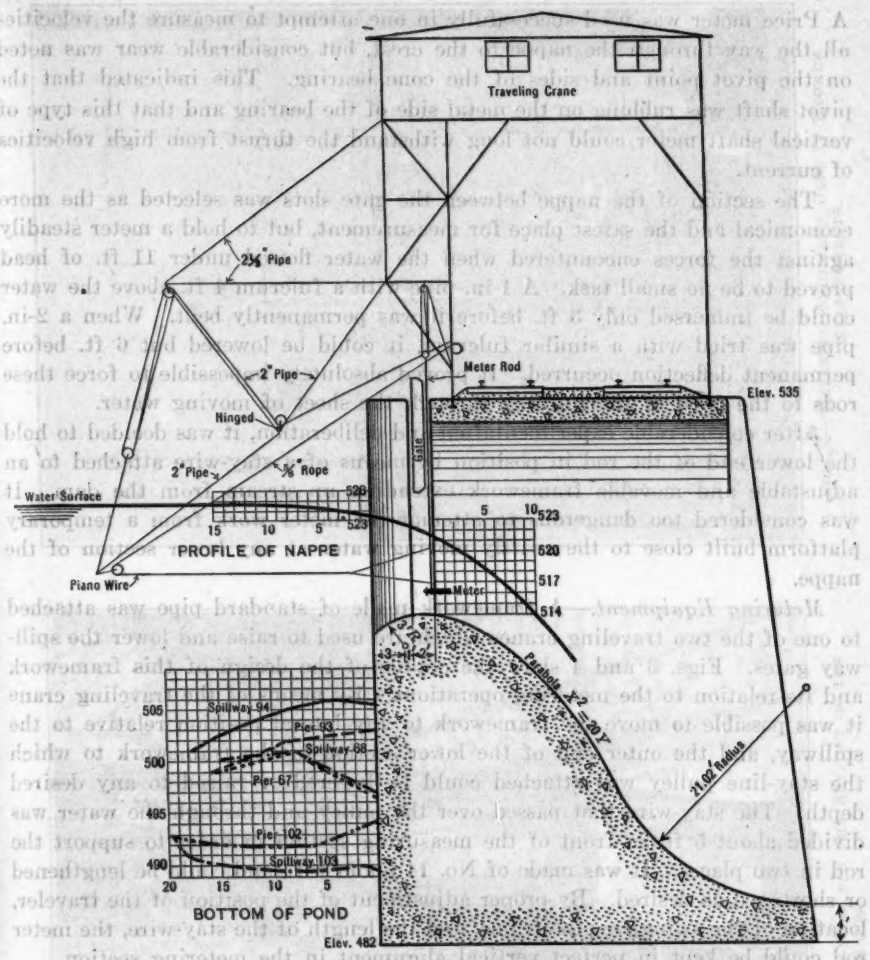



FIG. 3.—DETAILS OF SPILLWAYS IN THE MAIN DAM.

A Price meter was used successfully in one attempt to measure the velocities all the way through the nappe to the crest, but considerable wear was noted on the pivot point and sides of the cone bearing. This indicated that the pivot shaft was rubbing on the metal side of the bearing and that this type of vertical shaft meter could not long withstand the thrust from high velocities of current.

The section of the nappe between the gate slots was selected as the more economical and the safest place for measurement, but to hold a meter steadily against the forces encountered when the water flowed under 11 ft. of head proved to be no small task. A 1-in. pipe with a fulcrum 4 ft. above the water could be immersed only 3 ft. before it was permanently bent. When a 2-in. pipe was tried with a similar fulcrum, it could be lowered but 6 ft. before permanent deflection occurred. It proved absolutely impossible to force these rods to the crest of the spillway through the sheet of moving water.

After considerable experimentation and deliberation, it was decided to hold the lower end of the rod in position by means of a stay-wire attached to an adjustable and movable framework extending up stream from the dam. It was considered too dangerous to attempt the meter work from a temporary platform built close to the swiftly moving water at any lower section of the nappe.

*Metering Equipment.*—A framework made of standard pipe was attached to one of the two traveling cranes which are used to raise and lower the spillway gates. Figs. 3 and 4 show the details of the design of this framework and its relation to the metering operations. By means of the traveling crane it was possible to move the framework to any lateral position relative to the spillway, and the outer end of the lower section of the framework to which the stay-line pulley was attached could be lowered or raised to any desired depth. The stay-wire that passed over the pulley and through the water was divided about 5 ft. in front of the measuring section in order to support the rod in two places. It was made of No. 14 piano wire and could be lengthened or shortened as desired. By proper adjustment of the position of the traveler, location of the swinging pipe frame, and the length of the stay-wire, the meter rod could be kept in perfect vertical alignment in the metering section.

The meters were attached to the bottom of steel rods, 23.5 ft. long, which were either 1 or 1.25 in. in diameter, depending on which meter was in use. The stay-wires were connected to the rod by collars inside of which the rod was free to rotate. However, the rods were provided with keys to keep the meters and rod handles from rotating about the shaft. The meters were raised and lowered with the rod by hand or with the aid of a pulley and rope attached to the frame of the crane. The meters and rods could be lowered into position by passing them through the 5-in. space between the down-stream side of the gate and the up-stream face of the concrete deck of the dam. Guides at the top and bottom of the gate were used to plumb the rod at designated verticals in the measuring section.

*Current Meters.*—Three meters of the Ott type were used in making the different measurements, although the equipment permitted the operation of but one meter at a time. (See Fig. 5.) Most of the work was done with a

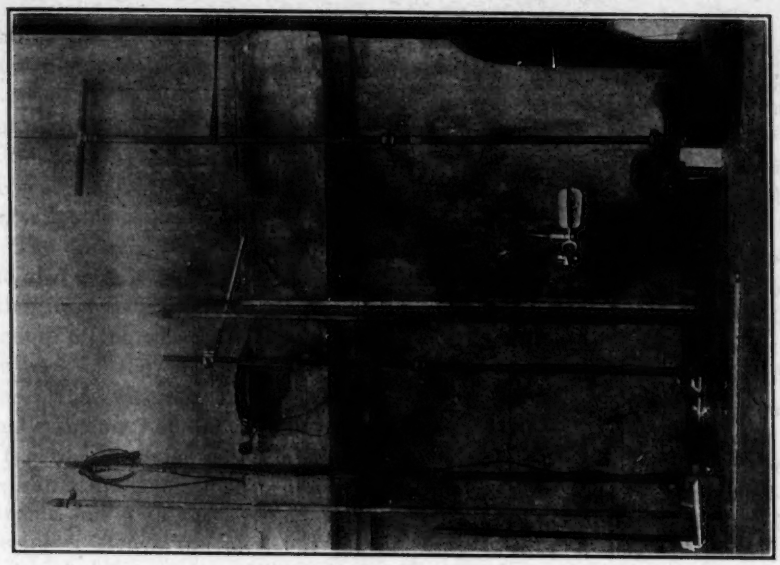


FIG. 5.—METERING EQUIPMENT USED IN KEOKUK MEASUREMENTS.

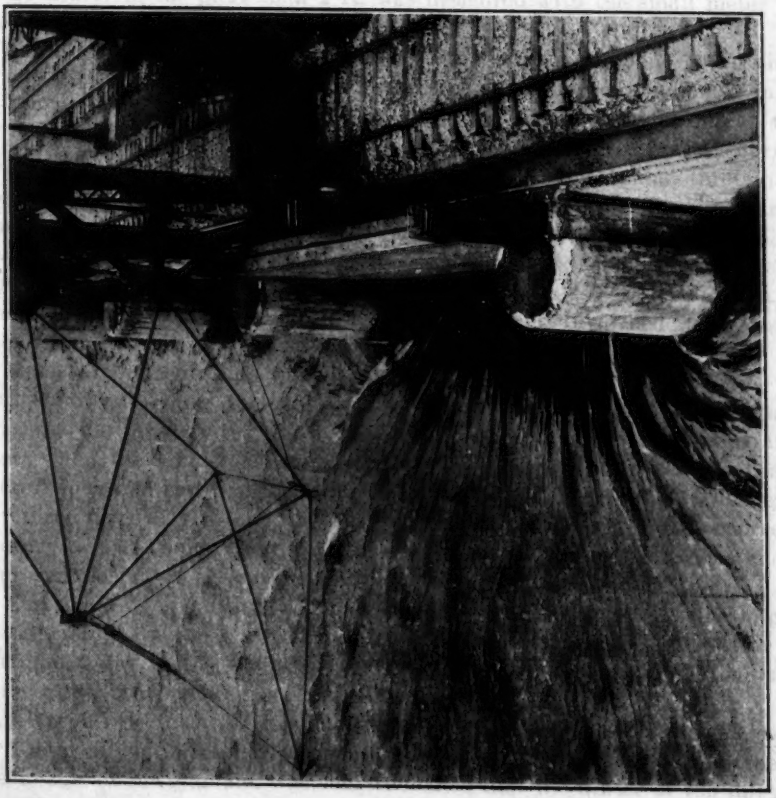
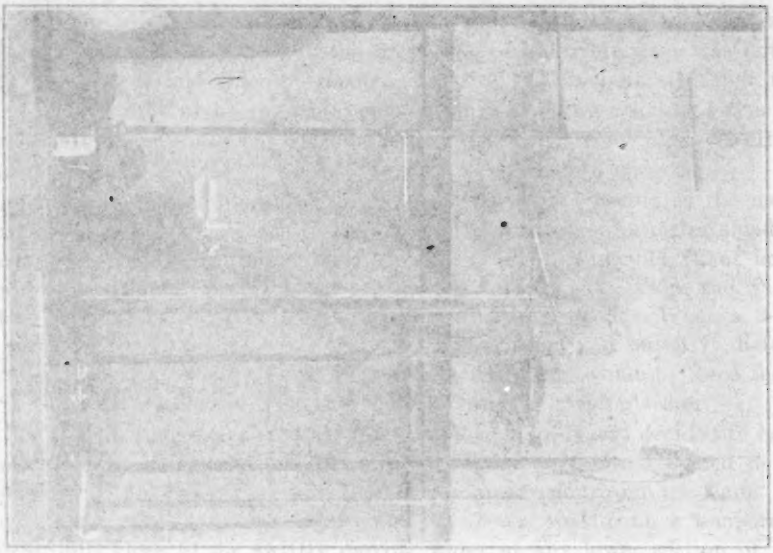
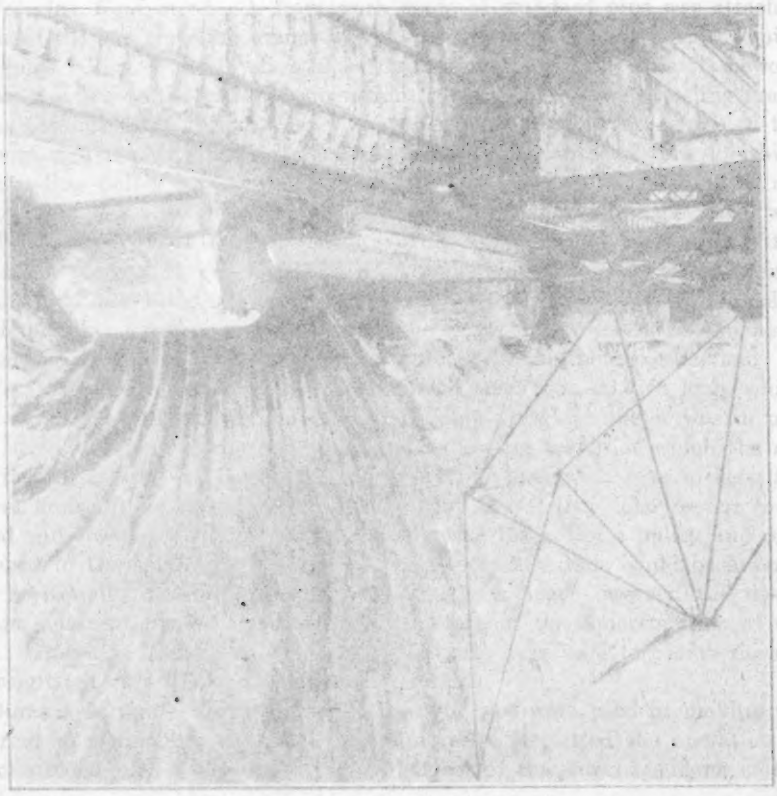


FIG. 4.—FRAMEWORK FOR HANDLING STAY-WIRE FOR THE SUPPORT OF THE CURRENT METER ROD.





small Ott meter (No. 2908) which had a propeller with two blades. Soon after the work started, this meter was equipped with a special tail-piece and a guard to protect it against striking the face of the concrete pier. With this instrument it was possible to make velocity measurements within 0.4 ft. of the side of the spillway. A steel rod, 1 in. in diameter, was used and this whole equipment seemed easier to handle and keep under control than that used with the larger meters.

The Mensing-Ott meter (No. 3246) was used with a 3-blade propeller, and also with a 2-blade propeller. With the 3-blade propeller it had characteristics in measuring angular currents which were superior to the others, but because of the wheel it was lowered with difficulty in the small space between the back of the gate and the concrete deck of the dam, and because of the larger rod, was not as easily handled in the swiftly moving water as the smaller meter. It did not prove practical to make measurements closer than 2 ft. from the pier with the larger meter because of the danger of the rod becoming unmanageable in moving from one section to another in the oblique current and striking the pier. At one time, when the rod got out of control, the meter struck the pier, but the propeller escaped damage because of the recess at the gate slot. When the larger meter was used, the part of the discharge nearer to the piers than 2 ft. was measured with the small meter provided with a steel guard.

While the vertical rod held the meter in a horizontal plane at all times, no attempt was made to keep the meter axis in a line perpendicular to the measuring section. The meters were supplied with longer tail-pieces than are ordinarily used, and were allowed to swing into the thread of the current. Throughout the major part of the spillway the vertical plane through the thread of the current was perpendicular to the metering section, but near the piers there was considerable contraction of the nappe, with resultant oblique flow. The horizontal angle assumed by the meter axis was measured for every point in order to correct the velocity measurements for this deflection. A handle clamped firmly to the rod near its upper end furnished the necessary index for measuring this deflection.

Much experimentation was done with various forms of tail-pieces and methods of attaching the stay-wire to the meter rod. On four different occasions the piano wires broke despite the care taken in their preparation. Once, when the lake was rather rough, the twist on the meter rod became severe enough to shear off the screws holding the key in place. During the first three days of the tests the bending of the rods was a daily occurrence. In every case of trouble, however, the man operating the rod was quick enough to prevent losing or seriously damaging any of the equipment.

The strain of operating the small meter in the high velocities was so great that during the tests it proved necessary to make a new steel shaft for this instrument due to the excessive wear that had taken place in the ball races of the guide-bearing near the propeller.

A special device was constructed to measure the vertical as well as the horizontal angle of the current in the nappe. It was operated from the upstream side of the gate and was constructed so that the center of the tail-piece

stood in the plane of the measuring section. The tail-piece was about 2 ft. long and was hung about 5 in. down stream from the 1-in. pipe which supported it. The up-stream end of the tail-piece was pinned to a ⅜-in. iron rod which passed inside the 1-in. pipe and extended through a hole in a tee at the top. All parts adjacent to the vane were thinned down and given streamline surfaces to offer the least resistance to flow. Handles for operating the equipment were screwed to the tee. The pivot point was located so that the weight of the tail-piece balanced the weight of the ⅜-in. iron rod. The stay-wires were connected to the pipe in a manner which left it free to rotate. Horizontal angles were indicated by the angle which the handles made with the face of the gate; the vertical displacement of the ⅜-in. rod through the tee at the top gave the data required to compute the sine of the vertical angle of the current. The equipment was very rugged, but, nevertheless, dependable and accurate.

*Rating of Current Meters.*—During the tests the water attained velocities of 13 to 26 ft. per sec. and struck the meter at angles with the axis of 0 to 22 degrees. It was impossible to secure ratings in velocities as high as most of those encountered, but after a survey of the possibilities at various rating stations, the meters were sent to the U. S. Bureau of Standards where they were calibrated in velocities as high as 16 ft. per sec. Since the bearing friction becomes a negligible force at high velocities of flow, the rating curves for all these meters were straight lines at velocities greater than 2.5 ft. per sec. They were thus extended to give the rating throughout the range of velocities encountered. The meters were rated both before and immediately following the measurements at Keokuk. A special rating of Meter No. 2908 was made with the guard mounted on the meter frame. While no part of this guard was within 2 in. of the propeller, this obstruction behind the wheel retarded the revolution approximately 2 per cent.

The meters were also rated with the water approaching the propeller at horizontal and vertical angles. These ratings showed little if any difference in the registration, with currents approaching at the same angles from various directions. In streams of great obliquity the three Ott meters under-registered the axial component of the currents by different amounts, the 3-blade meter having by far the best characteristics. Fig. 6 gives the amount of the under-registration in currents of high velocity as obtained from the ratings at various angles. In other words, it shows the correction (in percentage) to be added to the measured velocity in order to obtain the actual component of velocity along the axis of the meter. The meters used were rated by the U. S. Bureau of Standards during April and June, 1924.

*Discharge Measurements.*—Three discharge measurements (Tests 1, 2, and 8, in Table 1), were made when the spillway being metered was remotely isolated from others that were open. Column (4) of Table 1 gives the location of the tests. Test 1 was performed merely to familiarize the observers with the operation of the equipment.

Tests 3 to 7, inclusive, were performed on Spillway No. 68 in order to determine the effect of the operation of adjacent spillways on the discharge through the spillway under test. Thus, in Test 3, the adjacent spillway,

No. 69, was open. In Tests 4 and 5, one spillway on each side of No. 68 was opened. These two tests were performed under exactly the same spillway conditions, but with different meters, in order to check the accuracy of this work; also, because parts of Test 4 were performed on different days, it was thought desirable to repeat this test in a continuous run. In Test 6, the spillway observed was the end one of a series of five that were open, and in Test 7 the measurements were made on No. 68 when operating as the center spillway of a series of seven which were discharging fully.

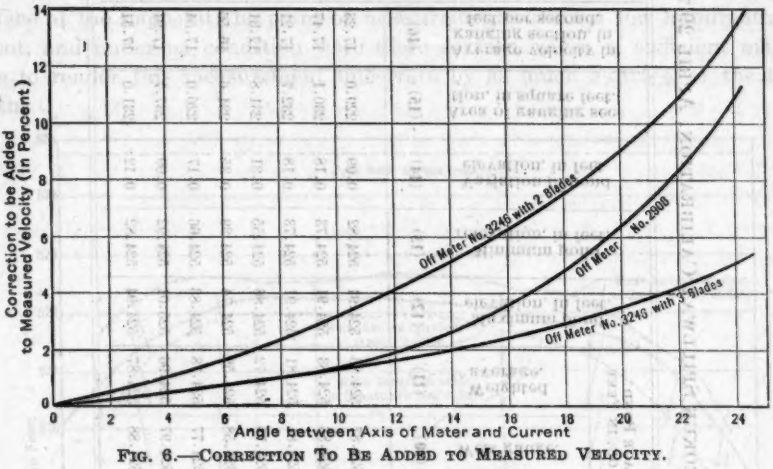



FIG. 6.—CORRECTION TO BE ADDED TO MEASURED VELOCITY.

Individual velocity measurements adequate in number and location to determine definitely the vertical variation in speed of current were taken in vertical lines located at the following distances, in feet, from the side of the spillway: 0.4, 1.0, 2.0, 3.5, 5.0, 7.0, 11.0, 15.0, 19.0, 23.0, 25.0, 26.5, 28.0, 29.0, and 29.6.

The meter revolutions ordinarily were measured at points 1 ft. apart in each vertical, except near the spillway crest, the water surface, and in the shallower verticals near the edge of the spillway, where observations were made at ½-ft. intervals. The period during which the meter revolutions were counted was generally between 30 and 60 sec.

The section in which the metering was done proved to be an ideal one in many respects. The water being accelerated as it passed over the crest, flowed with remarkable uniformity in smooth stream lines and was free from the characteristic pulsations in velocity and annoying vortices which so often accompany the phenomenon of flow in other channels. Many checks of observations were made at certain points after short lapses of time, with the invariable result of duplicating the previous readings exactly or within slight differences of not more than a fraction of a second. The variation in velocity from the water surface to the crest also was remarkably uniform. In many verticals, a long section of the velocity curve was almost a straight line. This insured the determination of this curve to a high degree of accuracy.

TABLE 1.—GENERAL SUMMARY OF RESULTS OF KEOKUK SPILLWAY CALIBRATION, APRIL 29 TO MAY 8, 1924.

| Test No. (1) | Date of test. (2) | Total duration of test, in hours and minutes. (3) | No. of Rate tested. (4) | Average elevation of spillway crest, in feet. (5) | Nos. of adjacent Gates open. (6) | Meter used on Stations 2 to 28. (7) | Meter used on Stations 0, 1, 29, and 29.6. (8) | Average Pond Elevation, in Feet. East gauge. (9) | Average Pond Elevation, in Feet. West gauge. (10) | Average Pond Elevation, in Feet. Weighted average. (11) | Maximum pond elevation, in feet. (12) | Minimum pond elevation, in feet. (13) | Variation in pond elevation, in feet. (14) | Area of gauging section, in square feet. (15) | Average velocity in gauging section, in feet per second. (16) | Measured discharge, in cubic feet per second. (17) | Head on crest of spillway, in feet. (18) | Coefficient of discharge, $C$, in $Q = CLH^{\frac{3}{2}}$. (19) | Discharge for Pond Elevation 524.80, in cubic feet per second. (20) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | April 29.. | 7–52 | 68 | 513.97 | None | Small 2-blade | From Test 8 | 524.86 | 524.87 | 524.86 | 524.91 | 524.82 | 0.09 | 229.0 | 17.23 | 3 946 | 10.89 | 3.66 | 3 913 |
| 2 | April 30.. | 5–30 | 68 | 513.97 | None | Large 2-blade | From Test 8 | 524.82 | 524.83 | 524.83 | 524.91 | 524.73 | 0.18 | 230.1 | 17.47 | 4 020 | 10.86 | 3.74 | 4 004 |
| 2 | May 1.. | 9–16 | 68 | 513.97 | 69 | Large 2-blade | Small 2-blade | 524.82 | 524.81 | 524.81 | 524.91 | 524.73 | 0.18 | 232.7 | 17.21 | 4 006 | 10.84 | 3.74 | 4 001 |
| 3 | May 2.. | 4–18 | 68 | 513.97 | 67–69 | Small 2-blade | Large 2-blade guarded | 524.73 | 524.75 | 524.72 | 524.86 | 524.55 | 0.31 | 231.8 | 17.31 | 4 002 | 10.75 | 3.78 | 4 046 |
| 4 | May 3.. | 4–31 | 68 | 513.97 | 67–69 | Large 3-blade | From Test 4 | 524.56 | 524.58 | 524.59 | 524.74 | 524.39 | 0.35 | 234.6 | 16.56 | 3 918 | 10.62 | 3.78 | 4 041 |
| 5 | May 5.. | 4–18 | 68 | 513.97 | 69–70 71–72 | Small 2-blade guarded | Small 2-blade guarded | 524.76 | 524.77 | 524.78 | 524.83 | 524.66 | 0.17 | 226.0 | 17.54 | 3 964 | 10.81 | 3.72 | 3 974 |
| 6 | May 6.. | 4–40 | 68 | 513.97 | 65–66–67 69–70–71 | Small 2-blade guarded | Small 2-blade guarded | 524.95 | 524.97 | 524.96 | 525.01 | 524.92 | 0.09 | 237.7 | 17.94 | 4 264 | 10.99 | 3.90 | 4 172 |
| 7 | May 8.. | 3–39 | 103 | 513.89 | None | Small 2-blade guarded | Small 2-blade guarded | 524.87 | 524.88 | 524.87 | 524.94 | 524.82 | 0.12 | 231.0 | 17.51 | 4 044 | 10.98 | 3.71 | 4 066 |

*Nappe Cross-Section.*—The physical dimensions of the spillways were measured at the dam. The upper surface of the nappe at the section where the meter propeller was operating was determined at each vertical by taking a rod reading with the propeller at the water surface. As a check on these readings the surface was also measured with a steel tape, at sections under both the up-stream and the down-stream side of the gate before and after each discharge measurement. The three sections thus obtained during Test 8 are shown in Fig. 7, the metering section being about midway between the other two. At times of little wind there was no difficulty in determining the water surface of the nappe at the point of measurement within a few hundredths of a foot, and under no condition were there surface waves of sufficient magnitude to render this measurement uncertain by as much as 0.5% of the total depth.

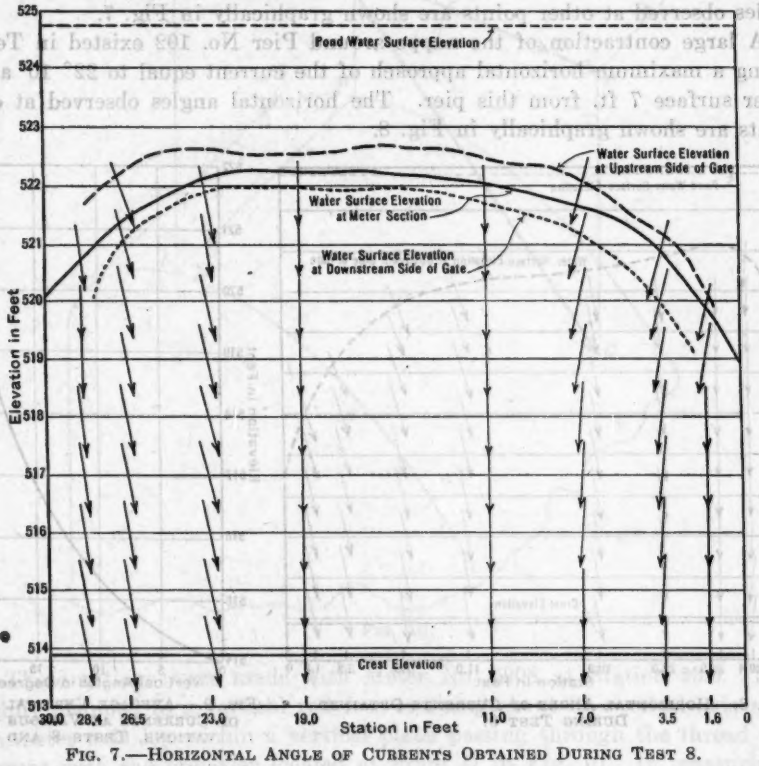



FIG. 7.—HORIZONTAL ANGLE OF CURRENTS OBTAINED DURING TEST 8.

*Pond-Water Level Measurements.*—As 3 to 9 hours were required to make a single measurement of discharge it was fortunate that the lake was large and little variation in pond level occurred during the tests. The maximum difference in pond level of 0.35 ft. occurred during Test 5, but the average maximum for the eight tests was only one-half this difference. (See Table 1.)

Elevations of the pond-water surface were recorded every 15 min. during the test by Gurley printing water-stage registers located on Piers No. 30 and

No. 90 and close to Pier No. 119. In addition, the water level at points near certain closed spillways between the two recording gauges were read with a tape and weight in order to detect the amount of depression in the pond-water surface approaching the open spillways under test.

*Obliquity of Currents in Nappe.*—Using the current direction meter, measurements of the vertical as well as the horizontal angle of the current were made in connection with Test 8 on Spillway No. 103 discharging alone. These readings were then repeated on this same spillway (as Test 9) with the nappe greatly contracted on one side, caused by the opening of Spillways Nos. 104 to 109 during the test. Readings were taken at most of the verticals used in the current-meter work and at intervals of 1 ft. in elevation.

In Test 8, the maximum horizontal angle of approach of the current (18° 30′) was observed at the surface 3.5 ft. from the pier. The horizontal angles observed at other points are shown graphically in Fig. 7.

A large contraction of the nappe around Pier No. 102 existed in Test 9, giving a maximum horizontal approach of the current equal to 22° 10′ at the water surface 7 ft. from this pier. The horizontal angles observed at other points are shown graphically in Fig. 8.

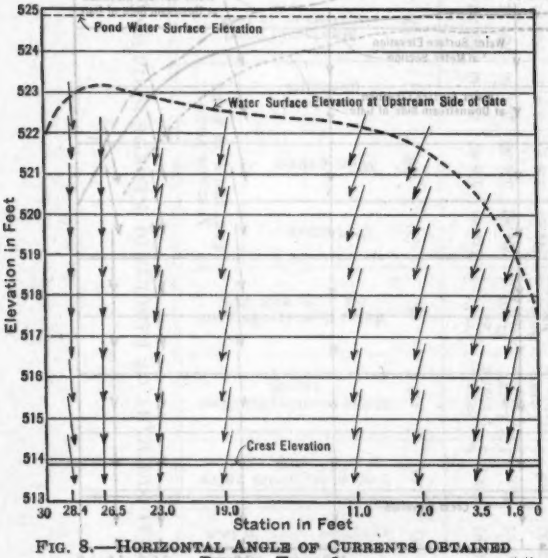



FIG. 8.—HORIZONTAL ANGLE OF CURRENTS OBTAINED DURING TEST 9.

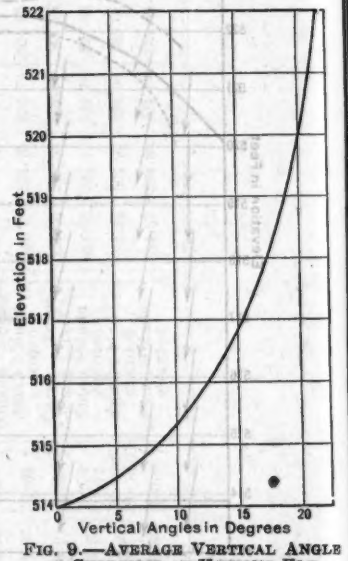



FIG. 9.—AVERAGE VERTICAL ANGLE OF CURRENTS AT VARIOUS ELEVATIONS, TESTS 8 AND 9.

The vertical angles at which the current approached the metering section were remarkably constant at a given elevation above the spillway crest. The average results from the observations in Tests 8 and 9 are shown in Fig. 9. Only twelve of the readings taken in Tests 8 and 9 gave values which were more than 3° from this curve, and in no case was there a departure of more than 5 degrees. This average curve was used as the basis for all computations involving the evaluation of the vertical angle of approach of the water toward the meter.

*Computation of Actual and Normal Current Velocities.*—Meters, held rigidly on a vertical rod that was free to swing with the current, registered only the component of the stream velocity in the horizontal plane passing through the meter. If the vertical angle of approach was significant, even the value of this component was under-registered by the Ott meter. The treatment of the measured current values in one of the fifteen verticals constituting Test 8, is illustrated by Fig. 10 and the data are shown in Table 2.

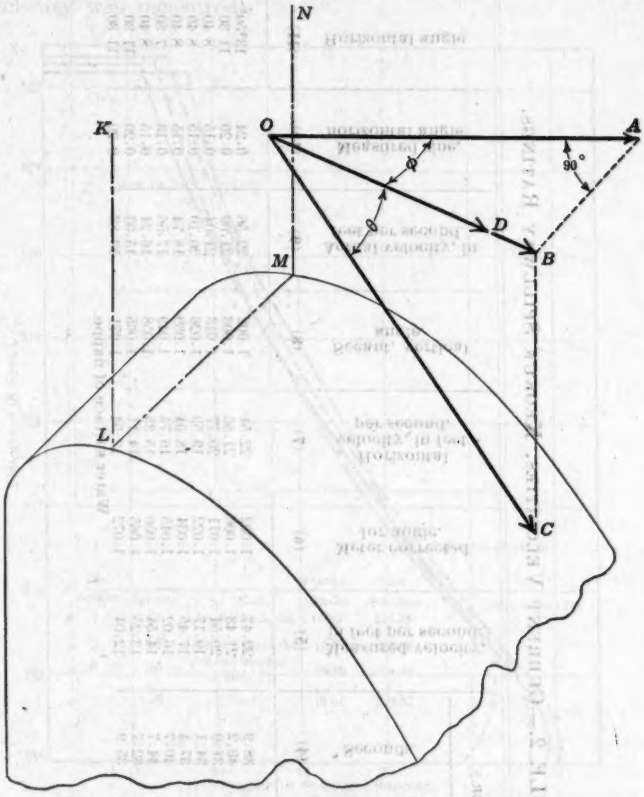



FIG. 10.

These ratings were made with Meter No. 2908 at Station 23.0. It was rigidly clamped to a vertical rod free to swing about its vertical axis so that the meter axis lay within a vertical plane passing through the thread of the current with the propeller located at Point *O* in Fig. 10. In measuring the velocity of the current passing in the direction, *O C*, the meter should have registered its component, *O B*, projected vertically into the horizontal plane passing through the axis of the meter. The ratings showed that as a matter of fact the Ott meters sometimes under-registered this component, obtaining the value, *O D*, which is less than *O B*. It was necessary, therefore, to correct some measured velocities by applying the small under-registration factors shown in Fig. 6. Thus, Column (7) in Table 2 is the product of Columns (5)

TABLE 2.—CURRENT VELOCITIES, KEOKUK SPILLWAY RATINGS.

| Elevation of meter, in feet. (1) | Meter | | Measured velocity, in feet per second. (5) | Meter corrected for angle. (6) | Horizontal velocity, in feet per second. (7) | Secant, vertical angle. (8) | Actual velocity, in feet per second. (9) | Measured sine, horizontal angle. (10) | Horizontal angle. (11) | Cosine, horizontal angle. (12) | Normal velocity, in feet per second. (13) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Vertical angle. (2) | Revolutions. (3) | Seconds. (4) | | | | | | | | |
| 514.28 | 3°3′ | 1 000 | 38.9 | 1.002 | 22.51 | 1.002 | 22.56 | 0.24 | 13°5′ | 0.971 | 21.86 |
| 514.67 | 6 20 | 1 000 | 40.2 | 1.006 | 21.56 | 1.006 | 21.69 | 0.20 | 11 30 | 0.980 | 21.13 |
| 515.17 | 9 00 | 750 | 31.9 | 1.011 | 20.77 | 1.013 | 21.04 | 0.15 | 8 40 | 0.989 | 20.54 |
| 516.17 | 13 00 | 750 | 34.1 | 1.022 | 19.59 | 1.026 | 20.10 | 0.15 | 8 40 | 0.989 | 19.87 |
| 517.17 | 15 40 | 750 | 37.4 | 1.034 | 18.04 | 1.039 | 18.74 | 0.15 | 8 40 | 0.989 | 17.84 |
| 518.17 | 17 30 | 750 | 40.7 | 1.045 | 16.74 | 1.049 | 17.56 | 0.13 | 7 30 | 0.991 | 16.69 |
| 519.17 | 19 00 | 750 | 44.7 | 1.056 | 15.85 | 1.058 | 16.24 | 0.15 | 8 40 | 0.989 | 15.18 |
| 520.17 | 20 10 | 500 | 32.7 | 1.065 | 14.11 | 1.065 | 15.03 | 0.20 | 11 30 | 0.980 | 13.33 |
| 521.17 | 21 00 | 500 | 35.9 | 1.072 | 12.91 | 1.071 | 13.83 | 0.20 | 11 30 | 0.980 | 12.65 |
| 521.77 | Water surface of nappe. | | | | | | | | | | |

and (6). The actual value of the current velocity, $OC$, was obtained by multiplying the horizontal component, $OB$, by the secant of the vertical angle, $\theta$. The average value of this angle was read from Fig. 9 and was tabulated in Column (2). The component of the stream velocity which was perpendicular to the gauging section is $OA$ (Column (13)), and was obtained by multiplying the horizontal velocity, $OB$ (Column (7)), by the cosine of the horizontal angle, $\phi$, the sine of which was observed at every point at which the velocity was measured.

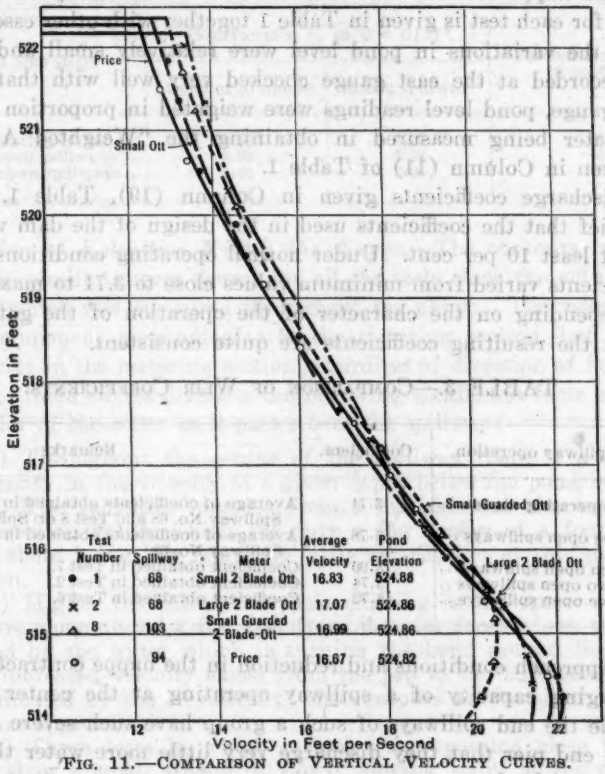



FIG. 11.—COMPARISON OF VERTICAL VELOCITY CURVES.

*Computation of Spillway Discharge.*—Table 2 is a sample showing the results obtained from the meter measurements in a vertical line 23 ft. from Pier No. 103 in Spillway No. 103 during Test 8. The results of all observations in each test were computed in this manner, and from such tables, curves were plotted showing the distribution of velocities in the different vertical lines. Such curves are shown in Fig. 11. The average velocity in each vertical was obtained by applying Simpson's rule to twenty equally spaced points on each vertical velocity curve. The values of mean velocity thus found were remarkably uniform across the spillway section. The maximum variation obtained, including tests with the most contracted nappes, was only 17 per cent.

When symmetrical conditions of flow existed there was little variation in depth of the nappe and current velocities in the middle half of the spillway, but the pier contractions caused a rapid decrease in depth at points closer to the face of a pier. It is chiefly for this reason that the distance between the vertical lines of measurement was decreased near the edge of the spillway.

The discharge of the spillway was computed from the data on average velocity and nappe depth in the manner commonly used in stream measurement; that is, by computing the average velocity, depth, area, and quantity of flow, in the nappe sections between verticals. A summary of the computed discharge for each test is given in Table 1 together with other essential data.

While the variations in pond level were relatively small and the average of that recorded at the east gauge checked very well with that recorded at the west gauge, pond level readings were weighted in proportion to the quantity of water being measured in obtaining the "Weighted Average Pond Level" given in Column (11) of Table 1.

The discharge coefficients given in Column (19), Table 1, verified the earlier belief that the coefficients used in the design of the dam were conservative by at least 10 per cent. Under normal operating conditions, the correct weir coefficients varied from minimum values close to 3.71 to maximum values of 3.90, depending on the character of the operation of the gates. Table 3 shows that the resulting coefficients are quite consistent.

TABLE 3.—COMPARISON OF WEIR COEFFICIENTS.

| Method of spillway operation. | Coefficient. | Remarks |
|---|---|---|
| One spillway operating alone..... | 3.71 | Average of coefficients obtained in Tests 1 and 2 on Spillway No. 68 and Test 8 on Spillway No. 103. |
| Middle of three open spillways ... | 3.78 | Average of coefficients obtained in Tests 4 and 5 on Spillway No. 68. |
| Middle of seven open spillways... | 3.90 | Coefficient obtained in Test 7. |
| End gate of two open spillways .. | 3.74 | Coefficient obtained in Test 2. |
| End gate of five open spillways... | 3.72 | Coefficient obtained in Test 6. |

Better approach conditions and reduction in the nappe contraction increase the discharging capacity of a spillway operating at the center of a larger group, while the end spillways of such a group have such severe contractions around the end pier that they discharge very little more water than a single spillway operating alone.

*Contraction of the Nappe Around the Piers.*—The piers reduced the discharge of water to a greater extent than that by which the area of unobstructed spillway was reduced by the concrete in the piers themselves. The weir coefficients in Table 4 were computed from the discharge flowing in the 8-ft. section remote from pier influence at the center of the nappe, in those tests in which the nappe was symmetrical. The reduction in discharge caused by the lateral contraction of the nappe as it passes around the piers with only one spillway open is more than twice that observed with many spillways open.

The form of the water surface of the nappe at the metering section which was observed in Tests 2, 3, 5, and 6, is shown in Figs. 12 and 13. Note the unsymmetrical form of the nappe surface in Tests 3 and 6 performed on a

spillway at the end of a set of open spillways as compared with the more balanced surface curves in the other tests. There was less contraction in the nappe of Test 7 with many gates open than in Tests 2 and 8 on a single open spillway. The deeper pond near the power-house end of the dam seemed to produce a slight difference in the nappe contraction at the two sides of the spillway in Test 8.

TABLE 4.—VARIATIONS IN WEIR COEFFICIENTS.

| Condition of operation. | COEFFICIENT, $C$, IN $Q = CLH^{\frac{3}{2}}$. | | Percentage reduction in coefficient. |
|---|---|---|---|
| | At center of nappe. | Entire spillway. | |
| One spillway operating alone..... | 3.92 | 3.71 | 5.4 |
| Middle of three open spillways... | 3.92 | 3.78 | 3.6 |
| Middle of seven open spillways... | 4.00 | 3.90 | 2.5 |

*Distribution of Velocities Within the Nappe.*—The contours in Figs. 12 and 13 and in similar curves drawn for all the tests show the actual velocity of the water which was measured throughout the nappe. The velocity shown is not the component perpendicular to the metering section, but the actual current velocity in the metering section regardless of direction of flow.

A study of these curves discloses the following significant facts with regard to the behavior of the water as it passes over the spillway:

(1) Throughout the center of the spillway there is a remarkable uniformity in the velocity at a given depth below the pond level.

(2) The velocities increase from a minimum at the water surface to maximum values which are only a few tenths of a foot above the crest, there being but relatively little retardation at this point due to friction.

(3) Higher velocities than those existing in the center of the spillway are encountered a few feet from the pier faces where the nappe is formed by the water which is turning the bend around the pier nose. This increased velocity exists from the top to the bottom of the nappe as indicated by the elevation of the contours as the sides of the spillway are approached.

(4) Those tests in which meters were operated close to the pier faces show a small stream of slowly moving water at the crest in the outside corners, indicating that the water followed with difficulty the contour of the spillway at these places.

In making comparisons between the several tests and various parts of the same spillway, cognizance should be taken of the fact that slight differences in the pond level which existed at the time of measurement in a given section may be responsible for small variations in contour position.

It is scarcely possible, from these figures based on slightly varying conditions, to note the significant facts disclosed in Fig. 14. In plotting this diagram the velocity readings taken at the middle of the spillway have been reduced to corresponding values such as would have existed with a uniform pond level at Elevation 524.80. A progressive increase in the number of open spillways seems to be accompanied by a progressive increase in the velocity

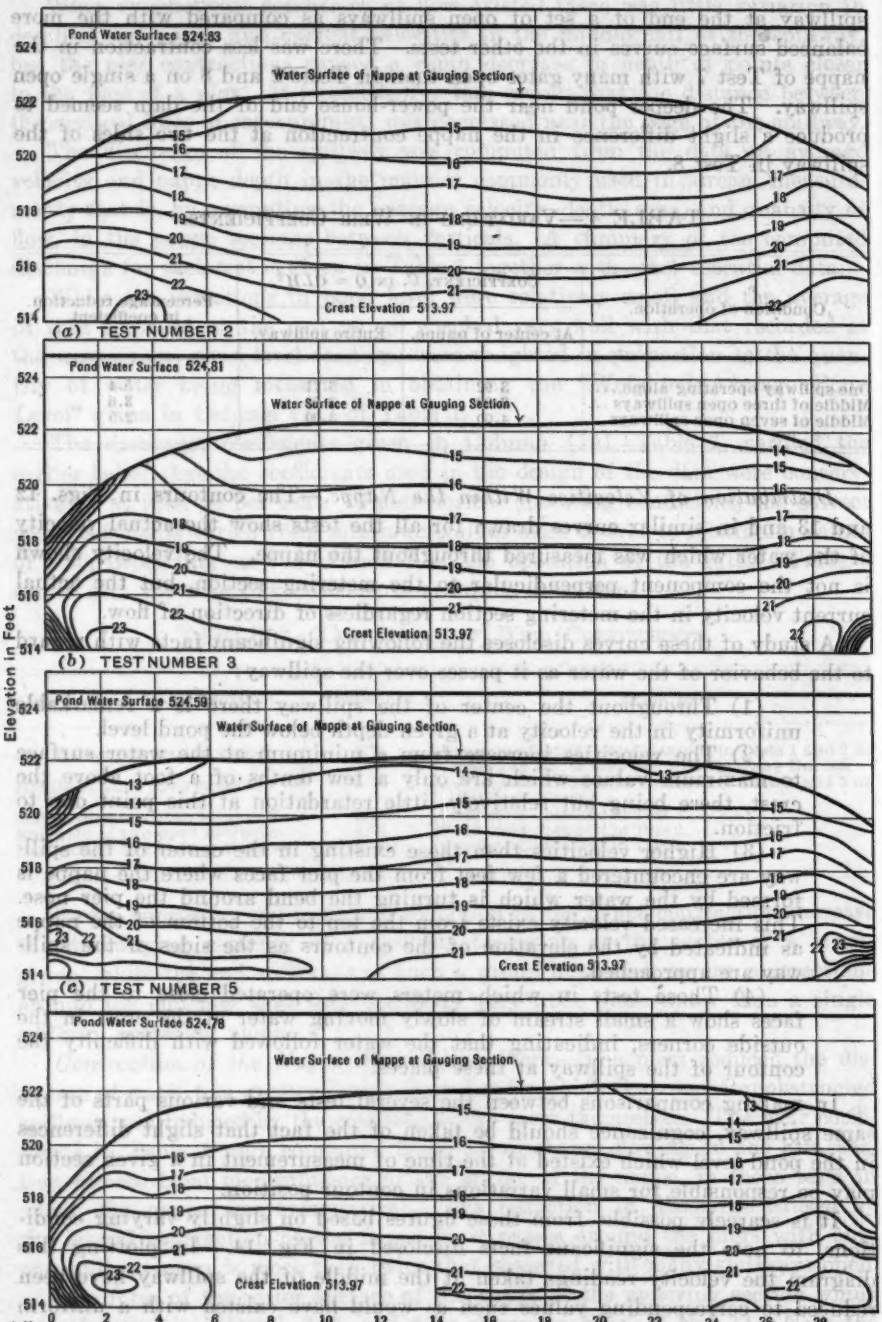

Pond Water Surface 524.83
Water Surface of Nappe at Gauging Section
Crest Elevation 513.97
(a) TEST NUMBER 2
Pond Water Surface 524.81
Water Surface of Nappe at Gauging Section
Crest Elevation 513.97
(b) TEST NUMBER 3
Pond Water Surface 524.59
Water Surface of Nappe at Gauging Section
Crest Elevation 513.97
(c) TEST NUMBER 5
Pond Water Surface 524.78
Water Surface of Nappe at Gauging Section
Crest Elevation 513.97
(d) TEST NUMBER 6
Elevation in Feet
Distance along Spillway Crest in Feet

FIG. 12.—ACTUAL VELOCITY CONTOURS, IN FEET PER SECOND.

and a depression of the nappe surface at the point of measurement. Thus, the larger spillway discharge coefficients with the greater number of open spillways may not be due entirely to the decrease in contraction of the nappe around the piers, but may also be produced by the higher velocities within the nappe itself when water is being drawn from a larger area in the pond, acquiring some initial velocity before reaching the spillway. This increase in velocity must be accompanied by a corresponding decrease in the pressure within the nappe. Considering the fact that the axis representing zero velocity is some distance to the left of the diagram, the difference in velocities is really of small consequence, hardly exceeding 5 per cent.

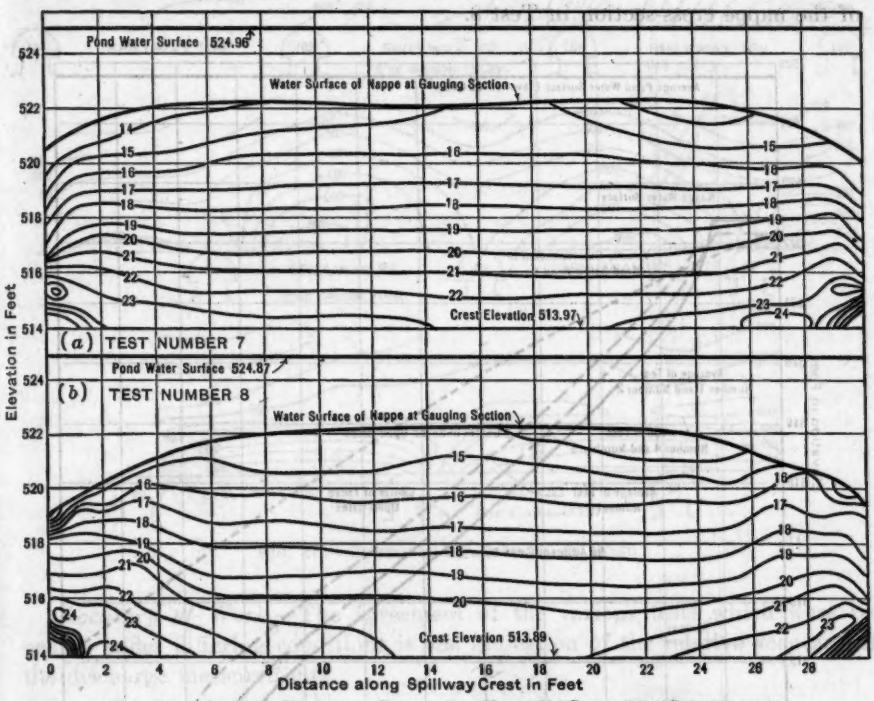

Pond Water Surface 524.96
Water Surface of Nappe at Gauging Section
Crest Elevation 513.97
(a) TEST NUMBER 7
Pond Water Surface 524.87
(b) TEST NUMBER 8
Water Surface of Nappe at Gauging Section
Crest Elevation 513.89
Elevation in Feet
Distance along Spillway Crest in Feet

FIG. 13.—ACTUAL VELOCITY CONTOURS, KEOKUK SPILLWAY CALIBRATION.

The curves of Fig. 14 indicate that the velocity at the surface of the nappe may exceed that acquired by the water in falling from the pond level to this point. This excess, although small indeed, was observed in many instances and may be created by the friction of the lower filaments of the nappe upon the slower moving filaments at the surface. It is possible, however, that this phenomenon may have been caused by an initial wind surface current, or a tendency on the part of the meter to over-register near the water surface, although observations generally were not made within 6 in. of the surface.

The higher nappe velocities near the piers result from lower pressures in the interior of the nappe, but even at these places the maximum nappe velocities did not attain the limit which would exist if the interior nappe pressure were reduced to zero.

*Condition of Stream Bed Up Stream from Spillways.*—Soundings were made to determine the elevation of the bed of the river within a distance of 25 ft. from the spillways upon which tests were made. These soundings were made with a steel tape from the platform of the traveling crane. Contours of the bed of the pond are shown in Fig. 15 and profiles at certain sections are shown in connection with Fig. 3. There is evidence of sedimentation on top of the irregular deposit of construction materials ahead of most of the spillways, although all this material was at least 7 ft. below the crest of the spillways under investigation. The nature of the stream bed ahead of Spillway No. 103 was obviously responsible for the slightly unbalanced form of the nappe cross-section in Test 8.

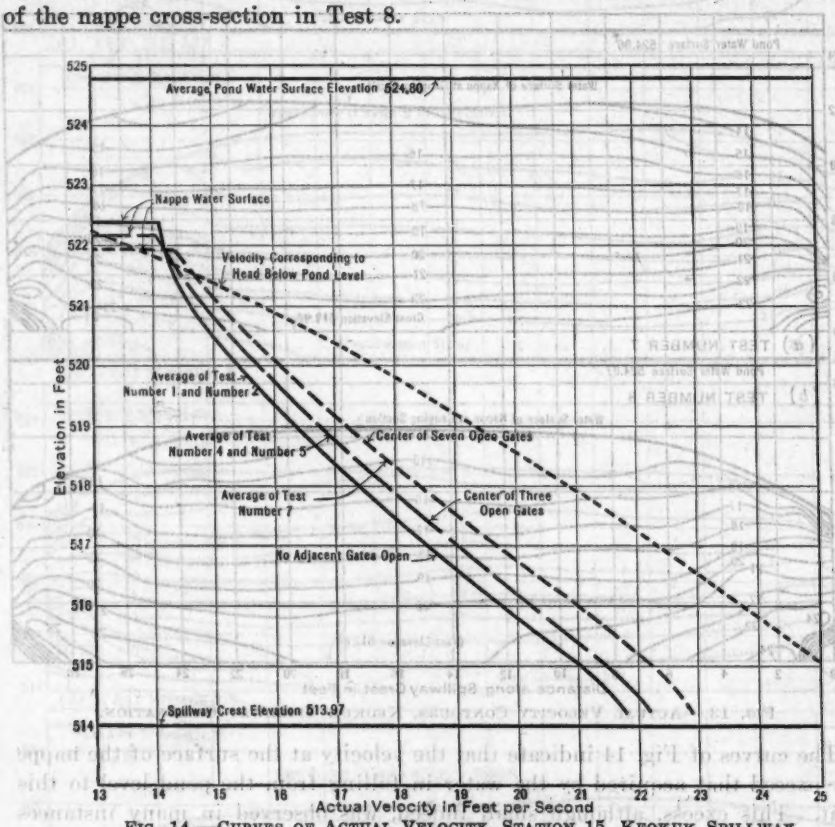



FIG. 14.—CURVES OF ACTUAL VELOCITY, STATION 15, KEOKUK SPILLWAY CALIBRATION.

*Surface Drop Near Open Spillway.*—Water surface elevations were taken in the immediate vicinity of Spillway No. 94 when discharging alone with no closely adjacent spillways open, in order to show accurately the shape of the depression of the water surface within 25 ft. of an open spillway. The result is shown by the contours in Fig. 16. It is interesting to note the change from the convex to concave contour as the nappe passes over the spillway. This work was done when the lake was very calm. A tape with a heavy sounding

weight attached at the zero end was lowered from the platform of the traveling crane and readings of the tape were taken with a level set up on the dam. Observations which were correct within 0.02 ft. were probably obtained in this manner.

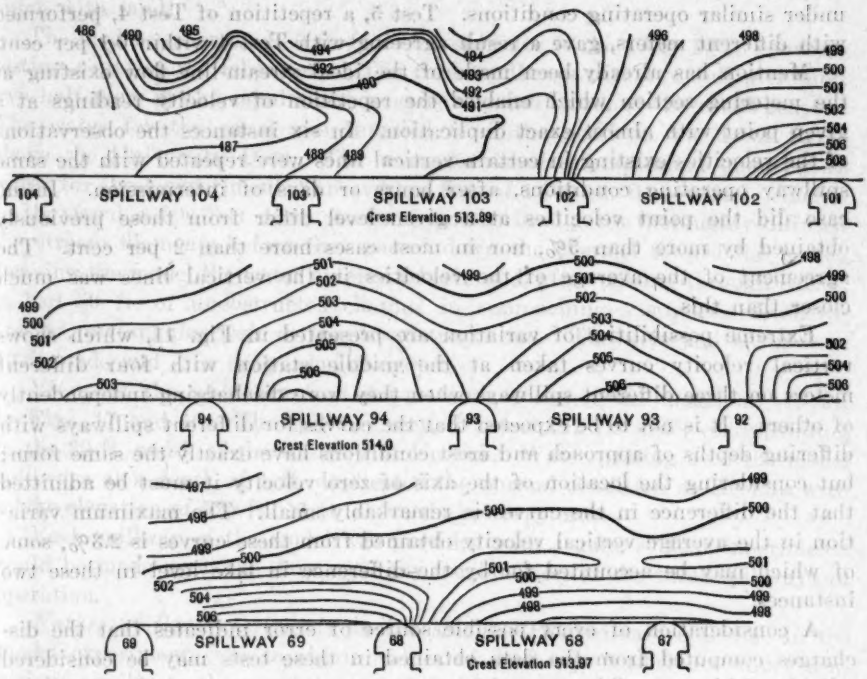



FIG. 15.—CONTOURS OF BOTTOM OF POND.

*Accuracy of Work.*—The agreement of the various tests which were repeated under differing conditions is one indication of the relative accuracy of the discharge measurements.

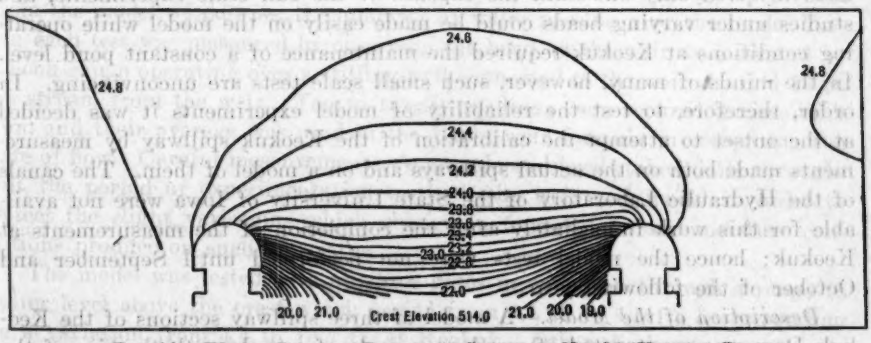



FIG. 16.—WATER SURFACE CONTOURS, SPILLWAY NO. 94, KEOKUK DAM.

Although Test 1 was regarded from the outset as a preliminary run, resulting in the more or less awkward and experimental handling of the equipment, its result departs 2.1% from Test 2 performed on the same gate with the same meter, and under the same conditions. Test 3 differs only 0.8% from Test 2, although it was performed on a different spillway with a different meter, but under similar operating conditions. Test 5, a repetition of Test 4, performed with different meters, gave a result agreeing with Test 4 within 0.1 per cent.

Mention has already been made of the ideal stream-line flow existing at the metering section which enabled the repetition of velocity readings at a given point with almost exact duplication. In six instances the observations of the velocities existing in certain vertical lines were repeated with the same spillway operating conditions, after hours or days of intermission. In no case did the point velocities at a given level differ from those previously obtained by more than 5%, nor in most cases more than 2 per cent. The agreement of the average of the velocities in the vertical lines was much closer than this.

Extreme possibilities of variation are presented in Fig. 11, which shows vertical velocity curves taken at the middle station with four different meters on three different spillways when they were discharging independently of others. It is not to be expected that the curves for different spillways with differing depths of approach and crest conditions have exactly the same form; but considering the location of the axis of zero velocity it must be admitted that the difference in the curves is remarkably small. The maximum variation in the average vertical velocity obtained from these curves is 2.3%, some of which may be accounted for by the difference in lake level in these two instances.

A consideration of every possible source of error indicates that the discharges computed from the data obtained in these tests may be considered reliable within a possible error of 2 per cent.

## Experiments on a Model of the Keokuk Spillways

The possibility of determining the proper discharge coefficients for the Keokuk spillways by means of small-scale experiments on a model of the dam was suggested before the field experiments at Keokuk were started. The model tests involved only one-sixth the expense of the full scale experiments, and studies under varying heads could be made easily on the model while operating conditions at Keokuk required the maintenance of a constant pond level. In the minds of many, however, such small scale tests are unconvincing. In order, therefore, to test the reliability of model experiments it was decided at the outset to attempt the calibration of the Keokuk spillway by measurements made both on the actual spillways and on a model of them. The canals of the Hydraulic Laboratory of the State University of Iowa were not available for this work immediately after the completion of the measurements at Keokuk; hence the model tests were not performed until September and October of the following year.

*Description of the Model.*—A model of three spillway sections of the Keokuk Dam was constructed of wood on a scale of one-eleventh the size of the

actual dam, a section of which is shown in Fig. 3. Great care was taken in duplicating exactly to scale the form of the crest, and the shape and spacing of the piers. The wood which was chiefly redwood, was planed and sandpapered to a smooth finish for use in the early experiments, although in later tests it was given, successively, a smoother finish with shellac and a rougher finish with sand.

The model was erected 80 ft. from the entrance to the 10 by 10-ft. concrete testing canal of the laboratory. The three spillways with two full piers and two half piers at the canal sides occupied the full width of the canal exactly. The water for the canal was supplied from the pond on the Iowa River above the University Dam. The river furnished an unlimited quantity of water for these experiments (an average flow of 1 500 cu. ft. per sec.). Baffles in the canal a few feet down stream from the head-gates thoroughly diffused the stream filaments before they passed down the channel toward the model. Leaving the model the water entered a basin and, after passing through baffles, it had 50 ft. of unobstructed channel in approaching a suppressed, sharp-crested, rectangular weir at the down-stream end. The weir and basin were 10 ft. wide and 7 ft. deep below the crest.

Views of the up-stream and down-stream faces of the model are shown in Figs. 17 and 18. The spillway sections are 2.727 ft. wide, corresponding to the 30-ft. width of the spillway at Keokuk. Water passing over the model spillways at a depth of 1 ft. duplicated the normal condition at Keokuk with a lake elevation of 525 ft., 11 ft. above the spillway crest.

Each spillway of the model was provided with a gate, so that experiments could be made upon the dam with any combination of the three spillways in operation.

*Nature of the Experiments on the Model.*—The primary objective in the model experiments was to obtain values of discharge coefficients under conditions similar to those which existed during the Keokuk measurements and, by varying the water level above the model, to determine values for these coefficients under abnormal pond-level conditions, such as existed during the early years of operation at Keokuk.

During each experiment the water level above the model was carefully observed by taking ten to twenty readings on a hook-gauge set so as to read the level in a stilling-well connected to the canal about 10 ft. up stream from the model. Likewise, the quantity of water passing over the model during each test was measured by observing the head on the weir as indicated by a hook-gauge operating over a stilling-well connected to the weir channel 16 ft. up stream from the weir. Ten to twenty readings were secured during each run, and their average was used in the Bazin weir formula in computing the rate of flow. Careful measurements were made and levels taken daily throughout the period of experimentation on both the weir and model in order to detect the slight variations which change in temperature and moisture conditions produce on such apparatus.

The model was tested with all three gates in operation through a range in water level above the crest which varied from 0.4 ft. to 1.5 ft., corresponding to a variation from 4.4 to 16.5 ft. at Keokuk. Other discharge experiments

were conducted when only single gates were opened with the spillway discharging under a head close to the normal head of 1 ft.

For these tests the model operated under the following conditions:

1.—Planed wood surface on model duplicating Fig. 2 with no deposit of silt against the dam.

2.—Planed wood surface on model with a deposit of sand against the dam duplicating the field conditions ahead of Spillway No. 68 as shown by the contours in Fig. 15.

3.—Shellac finish on the model with the deposit ahead of the dam.

4.—Sand finish on model with the deposit ahead of the dam.

In addition to the four series of discharge experiments, six other tests were made to determine the velocity distribution within the nappe when the model was being operated under conditions similar to those existing during the Keokuk measurements. A small Ott current meter and a Pitot tube were used for this purpose.

In order to compare the shape of the nappe flowing over the model spillway with that passing over the Keokuk Dam, during eight of the tests, measurements of the surface of the nappe were obtained across the section corresponding to that in which the meters were operated at Keokuk.

*Experiments on Effect of Condition of Model on Discharge.*—In building a model of the concrete spillways at Keokuk, the actual roughness of the surface probably should be scaled down according to some ratio as well as the dam itself. Since the surface of the concrete in the full-sized dam appeared to be remarkably regular for this class of work, it was thought that the surface of the model should be at least as smooth as wood could be planed. Accordingly, the first series of experiments were performed on the planed and sand-papered model as constructed in the canal with no surfacing of any kind. The resulting coefficients when the water was passed over all three spillways are shown by the points in Fig. 19 which define Curve 1.

A deposit of sand was next placed above the dam in an attempt to duplicate that existing above the Keokuk Spillways Nos. 67, 68, and 69, where most of the field measurements were made. Small stakes were cut to proper length and fastened at points corresponding to those places where the soundings had been taken for the preparation of Fig. 15. Wet sand was then packed against the dam and filled in between the stakes forming the most natural surface between the tops of the sticks, leaving them barely exposed. The sand was further compacted by inundation in passing water over the dam, and its surface remodeled until it was firm enough so that but little alteration of its contour was caused by further inundation or experimentation. The results from this series of experiments are shown by points defining the position of Curve 2 in Fig. 19.

For all heads on the model above 0.63 ft. the deposit of silt seemed to have caused a lowering in the discharge of the three spillways, although at a head of 1.00 ft. corresponding to the normal head of 11 ft. at Keokuk there was less than 0.6% more water passing over the model without the silt deposit than passed over with it.

The small amount of alteration in the coefficient caused by the deposit that now exists ahead of the Keokuk Dam was also observed in the Keokuk

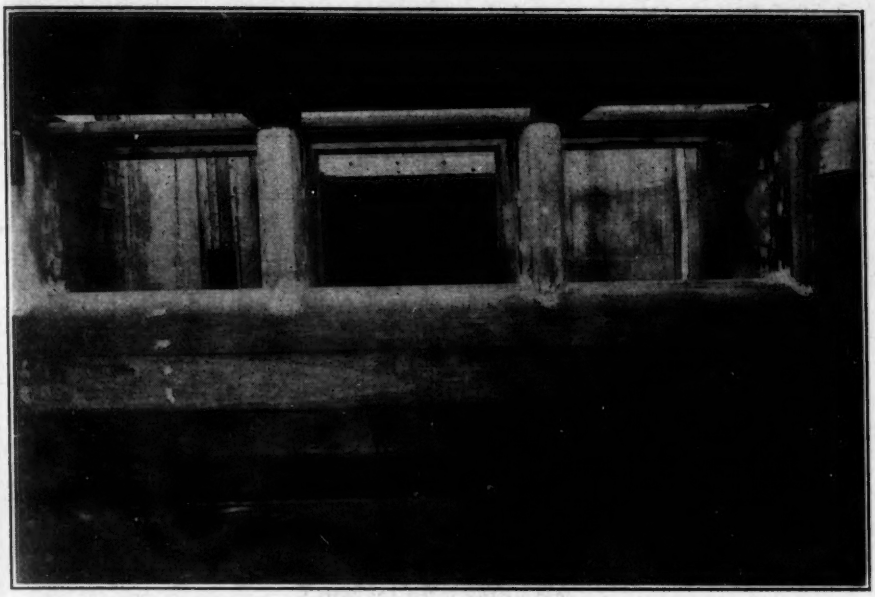


FIG. 17.—UP-STREAM FACE OF MODEL OF THREE SECTIONS OF THE KEOKUK SPILLWAY.

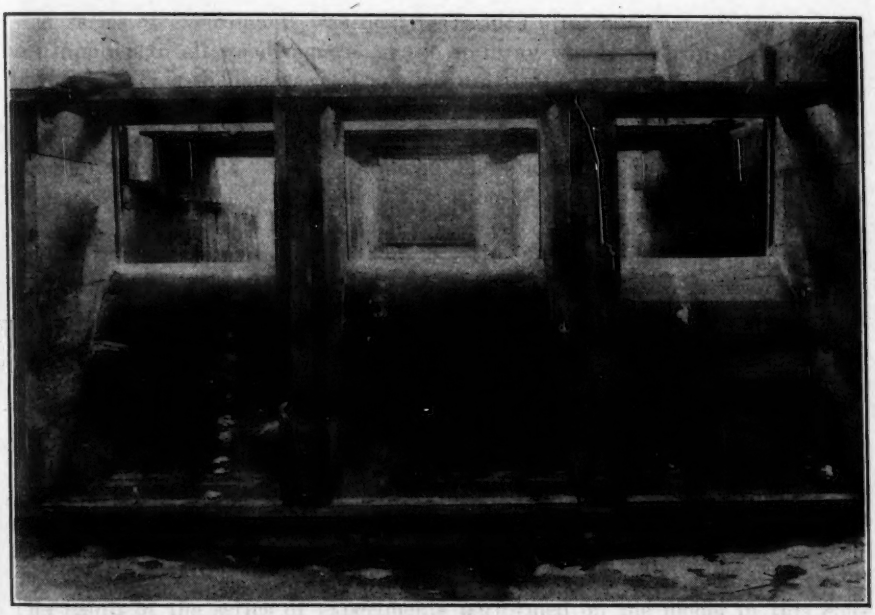
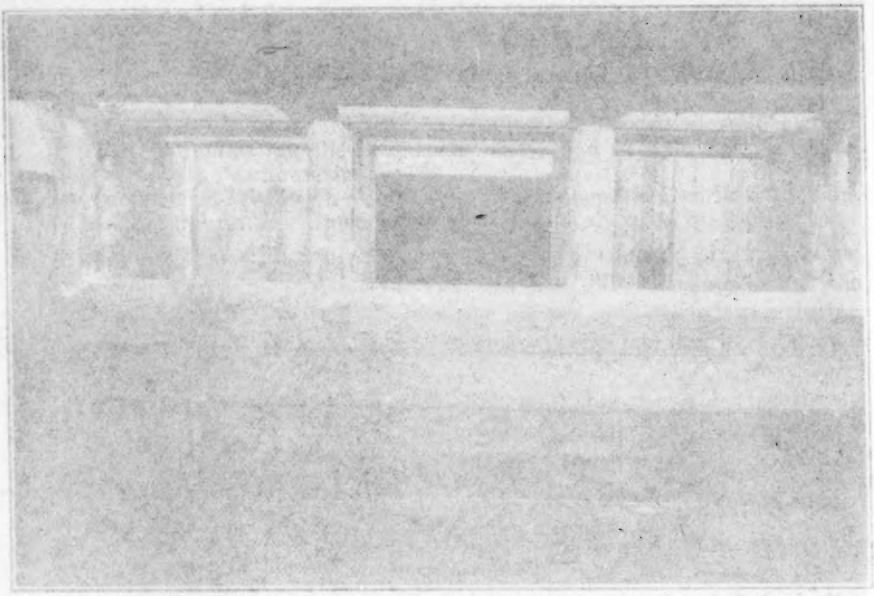
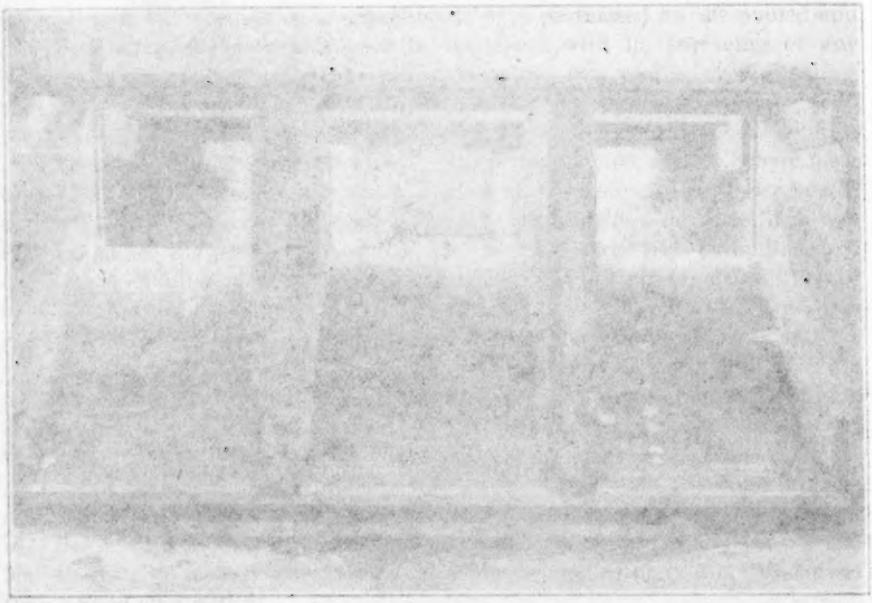


FIG. 18.—DOWN-STREAM FACE OF MODEL OF THREE SECTIONS OF THE KEOKUK SPILLWAY.





measurements, in which the results for Spillway No. 103 were in substantial agreement with the average of those obtained on Spillway No. 68, although the pond was 6 to 8 ft. shallower ahead of the latter spillway.

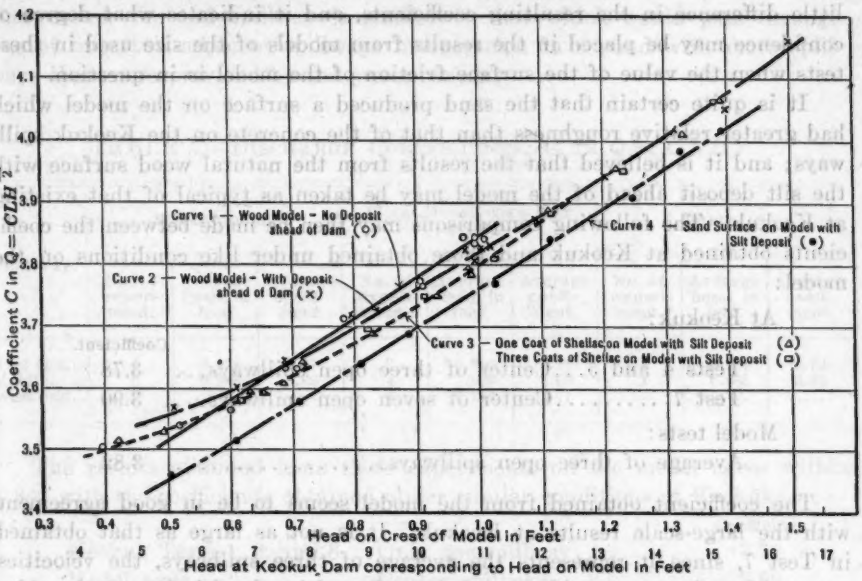

Coefficient C in $Q = CLH^{\frac{3}{2}}$
Curve 1 — Wood Model – No Deposit ahead of Dam (○)
Curve 2 — Wood Model – With Deposit ahead of Dam (×)
Curve 3 — One Coat of Shellac on Model with Silt Deposit (△)
Three Coats of Shellac on Model with Silt Deposit (□)
Curve 4 — Sand Surface on Model with Silt Deposit (●)
Head on Crest of Model in Feet
Head at Keokuk Dam corresponding to Head on Model in Feet

FIG. 19.—DISCHARGE COEFFICIENTS FOR MODEL OF KEOKUK SPILLWAY.

A series of experiments was next performed after one coat of shellac had been applied to all wooden parts of the spillway which came in contact with the water. This shellac was not applied until after the wood had thoroughly dried. Then, suspecting that one coat of shellac might not have produced the smoothest possible surface, two additional coats were applied, and another series of experiments were performed. There appeared to be no measurable difference in the results after the application of one or three coats of shellac, and the application of the shellac itself seemed to alter the discharge relatively little, as shown by Curve 3 in Fig. 19. When water was passing over the model with a head of 1 ft., the discharge seemed to be decreased about 0.4%, although at the greater heads the difference could not be detected. It is perhaps surprising that the smoother surface of the shellac caused a diminution of the discharge. However, this same effect of shellac has been noted in experimentation with other hydraulic apparatus, and it appears that water passes with less friction over a well-smoothed wooden surface (which soon becomes slippery under water) than over a shellac surface to which there may be some molecular adherence.

The shellac surface of the model was next treated with a fresh coat of shellac on which sand (passing a No. 28 Standard Tyler sieve but held on the No. 48 sieve) was thrown until the entire surface was thoroughly covered. The result of the series of experiments performed on the model in this condition is indicated by Curve 4 in Fig. 19. Except for the lower heads the increased friction of the sand surface caused a diminution of the discharge

amounting to approximately 1 per cent. However, when the head was small the difference was increased.

It was rather surprising that these various surface treatments produced so little difference in the resulting coefficients, and it indicates what degree of confidence may be placed in the results from models of the size used in these tests when the value of the surface friction of the model is in question.

It is quite certain that the sand produced a surface on the model which had greater relative roughness than that of the concrete on the Keokuk spillways; and it is believed that the results from the natural wood surface with the silt deposit ahead of the model may be taken as typical of that existing at Keokuk. The following comparisons may then be made between the coefficients obtained at Keokuk and those obtained under like conditions on the model:

At Keokuk:

| | | Coefficient. |
|---|---|---|
| Tests 4 and 5 | Center of three open spillways | 3.78 |
| Test 7 | Center of seven open spillways | 3.90 |

Model tests:

| | |
|---|---|
| Average of three open spillways | 3.82 |

The coefficient obtained from the model seems to be in good agreement with the large-scale results at Keokuk. It is not as large as that obtained in Test 7, since it represents the average of three spillways, the velocities approaching the two outside ones being affected by the friction on the sides of the testing canal. It is not as small as that obtained in Tests 4 and 5, since it probably represents the operation of a larger number of spillways, with all extraordinary contraction suppressed.

*Experiments on the Discharge of Only One Spillway.*—Since the discharge as computed over a sharp-crested weir has less certainty under low heads, and that low-head limit for these tests had been arbitrarily set at 0.4 ft., it was impossible by passing the water over the 10-ft. weir to measure accurately the discharge over a single spillway which was less than 3 ft. long when the pond level was less than 0.9 ft. above the crest of the model. Accordingly, most of the tests on the single spillways were performed under a head of about 1.0 ft. which duplicated the normal conditions existing at Keokuk. However, coefficients for the middle spillway of the three were obtained throughout a small variation in head. The discharge coefficients which were obtained are given in Table 5.

These results are in agreement with those obtained when three spillways were operated with regard to the relatively small effect which was produced by the deposit of silt ahead of the dam and the application of the shellac to the surface of the model.

The coefficients are lower than those obtained with the three spillways in operation, due to the extraordinary contraction existing around the piers when the single spillways draw their discharge from the entire width of the channel. The coefficients obtained from the two end spillways are identical as they should be with accurately constructed apparatus; and the discharge

through the center spillway is less than that through either of the end spillways due to the greater contraction of the nappe around both piers in this case. Neglecting the effect of friction along the sides of the canal, the discharge over the end spillway is similar to that which would pass through one of only two adjacent spillways in operation in a pond with more spillways than were provided by the present model.

TABLE 5.—DISCHARGE COEFFICIENTS, $C$, IN $Q = CLH^{\frac{3}{2}}$

| Spillway. | Wood Model. | | | Wood Model with Silt Deposit | | | Shellac. | | |
|---|---|---|---|---|---|---|---|---|---|
| | No. of experiment. | Average head, in feet. | Average coefficient. | No. of experiment. | Average head, in feet. | Average coefficient. | No. of experiment. | Average head, in feet. | Average coefficient. |
| East end... | 4 | 0.973 | 3.77 | 2 | 0.980 | 3.77 | 1 | 0.984 | 3.69 |
| Middle..... | 5 | 0.995 | 3.69 | 4 | 1.025 | 3.69 | 1 | 0.985 | 3.69 |
| West end.. | .. | ..... | ..... | 1 | 0.996 | 3.77 | .. | ..... | ..... |

The results obtained from these experiments on the model agree within 0.8% with the coefficients obtained under similar conditions at Keokuk:

| | | Coefficient. |
|---|---|---|
| Keokuk Tests 1, 2, and 8... | Only one spillway..... | 3.71 |
| Model tests..... | Center spillway..... | 3.69 |
| Keokuk Test 3..... | End of two open spillways... | 3.74 |
| Model test..... | End spillway..... | 3.77 |

*Variation in Coefficient with Change in Head.*—Fig. 19 shows considerable variation in the discharge coefficient with an increase or decrease in head from normal conditions. Experiments performed when water was discharging over only the middle spillway gave smaller coefficients increasing less rapidly as the head increases. The resulting curve (Fig. 20) seems to indicate that under low heads there is less difference in the coefficient when many spillways are in operation compared with that when a single spillway is open.

Coefficients which exceed 4.0 are not usually encountered, but it is evident that they may be secured when water discharges at great depths over well-designed spillways.*

That the coefficient would decrease rapidly for lower heads on the spillway was anticipated from previous measurements on the Keokuk spillway at low pond levels and from gaugings of the Mississippi River during certain floods when low lake levels existed. These other measurements (Table 6), while not comparable in accuracy with the work described in this paper, indicated that the discharge coefficient of the Keokuk spillway decreased to a value close to 3.0 when the nappe passed over the spillway at a shallow depth, duplicating the phenomena of flow over a broad-crested weir with a rounded approach. A comparison of the results from the model experiments with all measurements made at Keokuk is shown in Fig. 20.

* *Water Supply and Irrigation Paper No. 200*, Pl. XXXVI, U. S. Geological Survey; Report of Hydraulic Power Committee, National Elec. Light Assoc., 1923.

The model experiments are in good agreement with the 1924 measurements at Keokuk and are in as close agreement with the measurements listed in Table 6 as one could expect from the nature of these other determinations of the discharge coefficient.

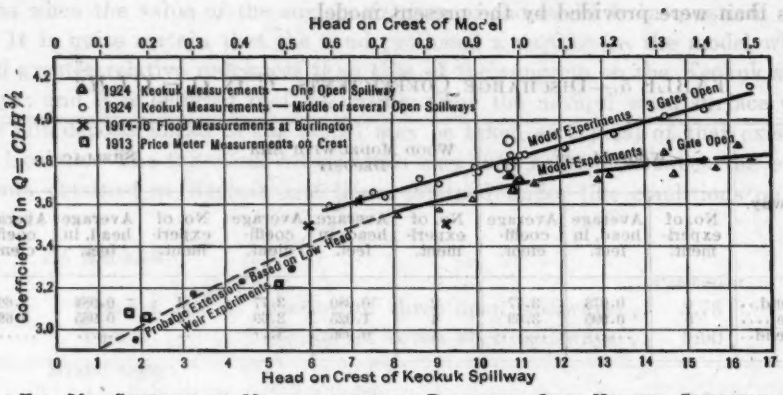

Head on Crest of Model
1924 Keokuk Measurements — One Open Spillway
1924 Keokuk Measurements — Middle of several Open Spillways
1914-15 Flood Measurements at Burlington
1913 Price Meter Measurements on Crest
Model Experiments
3 Gates Open
Model Experiments
1 Gate Open
Probable Extension Based on Low Head Weir Experiments
Coefficient C in Q = CLH 3/2
Head on Crest of Keokuk Spillway

FIG. 20.—SUMMARY OF MEASUREMENTS OF DISCHARGE OVER KEOKUK SPILLWAY.

*Measurements on the Form of the Nappe.*—In order to compare the form of the nappe passing over the model with that measured at the metering section at Keokuk, this exact section was located on the model and, by means of a pointed rod, about twenty readings to the water surface were taken to the nearest hundredth of a foot at varying intervals across the nappe.

TABLE 6.—MEASUREMENTS OF DISCHARGE OVER KEOKUK SPILLWAYS BEFORE 1924.

| Date. | Pond elevation, in feet. | Head, in feet. | Discharge per spillway, in cubic feet per second. | Coefficient in $Q = CLH^{\frac{3}{2}}$. | Remarks. |
|---|---|---|---|---|---|
| June 5, 1913.... | 515.63 | 1.71 | 206.3 | 3.06 | By R. H. Bolster with Price meter on crest. |
| June 6, 1913.... | 515.93 | 2.01 | 260.1 | 3.04 | By R. H. Bolster with Price meter on crest. |
| June 11, 1913... | 519.20 | 5.28 | 1 165.8 | 3.21 | By R. H. Bolster with Price meter on crest. |
| June 22, 1914... | 521.22 | 7.22 | 2 104 | 3.61 | Flood flow through 41.4 gates computed from current-meter measurement at Burlington, Iowa. |
| June 10, 1915... | 520.00 | 6.00 | 1 545 | 3.51 | Flood flow through 64 gates computed from current-meter measurement at Burlington, Iowa. |
| March 31, 1916. | 523.22 | 9.22 | 2 942 | 3.50 | Flood flow through 55.1 gates computed from current-meter measurement at Burlington, Iowa. |

The nappe surface was measured in this manner when the model with the planed wooden surface was operating with the sand deposit against the dam

and also without that deposit or sediment. Under each of these conditions, measurements were made on the nappe of the center spillway when operating alone, with all three spillways open, and with only one of the other spillways in operation. Readings were also taken to the surface of the nappe of one of the end spillways when operating alone.

A comparison of the results of the measurements on the model with the deposit of silt ahead of the dam and those taken on the Keokuk spillway under similar conditions is shown in Fig. 21. The model tests shown were conducted under the following circumstances: Test 45 was on Spillway No. 2, with Spillways Nos. 1 and 3 open; Test 46 on Spillway No. 2, with Spillway No. 1 open; and Test 47 on Spillway No. 2 and Test 48 on Spillway No. 1, with no other spillways open. The corresponding data for the field tests are given in Table 1. The depths of water in the nappe which were observed at Keokuk have been divided by the scale ratio of 11 in 1, in order that they might be plotted on these diagrams for comparison. In all cases the model seemed to represent quite faithfully the form of the larger nappe. In Fig. 21(*c*) the difference in average pond level between that existing at the time the model measurements were made and that existing during Test 5 at Keokuk, fully accounts for the separation of the two curves.

The four measurements of the nappe which were made when there was no sand deposit ahead of the dam gave water surface curves very similar to those shown in Fig. 21, except that the surface was lifted between 0.01 and 0.02 ft. uniformly across the entire spillway as if the deeper water ahead of the dam resulted in a very slight thickening of the nappe. A difference of this sort between Spillways Nos. 103 and 68 located in shallower water, was not noted in the tests at Keokuk.

In order to test the accuracy of using the middle third of the Keokuk spillways as an index of what flow would be obtained over an unobstructed crest, the piers were removed from the model spillway and an accurate determination was made of the cross-section of the nappe. Fig. 21 (*d*) gives an interesting comparison between the nappe cross-section at the location of the middle spillway of the unobstructed crest with that observed when one and three spillways were open and the piers were in place. The observations for Test 47 have been reduced to the same pond level of 1 ft. above the crest, which existed in the other two tests. It will be noted that the cross-section of the nappe in the center section of all the tests is practically identical, showing clearly that the flow in the center section of the spillway flanked by piers can be used for a reliable determination of the flow over an unobstructed crest. It is believed that this one experiment proves beyond a doubt the correctness of the analysis which resulted in the coefficient of 4.00 for the normal head on the unobstructed crest at Keokuk.

The amount of contraction of the nappe caused by the piers is very evident from Fig. 21 (*d*) and the fact that it decreases with the opening of adjacent spillways is quite significant. From this diagram it proved entirely possible to compute within 1% the value of the discharge coefficients previously obtained by flow measurements for the two cases—of a spillway discharging by itself and at the center of a group. The computation consisted

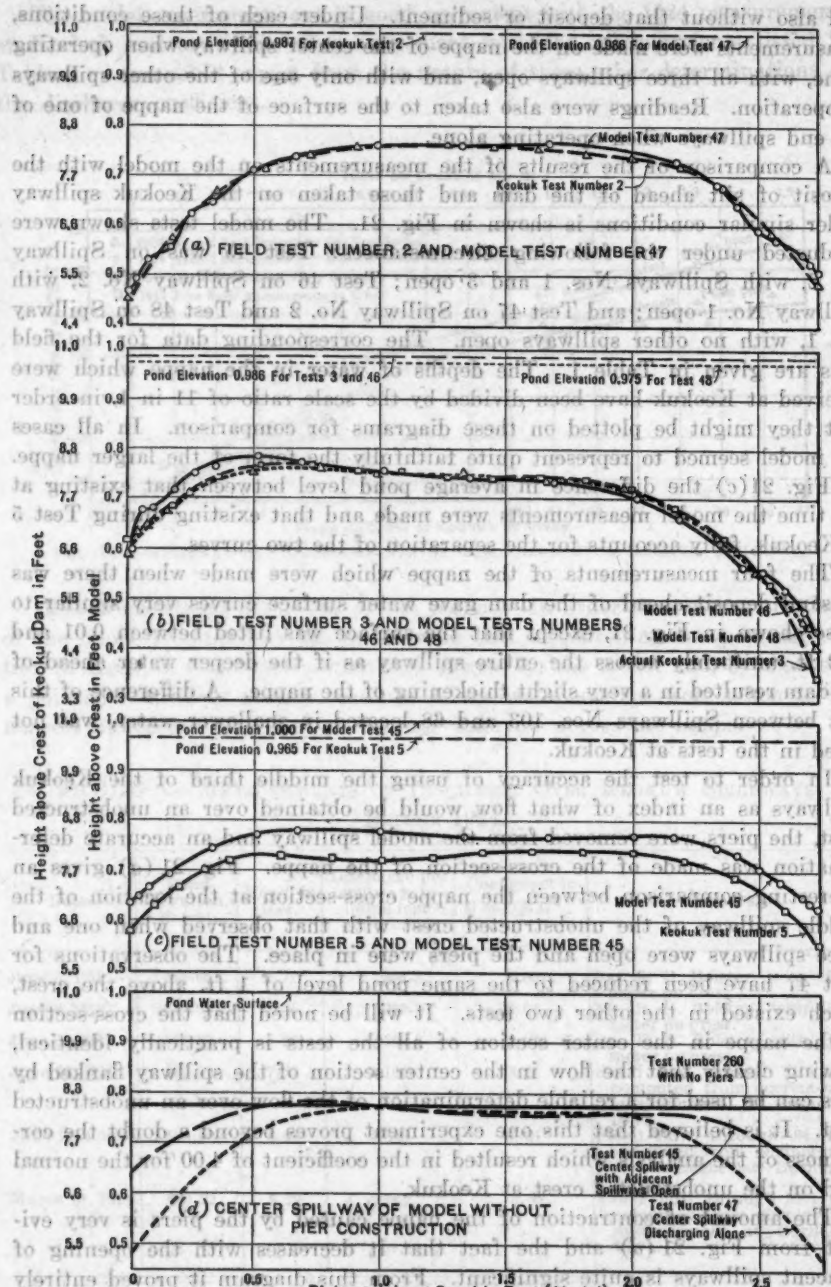

Pond Elevation 0.987 For Keokuk Test 2
Pond Elevation 0.986 For Model Test 47
Model Test Number 47
Keokuk Test Number 2
(a) FIELD TEST NUMBER 2 AND MODEL TEST NUMBER 47
Pond Elevation 0.986 For Tests 3 and 46
Pond Elevation 0.975 For Test 48
Model Test Number 46
Model Test Number 48
Actual Keokuk Test Number 3
(b) FIELD TEST NUMBER 3 AND MODEL TESTS NUMBERS 46 AND 48
Pond Elevation 1.000 For Model Test 45
Pond Elevation 0.965 For Keokuk Test 5
Model Test Number 45
Keokuk Test Number 5
(c) FIELD TEST NUMBER 5 AND MODEL TEST NUMBER 45
Pond Water Surface
Test Number 260 With No Piers
Test Number 45 Center Spillway with Adjacent Spillways Open
Test Number 47 Center Spillway Discharging Alone
(d) CENTER SPILLWAY OF MODEL WITHOUT PIER CONSTRUCTION
Height above Crest of Keokuk Dam in Feet
Height above Crest in Feet Model
Distance along Crest in Feet (Model)
Distance along Crest of Keokuk Spillway in Feet

FIG. 21.—COMPARISON OF NAPPE CROSS-SECTIONS.

of subtracting from the quantity of water passing over an unobstructed spillway that discharge which would have passed through the area above the actual nappe section and the level water surface of Test 260. The elements of the computation were as listed in Table 7.

TABLE 7.—VALUES USED IN COMPUTATIONS.

| Item. | No piers or obstructions. | Single spillway open. | Center of several open spillways. |
|---|---|---|---|
| Area of nappe, in square feet | 2.124 | 1.940 | 2.072 |
| Reduction by piers, in square feet | 0 | 0.184 | 0.072 |
| Percentage of reduction in area | 0 | 8.7 | 3.4 |
| Ratio of average velocity in reduced portion to average velocity in full nappe | .... | 0.78 | 0.76 |
| Reduction in discharge by piers, in percentage | 0 | 6.7 | 2.6 |
| Discharge coefficients in model experiments | 3.91 | 3.69 | 3.82 |
| Discharge coefficient computed from reduction of nappe by piers | 3.91 | 3.65 | 3.81 |

It is interesting to note the close agreement between the values of reduction in coefficient given in Table 7 and that computed for the Keokuk measurements in Table 3.

The form of the nappe at the crest of the spillway changed considerably when the head was raised above or lowered below the normal condition. The views in Figs. 22 and 23 with only the center spillway in operation show that the piers cause relatively little reduction of the nappe section when the head is low, but this increases to a very large amount when the head is 50% greater than normal. The increase in the magnitude of the pier obstruction with increasing head is probably responsible for the small increase in the discharge coefficient at high heads when a single gate is open as shown in Fig. 20.

*Measurements of Velocities in the Nappe Flowing Over the Model.*—A small Ott current meter (No. 2914) was first used in an attempt to measure the velocities existing in the center spillway of the model at points in the nappe which were in the section corresponding to that in which the meters were operated at Keokuk. Only two complete measurements throughout the nappe were made with this instrument before it was abandoned in favor of a small Pitot tube made from $\frac{1}{8}$-in. brass tubing. The meter had a propeller 0.18 ft. in diameter and when mounted on a $\frac{3}{4}$-in. rod, caused no apparent obstruction to the flow, since the metering section was some distance down the falling nappe. The diameter of the wheel of the meter, however, was almost one-fourth the depth of the nappe, and hence many points in a vertical section could not be taken with this instrument.

The Pitot tube had been previously manufactured and calibrated for use in measuring velocities in the nappe of a sharp-crested weir and proved a very excellent instrument for this kind of work. The use of a tube of this kind in the nappe requires watchfulness in order to prevent the entraining of air when operating near the water surface where the tube is likely to split the nappe and cause a crevice of air behind it. The behavior of the tube

when water approaches it at an oblique angle, as it does in the upper sections and sides of the nappe, is the main element of uncertainty involved in its use, but coefficients under these conditions may be obtained by calibration. There was a striking resemblance between the velocity distribution as observed in the model and that obtained at Keokuk under similar conditions, revealing all the characteristics previously mentioned.

In each test the total discharge of the spillway was calculated by treating these velocity measurements in the same manner as that by which the measurements at Keokuk were computed. This afforded a sort of check on the weir measurements. A summary of these results is given in Table 8.

TABLE 8.—SUMMARY OF DISCHARGE MEASUREMENTS ON CREST OF MODEL.

| Test No. (1) | Other spillways operating. (2) | Instrument used. (3) | Number of verticals. (4) | Number of points in vertical. (5) | Discharge by velocity reading. (6) | Discharge by weir. (7) | Percentage of difference. (8) |
|---|---|---|---|---|---|---|---|
| 31 | None | Meter | 8 | 6 | 9.67 | 9.74 | −0.9 |
| 68 | None | Meter | 8 | 7 | 10.13 | 9.93 | 2.0 |
| 69 | None | Pitot tube | 13 | 9 | 9.72 | 9.77 | −0.5 |
| 84 | All | " | 14 | 9 | 11.15 | 10.89 | 2.4 |
| 85 | No. 3 | " | 13 | 9 | 10.06 | 10.14 | −0.8 |

It is to be noted that in Column (7) of Table 8, Test 84 is the average of all three spillways which is probably less than that flowing over the center spillway measured by the Pitot tube. In Test 85 it is the average of the center and end spillways which is more than the quantity passing over the center spillway alone which was measured by the Pitot tube.

### EFFECT OF FORM OF PIERS ON DISCHARGE OVER SPILLWAY.

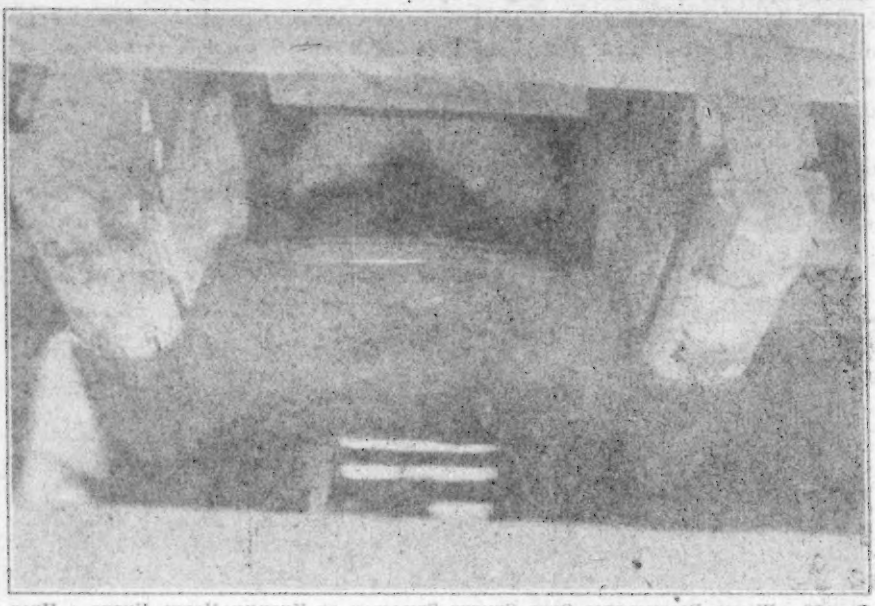
As a conclusion to the tests on the model the effect produced by three other pier forms was investigated in order to compare the behavior under these conditions with that for the model of the Keokuk spillway. These experiments were performed in November and December, 1925, and in May, 1926.

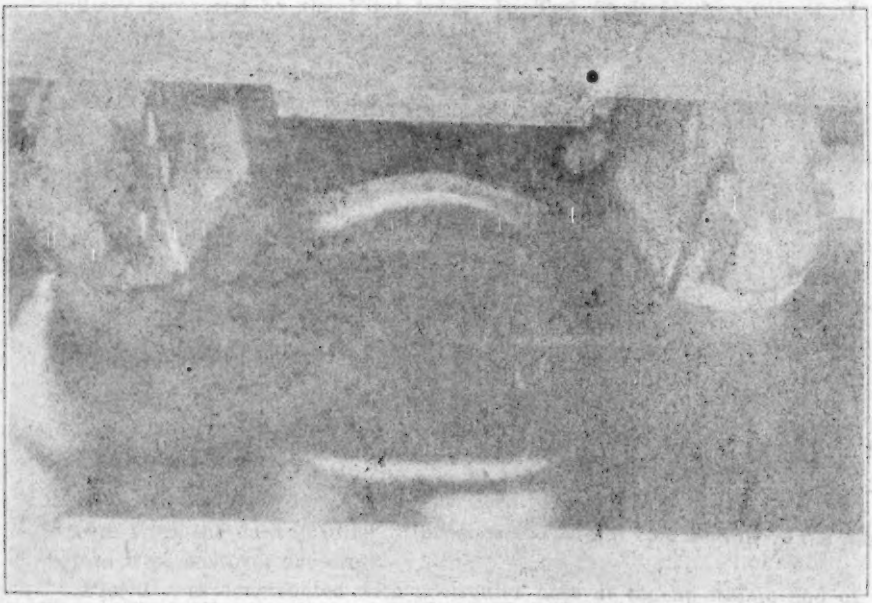
*Description of Pier Forms.*—The additional three types on which experiments were made are shown in Fig. 24 and are designated as Forms *B*, *C*, and *D*. Form *A* is the pier nose of the Keokuk model. The new piers were made of wood and were duplicated on the two full piers and two half piers of the model. Neither the form of the crest of the dam nor the silt deposit ahead of the dam was altered for these tests, but the sand finish which had been placed on the dam during the previous experiments with the Keokuk model was removed. Piers *B* and *C* were formed by extending the piers up stream from the dam in order to produce the lateral contraction of the nappe before it passed over the crest.

Pier *D* was constructed with a circular nose at the up-stream end of the Keokuk piers, but was formed with only one-half their radius. The further increase to full pier width was made gradually, thus producing a large part

FIG. 22.—WATER DISCHARGING OVER CENTER SPILLWAY OF KEOKUK MODEL UNDER A HEAD OF 0.6 FEET.

FIG. 23.—WATER DISCHARGING OVER CENTER SPILLWAY OF KEOKUK MODEL UNDER A HEAD OF 1.5 FEET.





of the lateral contraction of the nappe as it was falling over the crest of the dam. There was no provision for a gate slot in Pier *D*.

*Experiments and Their Results.*—The discharge over all three spillways and the several spillways operating singly was measured in the same manner as in the experiments on the Keokuk model. Finally, all the piers were removed and discharge measurements were made on the spillway, free from all obstructions, occupying the full width of the testing canal. The resulting coefficients when all three spillways were in operation are shown in Fig. 25 (*a*).

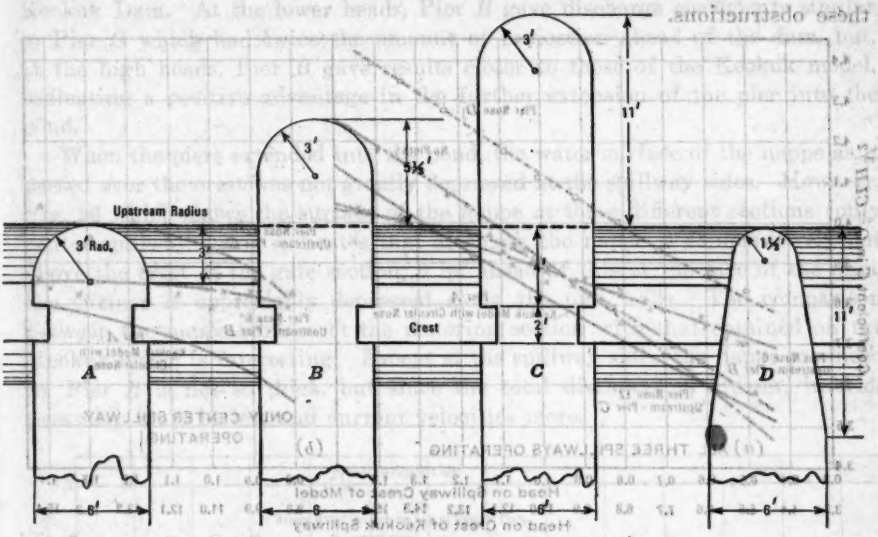



FIG. 24.—TYPES OF SPILLWAY PIER NOSES.

Pier *B*, the nose of which extended up stream from the face of the model dam 6 in. (or 5.5 ft. in the actual dam), gave higher coefficients than the Keokuk model when the head was less than 1.22 ft. In fact, it allowed a higher discharge through the spillway than Pier *C*, with a nose extending 12 in. up stream from the dam, for heads lower than 0.95 ft., which is close to the normal head at Keokuk. Because the coefficients obtained with Pier *B* seemed exceptionally large throughout the lower heads, the entire series of tests on this pier was repeated after a lapse of one week. The results confirmed those of the previous series in every respect. At heads between 0.95 ft. and 1.4 ft. Pier *C* allowed more water to pass than Pier *B*, but the maximum difference amounting to only 1%, is of little significance.

The *D* type of piers allowed considerably more water to pass than any of the other forms as shown by the coefficients in Fig. 25 (*a*), which were computed on the basis of the same net crest length between piers. About 8% more water could be passed over the Keokuk spillway provided the piers were shaped in this manner. It may not have been entirely consistent to compute the coefficients for Pier *D* on the basis of the crest length of the other models, since the flat part of the crest at its up-stream edge measured 2.953 ft. instead of 2.727 ft., which is the width between the piers after the water has passed

down the face of the dam. Had the coefficients for Pier *D* been computed on the basis of a crest length of 2.95 ft., they would have been very similar to those obtained with the Keokuk model throughout the entire range in head.

A series of experiments on the spillway with all piers removed gave coefficients which were higher than those obtained with Piers *A*, *B*, and *C* for all heads greater than 0.94 ft. It is interesting to note that below this head there is some indication that it is possible for a dam with piers to have a higher discharge coefficient per linear foot of open spillway than one without these obstructions.

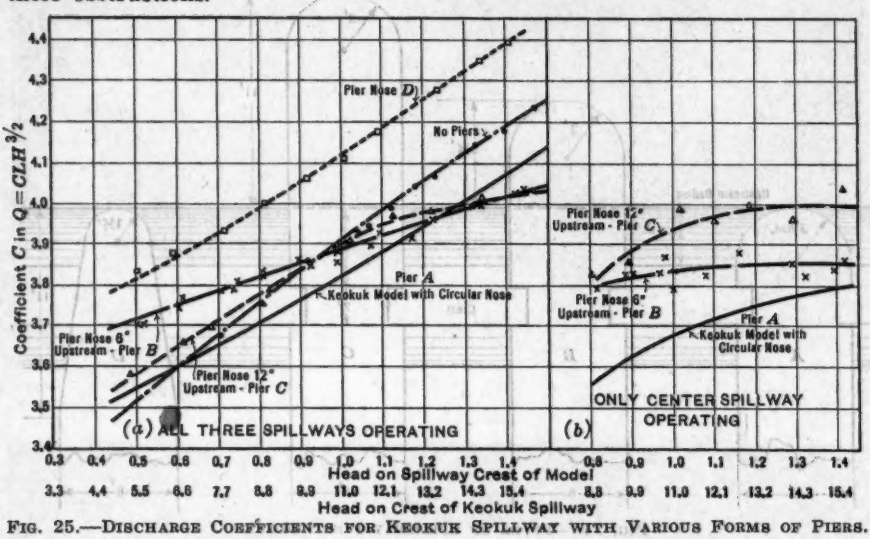

Coefficient C in Q = CLH^3/2
Pier Nose D
No Piers
Pier A
Keokuk Model with Circular Nose
Pier Nose 6" Upstream - Pier B
Pier Nose 12" Upstream - Pier C
(a) ALL THREE SPILLWAYS OPERATING
(b) ONLY CENTER SPILLWAY OPERATING
Head on Spillway Crest of Model
Head on Crest of Keokuk Spillway

FIG. 25.—DISCHARGE COEFFICIENTS FOR KEOKUK SPILLWAY WITH VARIOUS FORMS OF PIERS.

The Keokuk spillways are 30 ft. wide and the piers 6 ft. thick. The relative amount of obstruction caused by the different forms of piers may be expressed more clearly by tabulating the discharge over a dam of a certain gross length made up of these piers and spillways with that which would pass over a dam of the same gross length with no such obstructions (see Table 9).

TABLE 9.—DISCHARGE OVER DAM WITH PIERS EXPRESSED IN PERCENTAGE OF DISCHARGE OVER DAM OF SAME LENGTH WITH NO PIERS.

ALL SPILLWAYS IN OPERATION.

(Piers and spillways similar to Keokuk Dam assumed. Open spillway equals 0.83% of the gross length of the dam.)

| Type of pier. | Head = 5 ft. | Discharge in Percentage of That Through Unobstructed Spillway. | |
|---|---|---|---|
| | | Head = 11 ft.* | Head = 15 ft. |
| No piers | 100 | 100 | 100 |
| Keokuk Piers: | | | |
| Pier *A* | 84 | 81 | 81 |
| Pier *B* | 88 | 83 | 80 |
| Pier *C* | 83 | 83 | 80 |
| Pier *D* | 91 | 88 | 87 |

* Normal head at Keokuk.

The small differences between the discharge over the spillways with various pier forms when all spillways were in operation were greatly magnified when a single spillway was operating alone. Tests with the center spillway of the model operating while the other two were closed by gates were performed with Piers *B* and *C*, and had previously been made upon the Keokuk Pier *A*. The results are shown in Fig. 25 (*b*). The curves in this diagram show a decided increase in the discharge accomplished by projecting the pier ahead of the dam. Pier *C* allows about 7% more water to pass through a single spillway than passed over the spillway when operating as a model of the Keokuk Dam. At the lower heads, Pier *B* gave discharge coefficients similar to Pier *C* which had twice the amount of projection ahead of the dam, but, at the high heads, Pier *B* gave results closer to those of the Keokuk model, indicating a positive advantage in the further extension of the pier into the pond.

When the piers extended into the pond, the water surface of the nappe as it passed over the crest was not greatly depressed at the spillway sides. However, Fig. 26 which shows the surface of the nappe at three different sections (only center spillway open) indicates that although the nappe is comparatively flat above the crest at the gate section, 6 in. ahead of this at the face of the dam, the surface is appreciably depressed along the pier walls. The comparison between the nappe surface at the metering section with that obtained on the Keokuk model is interesting. Except at the spillway sides, the nappe produced by Pier *B* is not so thick, but since the total discharge is greater, interior pressures must be less and current velocities more.

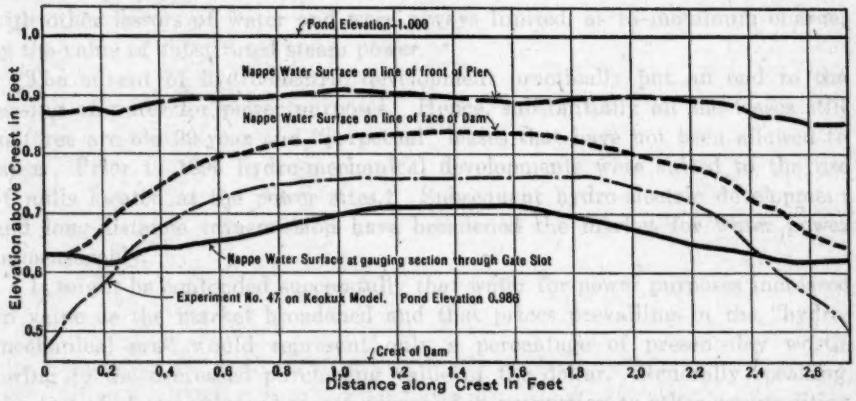



FIG. 26.—CENTER SPILLWAY WITH PIER *B*, COMPARED WITH NAPPE OF KEOKUK MODEL.

## ACKNOWLEDGMENTS

These experiments were financed by the Mississippi River Power Company and the Hydraulic Laboratory of the State University of Iowa.

S. M. Woodward, M. Am. Soc. C. E., of the Department of Mechanics and Hydraulics, State University of Iowa, made valuable suggestions in the preparation of this paper and with J. B. Spiegel, Assoc. M. Am. Soc. C. E., District Engineer, U. S. Geological Survey, visited Keokuk and inspected the

work during the progress of the experiments on the spillways. Mr. Paul Mercer, Assistant Engineer, Mississippi River Power Company, and George E. Shafer, Jun. Am. Soc. C. E., Research Assistant in Hydraulics at the State University of Iowa, assisted in making the measurements at Keokuk. The current meters were rated for these tests free of charge by the engineers of the U. S. Bureau of Standards. The following graduate students in Hydraulic Engineering at the State University of Iowa served as observers in performing the model experiments and assisted in the preparation of this paper: T. L. Herrick and J. W. Howe, Assoc. Members, Am. Soc. C. E., W. E. Smith, J. W. Hummer, Jr., Glen N. Cox, and G. H. Hickox, Juniors, Am. Soc. C. E., and Messrs. Arnold Neshein, Ned Ashton, H. C. Brockman, J. S. Meyers, and J. C. Ducommun.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

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# WATER POWER LEASES

By William H. Cushman,* M. Am. Soc. C. E.

The unreliability of the "Real Estate Sales" method of valuing water power is well known.† Leased water prices, on the other hand, do furnish consistent data and serve as a useful basis of comparison with values arrived at by any other method. This consistency has had a sound basis in that water tendered under lease more nearly represented market rates for power than the "forced sales", "bartering", and "condemnation proceedings" represented values of properties as a whole. In some instances owners were competing with other lessors of water and were always limited, as to maximum charge, by the value of substituted steam power.

The advent of hydro-electric development practically put an end to the leasing of water for power purposes. Hence, substantially all the leases still in force are old 99-year and "perpetual" leases that have not been allowed to lapse. Prior to 1895 hydro-mechanical developments were suited to the use of mills located at the power sites.‡ Subsequent hydro-electric development and long-distance transmission have broadened the market for water power immeasurably.

It might be contended successfully that water for power purposes increased in value as the market broadened and that prices prevailing in the "hydro-mechanical era" would represent only a percentage of present-day worth, owing to the decreased purchasing value of the dollar. Generally speaking, the cost of electric power has not advanced in proportion to other commodities. Irrespective of this possibly higher value as of to-day, the rates charged under these old leases will approximate the average values set forth in present-day appraisals made by the "capitalized net earnings" method.

The United States Census of 1880 includes a volume devoted to water power. The data furnished therein were compiled by different engineers of

Note.—Written discussion on this paper will be closed in May, 1929.

* Cons. Hydr. Engr., Watertown, N. Y.

† "Water-Power Appraisals", by William H. Cushman, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 1843.

‡ *Proceedings*, Am. Soc. C. E., February, 1928, Papers and Discussions, p. 625.

eminence, notably George F. Swain, Past-President, Am. Soc. C. E., who collected the New England data. These data show perhaps the most detailed and painstaking effort in the report.

In the list of leased waters included in the 1880 Census report (Table 1) the power available is stated as against the amount leased whenever both quantities are given. The rates are first quoted as in the report; then, such of them as are stated to be for less than 24 hours per day are reduced, in Column (5), to a 24-hour basis, but these latter are not submitted as a final valuation. All rates are given in terms of annual cost.

TABLE 1.—LEASES GIVEN IN 1880 CENSUS.

| Location. (1) | Available horse-power. (2) | Leased horse-power. (3) | Census rates. (4) | 24-hour rates. (5) |
|---|---|---|---|---|
| Lawrence, Mass. | 13 160 | 10 909 | $14.08 | $21.12 |
| Biddeford, Me. | 1 704 | 1 200 | 10.91 | 23.80 |
| Lewiston, Me. | 13 253 | 11 513 | 9.38 | 20.45 |
| Windsor Locks, Conn. | 18 370 | 2 467 | 14.48 | 14.48 |
| Holyoke, Mass. | 24 000 | ..... | 13.47 (8) | 13.47 |
| Turners Falls, Mass. | 16 000 | 4 000 | 7.50 | 7.50 |
| Bellows Falls, Vt. | 12 000 | 5 613 | 7.50 | 7.50 |
| Unionville, Conn. | 980 | 750 | 11.44 | 11.44 |
| Occum, Conn. | 390 | 180 | 20.00 | 20.00 |
| Barretts Junction, Mass. | 270 | 200 | 9.00 | 9.00 |
| Birmingham, Conn. | 1 725 | 1 260 | 20.00 | 40.00 |
| Ansonia, Conn. | 1 600 | 1 600 | 20.00 | 40.00 |
| Cohoes, N. Y. | 10 630 | 6 556 | 14.67 | 14.67 |
| Oswego, N. Y. | 1 708 | 1 201 | 13.11 | 13.11 |
| Rochester, N. Y. | ..... | 400 | 16.25 | 16.25 |
| Lockport, N. Y. | 2 100 | 828 | 10.84 | 10.84 |
| Niagara Falls, N. Y. | 70 000 | 4 000 | 7.00 | 7.00 |
| Passaic, N. J. | 1 800 | 882 | 33.33 | 66.66 |
| Paterson, N. J. | 2 150 | 2 150 | 35.25 | 70.50 |
| Trenton, N. J. | 3 070 | 500 | 49.02 | 49.02 |
| Raritan, N. J. | 216 | 216 | 32.20 | 32.20 |
| Fredericksburg, Va. | 1 360 | 330 | 15.00 | 15.00 |
| Manchester, Va. | ..... | 925 | 42.04 | 42.04 |
| Augusta, Ga. | 13 635 | 3 650 | 5.00 | 5.00 |
| Hamilton, Ohio. | ..... | 654 | 44.05 | 44.05 |
| Middletown, Ohio. | ..... | 656 | 28.94 | 28.94 |
| Franklin, Ohio. | 600 | 600 | 29.37 | 29.37 |
| Dayton View, Ohio. | 270 | 74 | 36.71 | 36.71 |
| Dayton, Ohio. | ..... | 360 | 30.21 | 30.21 |
| Dayton, Ohio. | ..... | ..... | 27.93 | 27.93 |
| Lawrence, Kans. | ..... | ..... | 20.00 | 20.00 |
| Average. | ..... | ..... | $20.92 | $25.43 |

Leases of water were made on the Merrimac River, at Lawrence and Lowell, Mass., and Manchester, N. H. In each of these cases a group of power users formed a company. They paid sums of money (undoubtedly for the cost of development) and thereafter an annual sum of $300 per "mill power" as a maintenance charge. A "mill power" was rated between 85 and 86 "gross", or theoretical, h.p. All these leases were limited to a use of 16 hours per day.

At Lawrence, more water was leased, at a later period, at the rate of $1 200 per "mill power" per annum, or $14.08 per annual h.p. This 16-hour power rate reduced to a 24-hour basis would be $21.12. At Lowell, additional "mill powers" were sold at $10 000 each; which assumed at 85 h.p., gives a sale value of $117.64 per h.p. Whether the additional rights at Lowell and Lawrence

were subject to prior rights of the original lessees is not stated. At Manchester, no additional rights were sold or rented after the original group developed the power. The $300 per year per "mill power" charged for maintenance, amounted to $3.47 per h.p. As no actual prices for leased water are stated as applying to Manchester and Lowell, they are omitted from Table 1.

At Biddeford and Saco, Me., a "mill power" was rated at 85.23 gross h.p. No leases were granted to any save the original lessees. The York Company (owners), however, leased "surplus" water at the rate of $3.00 per "mill power" per day of 11 hours, or $10.91 per annual h.p. On a 24-hour basis this would be $23.80 for "surplus" power.

At Lewiston, Me., The Union Water Power Company "leases power by the horse power, rates varying, according to circumstances, from $2.50 to $12.50 per h.p. per year, being cheaper for original corporations." This rate is for 11-hour power. "These prices are for the net power on the shafts, being reckoned in all cases as 75% of the gross power." These two prices reduced to gross horse power on a 24-hour basis would be $9.38 and $20.45, respectively.

At Windsor Locks, Conn., The Connecticut River Company developed from 1 800 to 1 900 h.p. and gave perpetual leases to water and land to its power users. "The rates usually charged vary according to circumstances, but the nominal water rental is $2.50 per sq. in. under 30-in. head." This is equivalent to an annual rate per gross horse power of $14.48, the industries using this power being mostly of the 10-hour per day class; evidently, this rate applied to 24-hour service if desired.

The Holyoke Water Power Company owned all the original water rights at Holyoke, Mass., and sold "mill powers" and land. A "mill power" consisted of 86.3 h.p. "The rental per mill power, which is often expressed as the rate charged at Holyoke, indicates but little as to the actual cost of the power to the manufacturer. It is merely a minimum reservation to insure a permanent income for keeping the power in order." The cost of power is involved with land and cannot be separately stated. "There are two classes of power in use at Holyoke, permanent and surplus." "Surplus", or second-class power, is sold at $2.50 per day, or night, per mill power. This rate for "surplus" power, reduced to an annual gross horse-power basis, for 24-hour power, would be $13.47.

At Turners Falls, Mass., "the Turners Falls Company owns a large tract of land available for manufacturing sites, which it sells. Power is disposed of to manufacturers under a perpetual lease. The usual rate has been $7.50 per annum per h.p., rental reserved, but there is no established rate for the future." These leases carry the right to use the water 24 hours per day. Only 4 000 h.p. were developed in 1882, whereas the dam was constructed in 1867 and presumably power leases dated back to about that time. The head at Turners Falls was 28 ft. and the low water in the average year would have produced 16 000 h.p. Less than one-third the available power was leased in 1882; the low rates quoted prevailed among "original owners", as happened, in the earlier stages of power use, on the Merrimac and at other points on the Connecticut River.

The Bellows Falls Canal Company at Bellows Falls, Vt., leased power to manufacturers. "The nominal rate charged is $450.00 per annum per mill power of 60 h.p." There was 12 000 h.p. available. In 1880, only 4 210 h.p. was in use and, in 1882, this had increased to 7 040 h.p. It is evident that this power was not yet fully utilized and was in the "original ownership" stage of valuation, as at Turners Falls. This accounts for the low prevailing rate of $7.50 per gross h.p.

The Union Water Power Company, Unionville, Conn., developed from 860 to 1 000 h.p. which it leased at $175.00 per mill power per year. A mill power consisted of 15.34 gross h.p., giving a rate equivalent to $11.41.

At Greenville, Conn., water rights were sold with a "reserved annual rental of $130 per 1 000 spindle power. About 1 600 h.p. is developed." From this insufficient data, it is impossible to compute the value of a gross horse power.

About 290 h.p. was sold at Occum, Conn., at $20 per annual h.p. Further details are not specified.

There was a development at Barretts Junction, Mass., for about 200 gross h.p., and it was sold to the Springfield Soapstone Company at $9 per annum per h.p. Further, the Company "wishes to sell the balance of its power at rates which would be probably the same for other concerns."

The Birmingham, Conn., development was 1 375 gross h.p. and the power was leased for 99-year terms, at the following rates:

| | |
|---|---|
| Permanent water per gross horse power | $20 |
| First surplus per gross horse power | 12 |
| Second surplus per gross horse power | 8 |

This power was sold on a 12-hour per day basis which would give the "permanent water", on a 24-hour basis, a rate of $40 per annual h.p.

At Ansonia, Conn., 1 600 h.p. was developed and leased at rates closely resembling these charged at Birmingham, namely,

| | |
|---|---|
| Permanent water per gross horse power | $20.00 |
| Surplus water per gross horse power, $16.67 to $8.33 (average) | 12.50 |

While not stated to be 12-hour power, the nature of the industries served would indicate such a limitation and 24-hour power would be equivalent to $40.

The Cohoes Company, Cohoes, N. Y., owned land and water rights and granted perpetual leases for water at $200 per mill power. A mill power is 13.63 gross h.p., which gives an annual rate of $14.67.

The Oswego Canal Company, Oswego, N. Y., gave leases for 999 years. A "run" of water consisted of 11¼ cu. ft. per sec., at 20-ft. head, or 20.7 h.p. The annual rental prices in 1880 were:

| | |
|---|---|
| First-class runs, $350.00 (equivalent to a rate of) | $13.11 |
| Second-class runs, $275.00 (average) | 10.30 |
| Surplus, $154.00 (average) | 5.77 |

These prices held true until 1912, at which time rates were raised to $15 and $10, respectively, for first and second-class runs.

On the west bank of the Oswego River, the Varick Canal Company leased water. A Court decreed that a "run" consisted of 33⅓ cu. ft. per sec. under a 10-ft. head, or 37.87 gross h.p. First-class runs leased for $300, or $7.92 per h.p. per day; second-class runs averaged $137.50, or $3.63 per h.p. per day. This was really a *pro rata* division of unused water among active owners. It lowered the cost per horse power below the original cost.

There was about 9 000 gross h.p. at Rochester, N. Y., and such of it as was rented was charged for at $25 per "effective" h.p. This price converted into the equivalent rate per gross horse power, using the turbine efficiency current in 1880 (65%), would give a rate of $16.25.

The Lockport Hydraulic Company, Lockport, N. Y., rented water at rates ranging from $150.00 to $200.00 per "run", which consisted of 12 "effective" h.p. Turbine efficiency was reckoned at 65%, reducing these two figures to $10.84 and $8.13, respectively.

The Niagara Falls Hydraulic Power and Manufacturing Company controlled the hydraulic rights at Niagara Falls, N. Y., in 1880. Water was leased to heavy consumers at $7 per h.p. To users of less than 500 h.p., the rate was $10, but the Power Company maintained the turbine plant and delivered this power at the turbine shaft.

The Dundee Land and Water Company owned about 800 gross h.p. at Passaic, N. J., which it leased at $700.00 per "run" consisting of 8½ cu. ft. per sec., at 22-ft. head, or 21¼ gross h.p. This price for 12-hour power equalled $32.94 per gross h.p., which on a 24-hour basis, would be $65.88.

At Paterson, N. J., there was 2 150 gross h.p. available, $750 being charged for a "run" of water consisting of 8½ cu. ft. per sec., at a 22-ft. head, or 21¼ gross h.p., equivalent to $35.25 per h.p. All power was shut off between 7:00 P. M., and 6:00 A. M., during the periods of low flow. All lessees used steam auxiliary power (indicating full use). This rate is apparently based on 12-hour service; 24-hour service would have been on a basis of $70.50 per gross h.p.

In Raritan, N. J., 216 gross h.p. was available. Water was leased at from $400 to $300 per sq. ft. of opening under 30 in. of head, that is, 7.81 cu. ft. per sec. This at 14-ft. head would produce 12.42 gross h.p. which, at $400, would be equivalent to $32.20 per h.p. and at $300 to $24.15 per h.p.

The Trenton Water Power Company, Trenton, N. J., leased water at $4 per sq. in. of opening under 3-ft. head, which is equal to 8.62 cu. ft. per sec. At 12-ft. head this would produce 11½ gross h.p., costing $576 (144 × $4) and gives an annual rate of $49.02 per h.p.

The Fredericksburg Water Power Company in Fredericksburg, Va., leased to five mills, an aggregate of 330 h.p. at rates ranging from $5 to $15 per h.p. per annum (not specified as to whether this was gross horse power or otherwise). Here, 1 360 h.p. was available, of which only 330 h.p. was under lease.

The City of Manchester, Va., leased for 50-year periods, water at $4 per sq. in. per annum, under 3-ft. head, that is, 8.62 cu. ft. per sec. When used at 14-ft. head this would produce 13.7 gross h.p., which, at $576, would give $42.04 per h.p. and at 20-ft. head (19.6 h.p. at $576), $29.33 per h.p.

The City of Augusta, Ga., leased water at $5.50 per h.p. (not further specified, but probably gross horse power). Note that 13 635 h.p. was available, of which 3 650 h.p. was leased.

In Ohio, the following rates apply: At Hamilton, water was leased at prices equivalent to $44.05 and $19.09; at Middletown, the rate was $28.94 per gross h.p.; at Franklin, it was $29.37 per gross h.p.; at Dayton View, water was leased for 99-year periods at the gross horse-power rate of $36.71; at Dayton (Mad River), the cost was $30.21 per gross h.p.; and at Dayton (Miami Canal), water was leased at $27.93 and at $24.75 per annual gross h.p.

Power at Appleton, Wis. (not given in Table 1), was leased with the necessary land at the nominal rates of $1 to $2 per h.p., in blocks of from 500 to 1 000 h.p.; $3 to $4 for blocks of from 100 to 300 h.p.; and $2 to $3 for a 50-h.p. site with ½ acre of land.

In Kaukauna, Wis., 100 to 300-h.p. sites were tendered at from $2 to $5 per annual h.p. About 25 000 h.p. was available and evidently power was being tendered as an inducement to manufacturers to locate there. This site is not included in Table 1.

Water power at Lawrence, Kans., was leased at $20 per h.p. In Table 1, Column (5) contains some actual 24-hour prices, but others are "projected" rates based on prices for shorter periods of service per day. These latter prices are submitted for what they are worth, as it does not follow that "night shift" water might be delivered in one instance, or might be sold at reduced rates in another.

The leases listed in Table 1 may be separated into groups. From the locations where the larger amounts of power were available a group may be selected embracing the instances where only a small proportion of the water was under lease, that is, Turners Falls, Bellows Falls, Niagara Falls, Augusta, and the Wisconsin powers (not tabulated). In these instances it is evident that the unused power was in effect a bid for increased patronage and does not reflect average value in an active market.

Another group represents "day-shift" power users, where water power was in sharp demand and more closely approached the cost of steam-produced power, namely, Birmingham, Ansonia, Passaic, Paterson, and Trenton. These power costs, on a 24-hour basis, are greater than the average market rate and, in most instances, are in textile mill territory.

Ignoring other instances where relatively small quantities of water were leased, a group may be selected, including Lawrence, Biddeford, Lewiston, Windsor Locks, Holyoke, Cohoes, and Oswego, that represents sizable developments in active markets. While some of this group represent textile mill territory and others paper mill sections, the "Census Rates" column (Column (4)) shows an average price of $12.87, whereas the "24-Hour Basis" column (Column (3)) shows an average of $17.30.

In the locations where 24-hour power is leased unqualifiedly, that is, Windsor Locks, Holyoke, Cohoes, and Oswego, the prices agree closely and average $13.93 per annual gross h.p. These instances would seem to represent fairly the 1880 value of leased water of a class comparable with that used in present-day hydro-electric developments. It is to be remembered that the

Holyoke price, $13.47, is for "surplus" water; this rate tends to lower the average price of $13.93 unduly.

In instances, turbine efficiencies of 65% were used. It is a matter of record that at one of the cities (Cohoes), hydro-electric development has taken the place of hydro-mechanical use and that holders of original leases were apportioned electric power equal to their holdings developed at 65% efficiency; this would bring the average annual cost of $13.93 up to $21.43 for "shaft" horse power.

At Oswego a revision of leased water prices was made in 1912 whereby the old rate of $13.11 for "first-class water" was raised to $15.00. This being the only instance in this record of revision of price subsequent to 1880, it is worthy of note. In 1912, turbine efficiencies were ordinarily assumed to be 80% which would give a value of $18.75 to water sufficient to produce 1 h.p. on the shaft.

While all these leases represented the right to use water, it must be remembered that a certain amount of "development" was necessarily included. In some cases dam and head-gates represented the lessors' entire investment in development; but, in other cases, long canals, flumes, and penstocks were furnished and, to an extent, "operating costs" entered, as represented by upkeep, supervision of distribution, etc.

If a flat deduction is made, for partial development at so much per shaft horse power, from the average rate for leased water, it would seem that the difference would fairly represent the value per horse power of undeveloped water rights.

Without special emphasis on any of the foregoing rates and for purposes of comparison between "leased water" prices and valuations reached by the "capitalized net earnings" method, assume that the net value of water sufficient to produce an annual gross horse power is $15.00. Water power being customarily expressed in turbine shaft horse power, this $15.00, at 80% efficiency, would become $18.75, as representing the value ordinarily capitalized. Then, $18.75 capitalized at 8% gives a value of $234.37, or at 10%, a value of $187.50.

These values agree fairly well with those developed by the "capitalized net earnings" method, which run from $175 to $275 per h.p. according to the surrounding circumstances.

Holyoke price, $13.47, is for "surplus" water; this rate tends to lower the average price of $13.39 unduly.

In instances, turbine efficiencies of 65% were used. It is a matter of record that at one of the cities (Odessa), hydro-electric development has taken the place of hydro-mechanical use and that holders of original leases were apportioned electric power equal to their holdings developed at 65% efficiency; this would bring the average annual cost of $13.39 up to $21.13 for "shaft" horse power.

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# PAPERS AND DISCUSSIONS

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## PROGRESS REPORT OF THE SPECIAL COMMITTEE ON STEEL COLUMN RESEARCH*

To the Board of Direction,
American Society of Civil Engineers:

The study of column test data† which the Committee had partly completed and reported upon in January, 1926, made it appear desirable to conduct some experiments along certain lines on relatively small columns before completing its report. Accordingly, certain tests were planned and carried out during the past two years, which have yielded some results of value and which will be considered in this report.

It will be desirable, as a matter of convenience, to describe these tests and present the principal results before going on with the discussion in the order intended in the first report. This report, therefore, will consist of two parts: Part I is a description of the tests referred to, and Part II a continuation of the general report, as outlined in the Progress Report of 1926.

The tests described in Part I were made in the laboratories of the University of Wisconsin, under the direction of Professor M. O. Withey, of the Department of Mechanics, to whom the Committee is much indebted for the care and thoroughness with which this work has been done, and for the preparation of much of the material embodied in the report. Acknowledgment is also made of the valuable services of the following Research Assistants of the same Department: Messrs. R. R. Schrader, K. F. Wendt, E. A. Abendroth, H. T. Homewood, and E. R. Summers. Besides the use of its laboratories, the University of Wisconsin has contributed the test columns and considerable service by its Research Staff. The United States Forest Products Laboratory, through J. A. Newlin, M. Am. Soc. C. E., has assisted effectively by loaning the use of its 1 000 000-lb. testing machine and roller-bearing blocks. Engineering Foundation has co-operated in this work from the beginning, and a considerable part of the budget for 1927 has been secured through that organization.

* Presented at the Annual Meeting, January 16, 1929.
† *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1485.

## PART I.—TESTS OF COLUMNS MADE DURING 1925-27

*1.—Object.*—The principal object in mind in making these tests was to try out certain promising methods of studying the effectiveness of lacing-bars and other lateral elements. However, as the work developed, considerable valuable information was obtained regarding the stresses in such elements, the crookedness of certain columns, the relation of strength to yield point of material, the properties of the material at various points in a cross-section, the effect of form of section on eccentrically loaded columns, and other features. These various items will be discussed in their proper place after the tests have been described in general.

*2.—Scope.*—These column tests may be conveniently divided into three series, as follows

(A) Eleven columns tested under oblique loading, causing a definite amount of shearing stress on the column.
(B) Ten rolled H-columns tested under eccentric loads.
(C) Two columns similar to certain columns of Series (*A*), tested under eccentric loads.

*3.—Test Columns.*—The general dimensions and properties of these columns are given in Tables 1 and 2. Details of columns, including location of gauge lines, are given in Figs. 1 to 7. Details of Columns Nos. 4, 5, 5*A*, 5*B*, and 6, which were latticed channel columns, are given in Fig. 4, and details of Columns Nos. 7, 8, and 9, which were batten-plate columns, are shown in Figs. 5, 6, and 7, respectively. Columns Nos. 4, 5, 5*A*, 5*B*, and 6 differ only in the size of the lacing-bars, those used in Column No. 4 being very light, and those in Column No. 6, extra heavy. Fig. 8 gives the locations of gauge lines for the H-columns of Series (*B*). Columns Nos. 5*A* and 8*A* of Series (*C*) were similar to Columns Nos. 5 and 8, respectively, of Series (*A*).

The actual sectional areas were computed from the weights, deducting the calculated weights of rivet heads, lacing-bars, and other details. The values of $r$ were determined by measurement at each end of the column, and the two results were averaged.

All the columns listed in Tables 1 and 2 were contributed by the University of Wisconsin. All fabricated columns were manufactured by the American Bridge Company; and all H-columns were obtained from the Bethlehem Steel Company. The material and shopwork were specified to conform to the General Specifications for Railway Bridges of the American Railway Engineering Association.

*4.—Apparatus.*—Tests were made at the University of Wisconsin, some on a 1 000 000-lb. machine belonging to the United States Forest Products Laboratory, at Madison, and others with the 600 000-lb. testing machine in the Materials Testing Laboratory of the University An important feature of the testing apparatus without which these tests could not have been readily made was the bearing-block arrangement placed between the ends of the column and the testing machine. These blocks furnished roller-bearing devices in

such a manner as to provide nearly frictionless end bearings to the columns. Fig. 9 shows a cross-section of one of the bearing blocks which explains the arrangement. Strain measurements over 8-in. and 20-in. gauge lengths were made with Berry strain-gauges read to 0.0001 in. For 2-in. gauge lines, an Olsen strain-gauge was read to 0.00003 in. Deflections were read by wire-wound dial deflectometers to 0.0001 in. The yield point was determined by the drop of the beam, and checked by the movement of the extensometer dial. The proportional limit was determined by the deviation point of the stress-strain curve from a straight line.

The frictionless character of the end bearings was indicated in various ways throughout the testing program, particularly by the sensitiveness of the column to small changes in position of the ends, and by the very close actual agreement of lateral deflection under eccentric loads with the theoretical, assuming hinged ends. However, in order to assure itself on this point, the Committee obtained direct evidence of the actual amount of the friction. The two blocks were placed face to face in the testing machine and a load applied directly to them. The combination of the two blocks was then turned through a considerable angle by means of a projecting rod attached to the blocks. Assuming that the center of curvature of the roller bearing of each block was precisely at the face of the block, the combination of the two could be turned through an angle with no resistance except friction. The maximum value of the load, $P$, was 500 000 lb., as compared to a maximum load used in the column tests of about 400 000 lb. It was found that up to a certain point the force required to turn the blocks increased with the amount of the movement, but beyond this point, the blocks continued to roll at very nearly constant force. The results of the tests, given in Table 3, are the averages of three to eight sets of readings. The friction coefficient is found by dividing the torque by the product of the diameter of the circle of roller centers times the imposed load, $P$. The angular movement of 0.00006 radians was equal to one division of the bubble used, which was the smallest movement noted.

In the oblique loading tests, it is found from calculation that the angle of rotation for Column No. 4, with $P/A = 14\,570$ lb. per sq. in. ($P = 167\,000$) and 5% shear, was about 0.002; and for Column No. 2, with $P/A = 14\,170$ lb. per sq. in. ($P = 200\,000$), it was about 0.0011. Comparing these values with the results given in Table 3 it would appear that the friction coefficient was in most cases considerably less than 0.002, and this may be taken as about the maximum probable value for any of the tests.

The effect of a friction coefficient of 0.002 can be judged by calculating the eccentricity necessary to overcome this friction. The frictional moment for a single bearing block would be $P \times 6 \times 0.002 = 0.012\ P$, requiring an eccentricity of only 0.012 in.; that is, the effective eccentricity would be 0.012 in. less than that given in the tests. Inasmuch as these values were of the magnitude of 1 in., or more, the error due to friction would be very small—generally less than 1 per cent.

TABLE 1.—DIMENSIONS AND PROPERTIES OF COLUMNS OF SERIES (*A*) AND (*C*).

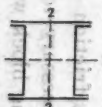
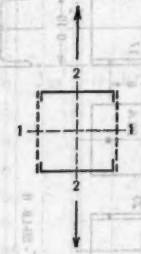
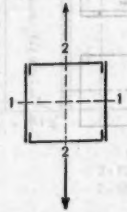
| Column No. (1) | Type. (2) | Make-up. (3) | Length, in inches. (4) | Area, in Square Inches. Nominal. (5) | Area, in Square Inches. Actual. (6) | $r_1$, in Inches. Nominal. (7) | $r_1$, in Inches. Actual. (8) | $\frac{l}{r_1}$. (9) | $I_2$, in inches.[4] (10) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | | 5 by 3½ by 5/16-in. angles<br>10 by 5/16-in. plates | 77 | 13.37 | 13.19 | 4.10 | 4.20 | 18.3 | 56.9 |
| 2 | | 10-in., 49.5-lb. H column | 89.5 | 14.37 | 14.15 | 4.28 | 4.26 | 21.0 | 87.8 |
| 3 |  | 12 by ¼-in. plates<br>10-in., 15-lb. channels | 89.5 | 14.92 | 14.67 | 4.42 | 4.43 | 20.2 | 210.9 |
| 3A | | 12 by ¼-in. plates<br>10-in., 15-lb. channels | 89.5 | 14.92 | 14.43 | 4.42 | 4.42 | 20.3 | 188.8 |

TABLE 1.—(*Continued.*)

| Column No. | Type. | Make-up. | Length, in inches. | Area, in Square Inches. | | $r_1$, in Inches. | | $\frac{l}{r_1}$. | $I_2$, in inches.[4] |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Nominal. | Actual. | Nominal. | Actual. | | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| 4 |  | 10-in., 20-lb. channels<br>2¼ by $\frac{3}{16}$-in. angle-bars<br>¼-in. splice-plates | 119 | 11.72 | 11.52 | 5.44 | 5.89 | 22.1 | 154.3 |
| 5 | | 10-in., 20-lb. channels<br>2½ by ¼-in. angle-bars<br>¼-in. splice-plates | 119 | 11.72 | 11.30 | 5.44 | 5.44 | 21.9 | 151.4 |
| 5A | | 10-in., 20-lb. channels<br>2¼ by ¼-in. angle-bars<br>¼-in. splice-plates | 119 | 11.72 | 11.62 | 5.44 | 5.40 | 22.0 | 155.5 |
| 5B | | 10-in., 20-lb. channels<br>2¼ by ¼-in. angle-bars<br>¼-in. splice-plates | 119 | 11.72 | 11.38 | 5.44 | 5.44 | 21.9 | 152.6 |
| 6 | | 10-in., 20-lb. channels<br>2½ by ⅜-in. angle-bars<br>⅜-in. splice-plates | 119 | 11.72 | 11.30 | 5.44 | 5.36 | 22.2 | 151.4 |
| 7 |  | 10-in., 20-lb. channels<br>¼ in. splice-plates<br>Two 9 by 12 by ¼-in. batten-plates | 119 | 11.72 | 11.84 | 5.44 | 5.44 | 21.9 | 158.7 |
| 8 | | 10-in., 20-lb. channels<br>¼-in. splice-plates<br>Four 9 by 12 by ¼-in. batten-plates | 119 | 11.72 | 11.84 | 5.44 | 5.43 | 21.9 | 158.7 |
| 8A | | 10-in., 20-lb. channels<br>¼-in. splice-plates<br>Four 9 by 12 by ¼-in. batten-plates | 119 | 11.72 | 11.30 | 5.44 | 5.41 | 22.1 | 151.5 |
| 9 | | 10-in., 20-lb. channels<br>¼-in. splice-plates<br>Eight 9 by 12 by ¼-in. batten-plates | 119 | 11.72 | 11.88 | 5.44 | 5.44 | 21.9 | 159.2 |

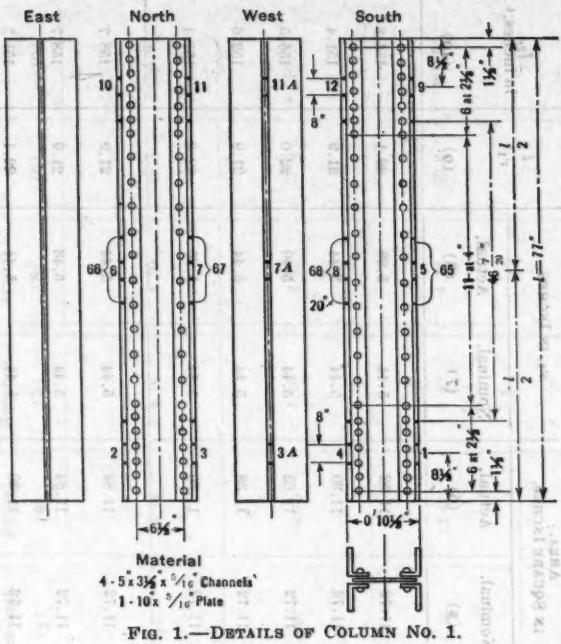



FIG. 1.—DETAILS OF COLUMN NO. 1.

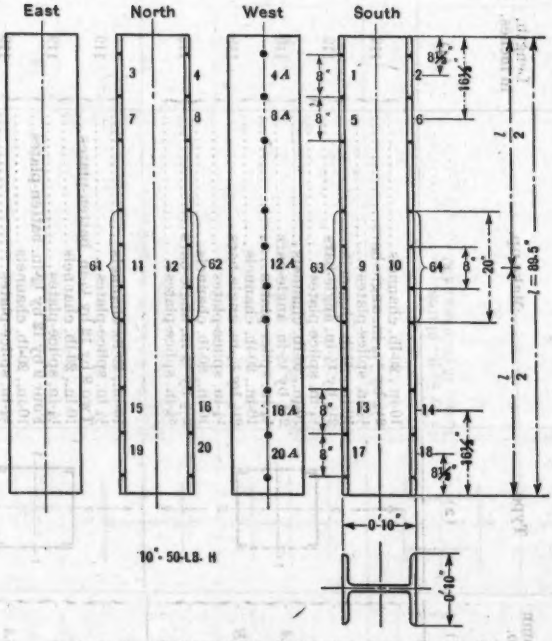



FIG. 2.—DETAILS OF COLUMNS NOS. 2 AND 2A.

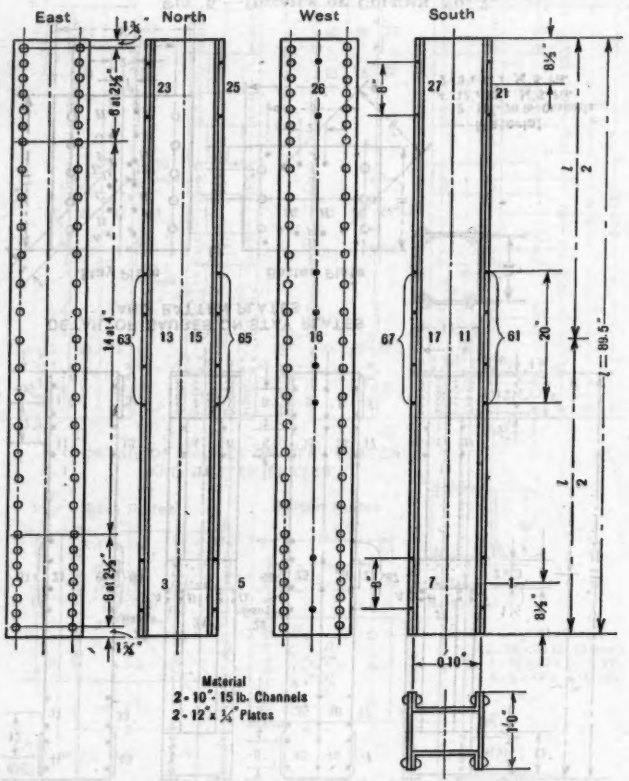



FIG. 3.—DETAILS OF COLUMNS NOS. 3 AND 3A.

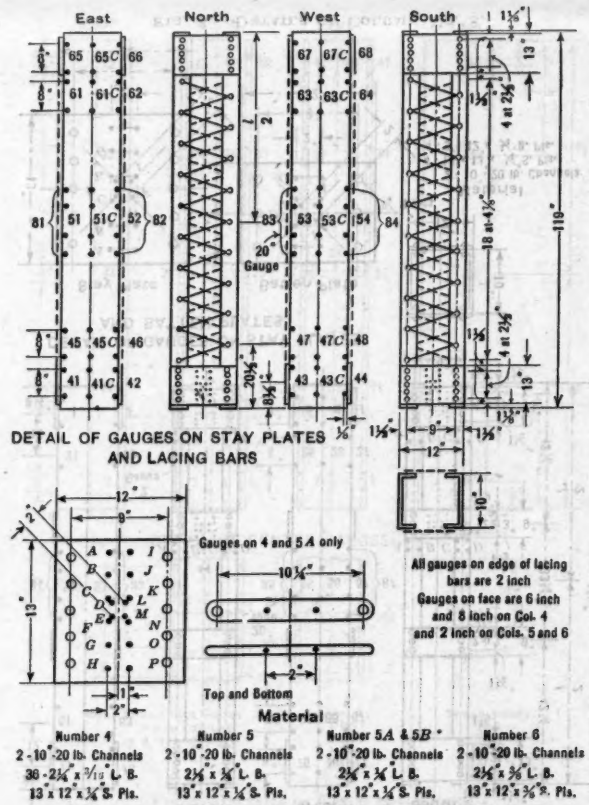

East
North
West
South
DETAIL OF GAUGES ON STAY PLATES AND LACING BARS
Gauges on 4 and 5A only
All gauges on edge of lacing bars are 2 inch
Gauges on face are 6 inch and 8 inch on Col. 4 and 2 inch on Cols. 5 and 6
Top and Bottom
Material
Number 4
2 - 10" - 20 lb. Channels
36 - 2¼" x 3/16" L B.
13" x 12" x ¼" S. Pls.
Number 5
2 - 10" - 20 lb. Channels
2½" x ¼" L. B.
13" x 12" x ¼" S. Pls.
Number 5A & 5B
2 - 10" - 20 lb. Channels
2½" x ¼" L B.
13" x 12" x ¼" S. Pls.
Number 6
2 - 10" - 20 lb. Channels
2½" x 3/8" L B.
13" x 12" x 3/8" S. Pls.

FIG. 4.—DETAILS OF COLUMNS NOS. 4, 5, 5A, 5B, AND 6.

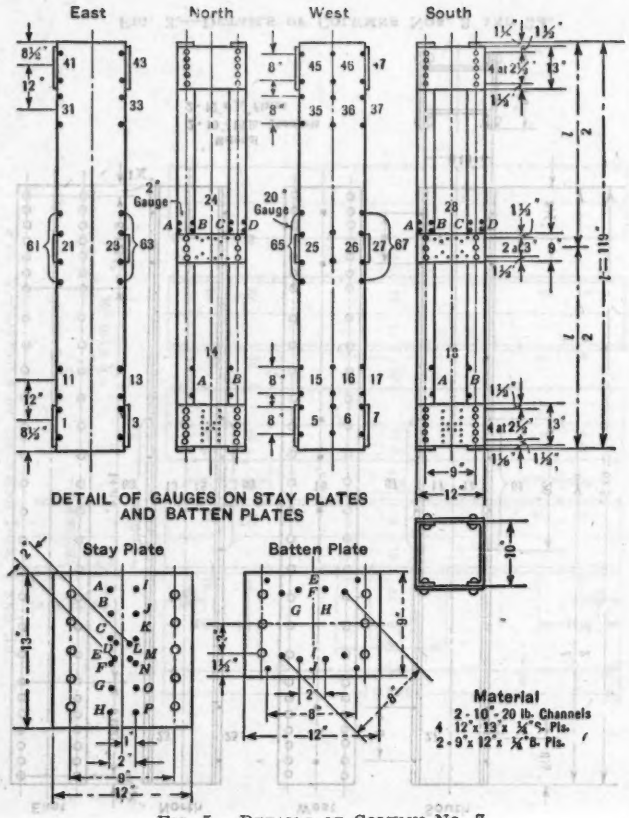



FIG. 5.—DETAILS OF COLUMN NO. 7.

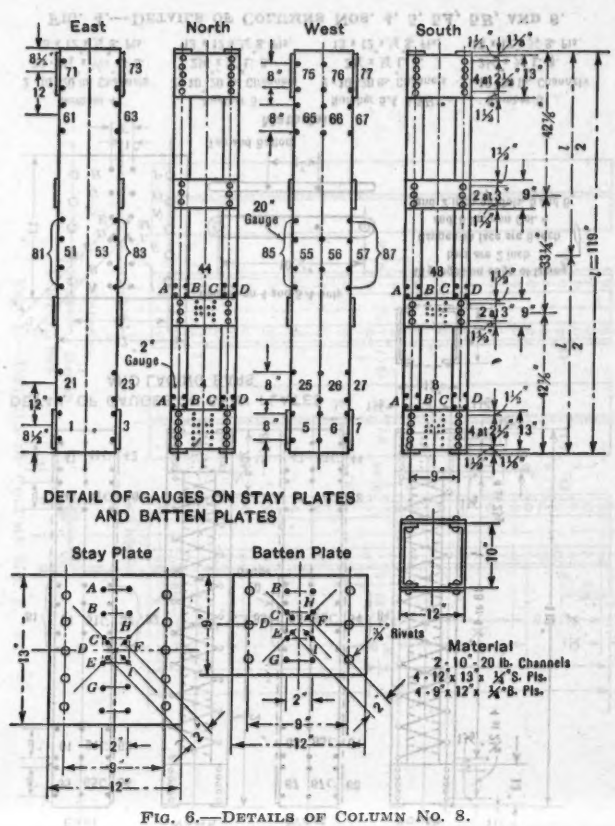



FIG. 6.—DETAILS OF COLUMN NO. 8.

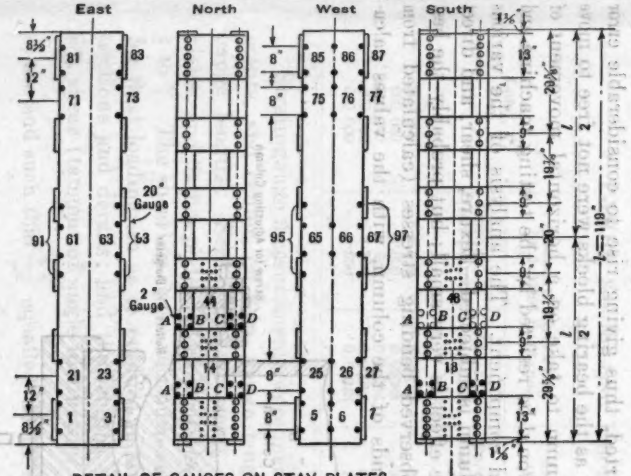



FIG. 7.—DETAILS OF COLUMN NO. 9.

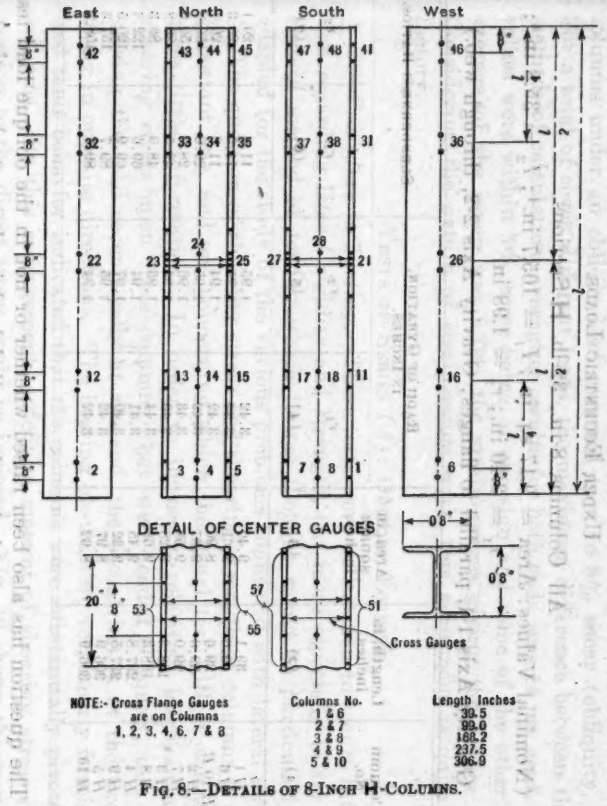

East
North
South
West
DETAIL OF CENTER GAUGES
Cross Gauges
NOTE:- Cross Flange Gauges are on Columns 1, 2, 3, 4, 6, 7 & 8
Columns No. | Length Inches
1 & 6 | 39.5
2 & 7 | 99.0
3 & 8 | 168.2
4 & 9 | 237.5
5 & 10 | 306.9

FIG. 8.—DETAILS OF 8-INCH H-COLUMNS.

TABLE 2.—Dimensions and Properties of Columns of Series (*B*) Tested Under Eccentric Loads.

All Columns 8-in., 32-lb. H-Sections.

(Nominal Values: Area = 9.17 sq. in.; $I_1 = 105.7$ in.$^4$; $I_2 = 35.8$ in.$^4$; $r_1 = 3.40$ in.; $r_2 = 1.98$ in.

Gravity Axis 1-1, parallel to flanges; Gravity Axis 2-2, through web.)

| Column No. | Length, in inches. | Area, in square inches. | Radii of Gyration, in Inches. | | Slenderness Ratios. | |
|---|---|---|---|---|---|---|
| | | | $r_1$. | $r_2$. | $\frac{l}{r_1}$. | $\frac{l}{r_2}$. |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| *H* 1 | 39.1 | 9.40 | 3.42 | 1.95 | 11.4 | 20.1 |
| *H* 6 | 39.6 | 9.61 | 3.44 | 1.95 | 11.5 | 20.3 |
| *H* 10 *B* | 39.6 | 8.96 | 3.42 | 1.94 | 11.6 | 20.4 |
| *H* 2 | 99.0 | 9.37 | 3.42 | 1.95 | 29.0 | 50.8 |
| *H* 7 | 99.0 | 9.63 | 3.43 | 1.96 | 28.9 | 50.6 |
| *H* 3 | 168.3 | 9.28 | 3.40 | 1.94 | 49.5 | 86.8 |
| *H* 8 | 168.3 | 9.63 | 3.44 | 1.95 | 48.9 | 86.3 |
| *H* 4 | 237.8 | 9.45 | 3.41 | 1.94 | 69.6 | 122.4 |
| *H* 9 | 237.5 | 9.32 | 3.40 | 1.97 | 69.9 | 120.5 |
| *H* 5 | 306.9 | 8.97 | 3.42 | 1.95 | 89.7 | 157.3 |
| *H* 10 | 306.9 | 8.92 | 3.42 | 1.94 | 89.7 | 158.2 |

The question has also been raised whether or not in the oblique load tests it could be safely assumed that the applied load is always truly vertical, or whether a horizontal component of considerable magnitude may not be developed as the column becomes distorted, thus giving rise to considerable error in the calculated shear. Inasmuch as the bearing blocks were not free to move laterally, any tendency of the column to take on a horizontal movement of one end with respect to the other would be resisted by the testing machine and thus give rise to a horizontal load component. The analysis of the various kinds of distortion to which the column is subjected—flexure, shear, and direct stress—indicates that such possible effect is very small; but probably the best answer is a comparison of the observed bending stresses (calculated from strain-gauge readings) near the ends of the column, with the values calcu-

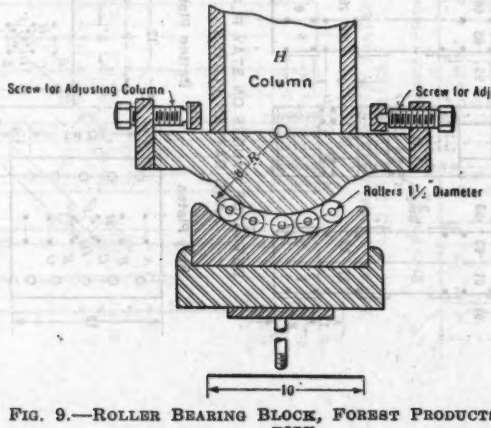



Fig. 9.—Roller Bearing Block, Forest Products Laboratory.

lated from the assumed shear. Such comparison has been made for four columns under an oblique load assumed to give 5% shear (obliquity, 0.05). For a total of seven different loads, the maximum difference between the two methods of calculation, averaging results for both ends, was 14 per cent. Most values were within 3%, some being of one sign and some of the other. The average of the seven was 2.7%, the strain-gauges giving the higher value. Considering the nature of strain-gauge work, this agreement is quite satisfactory.

### Tests of Series (*A*): Oblique Loading

*5.—Procedure in Testing.*—These tests were undertaken to secure information regarding lateral rigidity and strength, but more especially to try out a certain method of testing which it was thought would be particularly well adapted for the study of the various problems connected with lateral strength.

The question of shearing stress and shearing strength of columns is a very important one, and one concerning which the usual test with a central load gives little information. In order to impose a relatively large amount of shearing stress upon these columns, they were loaded obliquely, that is, by placing the load eccentrically at the end, the eccentricities at the two ends being in opposite directions. The bearing blocks were very nearly frictionless, and their behavior indicated that the columns were substantially pivoted end columns. By this arrangement, a definite amount of shearing stress in proportion to the direct stress could be imposed. Fig. 10 shows the manner of placing the column in the testing machine.

TABLE 3.—Rolling Friction of Bearing Blocks.

| Load, *P*. | Angular Movement of 0.00006 Radians. | | Angular Movement of 0.0006 Radians. | | Continuous Rolling. | |
|---|---|---|---|---|---|---|
| | Torque, in foot-pounds | Friction coefficient. | Torque, in foot-pounds. | Friction coefficient. | Torque, in foot-pounds. | Friction coefficient. |
| 100 000 | 1 610 | 0.00134 | 2 380 | 0.00198 | 3 040 | 0.00253 |
| 200 000 | 2 000 | 0.00083 | 3 040 | 0.00126 | 5 020 | 0.0021 |
| 300 000 | 2 420 | 0.00064 | 4 900 | 0.00136 | 9 100 | 0.0024 |
| 400 000 | 2 760 | 0.00058 | 6 330 | 0.00132 | 16 800 | 0.0035 |
| 500 000 | 2 550 | 0.00043 | 7 400 | 0.00123 | ..... | ..... |

Strain-gauge measurements were made at all corners near the ends, quarter-points, and center, on 8-in. gauge lengths, and also at the center on 20-in. gauge lengths. Strain measurements were also made on lacing-bars, stay-plates, and batten-plates. The locations of the gauge lines are shown in Figs. 1 to 7. The shearing stress was purposely made large, 5% of the direct stress for final loading, so as to produce relatively large stresses in the lateral connections and details; and the lengths of the columns were chosen so that a 5% shear (tangent of angle inclination = 0.05) would require an eccentricity at the end such that $\frac{e\,c}{r^2}$ equalled about 0.6, in which, $e$ is the eccentricity; $c$, the distance to extreme fiber; and $r$, the radius of gyration. Such eccentricity caused a maximum fiber stress at the end of about 1.6 times the aver-

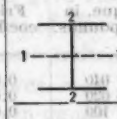
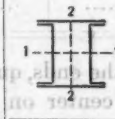
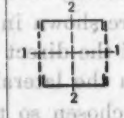
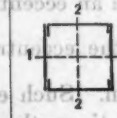
age, or 1.6 times $P/A$. In all cases, the plane of obliquity was parallel to Axis 2-2 (Table 1), the roller axes being parallel to Axis 1-1.

The columns were first centered in the testing machine by measurement and then by trial loadings until the strain-gauge measurements and deflection apparatus showed practically no eccentricity under a considerable load. After being centered, the ends were shifted a calculated amount to produce the desired obliquity.

Some of the columns were tested under 2% shear up to a maximum fiber stress of about 14 000 lb. per sq. in., and then tested to failure under 5% shear. Others were tested only under 5% shear.

*6.—Maximum Stresses at Ultimate Load.*—Table 4 gives the results of these tests relative to the ultimate loads and the maximum fiber stresses at the ends of the columns.

TABLE 4.—ULTIMATE LOADS AND MAXIMUM FIBER STRESSES AT ENDS OF COLUMNS UNDER 5% SHEAR.

| Column No. | Type. | Applied Loads, $\frac{P}{A}$, in Pounds per Square Inch at: | | Calculated Fiber Stresses at Ends, in Pounds per Square Inch at: | | Average Tensile Properties of Material, in Pounds per Square Inch. | |
|---|---|---|---|---|---|---|---|
| | | First scaling. | Ultimate. | First scaling. | Ultimate. | Yield point. | Proportional limit. |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| 1 | | 22 000 | 28 000 | 34 500 | 43 900 | 38 300 | 33 500 |
| 2 |  | 19 500 | 26 700 | 31 400 | 42 900 | 36 100 | 33 700 |
| 3 |  | 18 000 | 25 900 | 28 900 | 41 500 | 35 000 | 33 200 |
| 3A | | 18 000 | 24 900 | 28 800 | 39 900 | 36 500 | 33 400 |
| 4 |  | 18 000 | 22 800 | 29 000 | 36 800 | 32 000 | 26 400 |
| 5 | | 18 600 | 22 900 | 29 900 | 36 800 | 32 000 | 26 400 |
| 5A | | 22 000 | 22 800 | 35 400 | 35 900 | 32 300 | 30 000 |
| 6 | | 17 500 | 24 000 | 28 200 | 38 700 | 32 000 | 26 400 |
| 7 |  | ..... | 10 000 | ..... | 16 000* | 32 700 | 27 400 |
| 8 | | ..... | 15 000 | ..... | 24 100* | 32 200 | 28 400 |
| 9 | | 19 000 | 20 700 | 30 500 | 33 200* | 31 700 | 27 800 |

* The maximum unit stresses in the batten-plate columns occurred in the unsupported channels near plates, see Table 13.

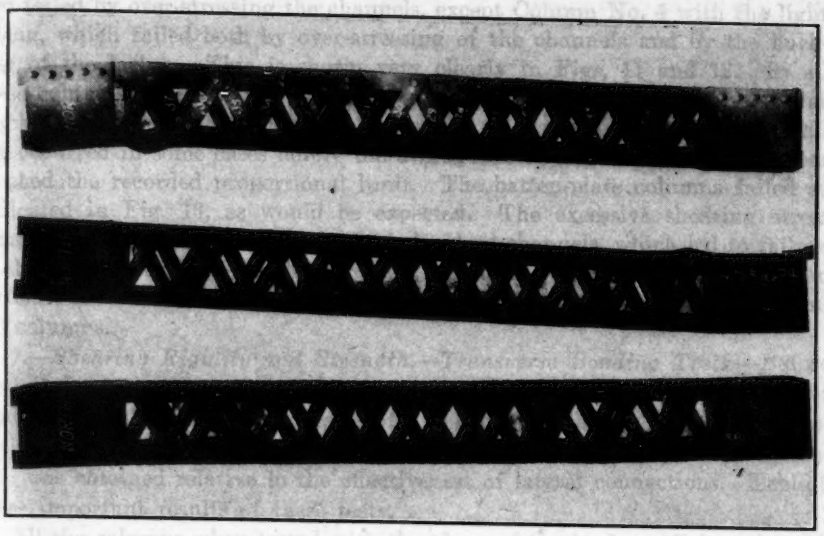


FIG. 11.—FAILURES OF COLUMNS NOS. 4, 5A, AND 6, TESTED UNDER OBLIQUE LOADS.

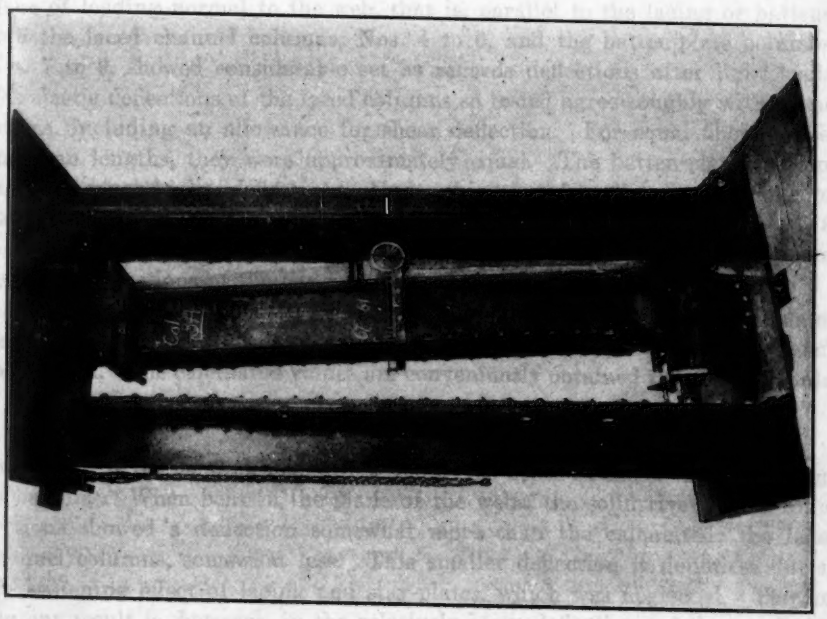


FIG. 10.—APPARATUS USED TO MEASURE DISTORTIONS AND DEFLECTIONS IN OBLIQUE LOADING TESTS. NOTE, ALSO, METHOD OF FAILURE OF COLUMN NO. 3A.

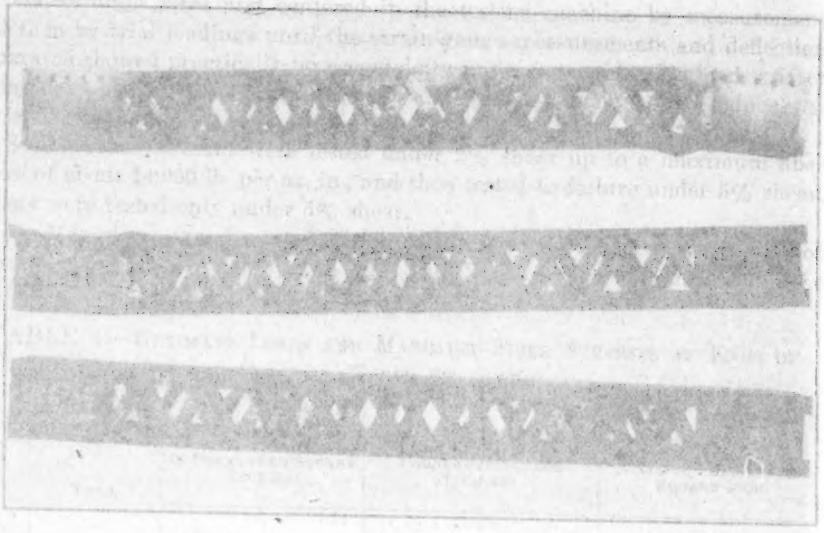


FIG. 11.—LINKS OF BELTS TESTED, SHOWING CHARACTER OF FAILURE.

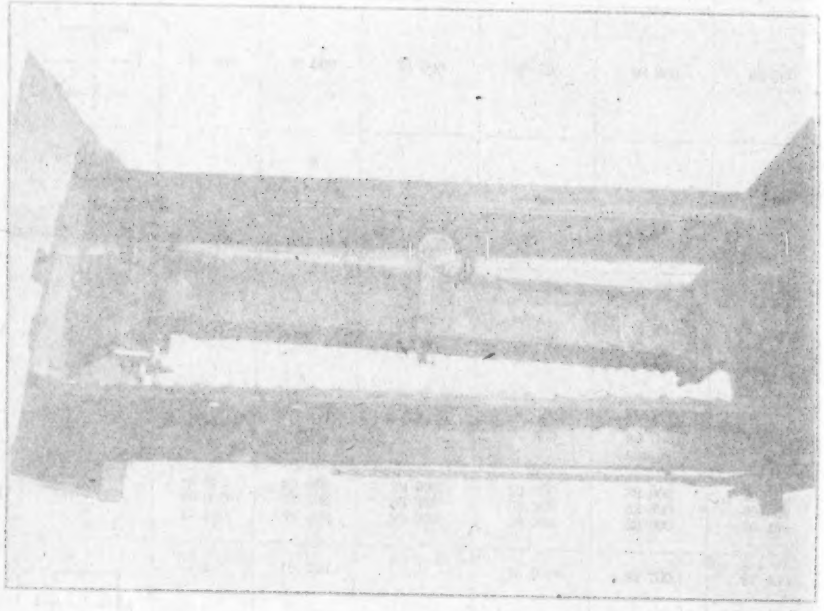


FIG. 12.—METHOD OF APPLYING LOAD TO LINKS OF BELTS.

Columns Nos. 1, 2, 3, and 3A, composed of solid rolled or riveted sections, failed in the manner indicated in Fig. 10. The latticed channel columns also failed by over-stressing the channels, except Column No. 4 with the light lacing, which failed both by over-stressing of the channels and by the buckling of the lacing. This is shown very clearly in Figs. 11 and 12. In all these columns, however, the calculated maximum fiber stress at the ends was much beyond the yield point of the material. As Table 4 shows, the first scaling occurred in some cases before the average unit stress in the extreme fiber reached the recorded proportional limit. The batten-plate columns failed as indicated in Fig. 13, as would be expected. The excessive shearing stress caused large bending moments in the individual channels, which led to failure at the first rivet of the end batten-plate. Column No. 9, however, carried an ultimate load almost as great as the maximum sustained by the latticed channel columns.

7.—*Shearing Rigidity and Strength.—Transverse Bending Tests.*—Before testing the columns as columns, they were tested as simple beams under center loads up to calculated fiber stresses of 1 000 to 6 000 lb. per sq. in. By comparing the deflections and sets when bent in different directions, some information was obtained relative to the effectiveness of lateral connections. Table 5 gives important results of these tests.

All the columns when tested with the plane of the load parallel to the web exhibited small sets and the deflections agree reasonably well with calculated values if due allowance is made for shear deflection. When tested with the plane of loading normal to the web, that is, parallel to the lacing or battens, both the laced channel columns, Nos. 4 to 6, and the batten-plate columns, Nos. 7 to 9, showed considerable set as regards deflections after light loads. The elastic deflections of the laced columns so tested agree roughly with calculations, including an allowance for shear deflection. For equal fiber stresses and span lengths, they were approximately equal. The batten-plate columns exhibited much higher deflections. Under this type of loading, basing the comparison on the same fiber stresses, the elastic deflections of Columns Nos. 7, 8, and 9 were, respectively, seven, three, and two times as much as the corresponding deflections of the laced columns.

It is of interest to compare the observed deflections with the calculated values, considering bending distortion only. Table 6 gives some results of such calculation. The calculated values are conveniently obtained from the formula,

$$\text{Deflection} = \frac{1}{6}\,\frac{f\,l^2}{E\,h}$$

in which, $f$ is the bending fiber stress and $h$, the depth of the beam in the plane of bending. When bent in the plane of the webs, the solid riveted and rolled sections showed a deflection somewhat more than the calculated; the laced channel columns, somewhat less. This smaller deflection is doubtless due to the stiffening effect of lacing and stay-plates, which was neglected. The important result is, however, in the relatively large deflections of these columns (from 1.6 to 2.0 times the calculated values) when bent in the plane of the lacing. This excess is due obviously to the contribution of the lacing and

stay-plates to the deflection, or, as it may be considered, the deflection due to shear; that is, the deflection due to shear is, in this case, from 60 to 100% as much as the deflection due to moment. The differences are naturally still larger in the case of the batten-plate columns. Inasmuch as most column formulas are based, to a greater or less extent, on theories which ignore shearing distortion, this question is of considerable importance. It is discussed further in Part II.

TABLE 5.—Comparison of Deflections and Sets at Mid-Length of Columns Under Small Transverse Loads.

| Column No. | Length of span, in inches. | Center load, in pounds. | Extreme fiber stress, in pounds per square inch. | Position of Load Plane. | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Parallel to Web. | | | | Normal to Web. | | | |
| | | | | Direct. | | Inverted. | | Direct. | | Inverted. | |
| | | | | Deflection, in inches. | Set, in inches. | Deflection, in inches. | Set, in inches. | Deflection, in inches. | Set, in inches. | Deflection, in inches. | Set, in inches. |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| 1 | 72.0 | 7 820 | 2 950 | 0.0118 | 0.0000 | 0.0118 | 0.0000 | ..... | ..... | ..... | ..... |
| 2 | 84.0 | 7 620 | 3 060 | 0.0148 | −0.0006 | 0.0146 | −0.0005 | ..... | ..... | ..... | ..... |
| 3 | 84.0 | 8 000 | 3 100 | 0.0169 | *0.0035 | ..... | ..... | ..... | ..... | ..... | ..... |
| | | 7 920 | 3 070 | ..... | ..... | 0.0135 | 0.0005 | ..... | ..... | ..... | ..... |
| 3A | 84.0 | 9 820 | 3 820 | 0.0169 | 0.0002 | 0.0144 | 0.0000 | ..... | ..... | ..... | ..... |
| 4 | 114.0 | 3 300 | 3 060 | 0.0197 | 0.0002 | ..... | ..... | ..... | ..... | ..... | ..... |
| | | 6 120 | 3 120 | ..... | ..... | ..... | ..... | 0.0353 | 0.0084 | 0.0367 | 0.0079 |
| 5 | 114.0 | 3 300 | 3 120 | 0.0203 | 0.0012 | ..... | ..... | ..... | ..... | ..... | ..... |
| | | 6 120 | 3 130 | ..... | ..... | ..... | ..... | 0.0406 | 0.0122 | 0.0364 | 0.0075 |
| | | 6 290 | 3 260 | ..... | ..... | ..... | ..... | 0.0402 | 0.0060 | ..... | ..... |
| 5A | 117.5 | 5 780 | 2 980 | ..... | ..... | ..... | ..... | ..... | ..... | 0.0387 | 0.0068 |
| | | 4 200 | 3 980 | 0.0294 | 0.0008 | ..... | ..... | ..... | ..... | ..... | ..... |
| 5B | 117.5 | 9 430 | 4 920 | ..... | ..... | ..... | ..... | 0.0496 | 0.0023 | 0.0531 | 0.0075 |
| | | 6 160 | 5 940 | 0.0433 | 0.0008 | ..... | ..... | ..... | ..... | ..... | ..... |
| 6 | 114.0 | 3 320 | 3 140 | 0.0209 | 0.0001 | ..... | ..... | ..... | ..... | ..... | ..... |
| | | 6 120 | 3 180 | ..... | ..... | ..... | ..... | 0.0306 | 0.0070 | 0.0320 | 0.0061 |
| 7 | 117.5 | 4 320 | 4 000 | 0.0325 | 0.0025 | ..... | ..... | ..... | ..... | ..... | ..... |
| | | 1 980 | 1 000 | ..... | ..... | ..... | ..... | 0.0678 | 0.0024 | 0.0688 | 0.0029 |
| 8 | 117.5 | 4 300 | 4 000 | 0.0278 | 0.0004 | ..... | ..... | ..... | ..... | ..... | ..... |
| | | 3 980 | 2 010 | ..... | ..... | ..... | ..... | 0.0600 | 0.0041 | ..... | ..... |
| | | 4 000 | 2 020 | ..... | ..... | ..... | ..... | ..... | ..... | 0.0645 | 0.0065 |
| 8A | 117.5 | 6 170 | 6 000 | 0.0468 | 0.0010 | ..... | ..... | ..... | ..... | ..... | ..... |
| | | 3 750 | 1 990 | ..... | ..... | ..... | ..... | 0.0628 | 0.0056 | 0.0628 | 0.0070 |
| | | 6 000 | 3 020 | ..... | ..... | ..... | ..... | 0.0505 | 0.0055 | ..... | ..... |
| 9 | 117.5 | 5 970 | 3 000 | ..... | ..... | ..... | ..... | ..... | ..... | 0.0496 | 0.0061 |
| | | 4 340 | 4 000 | 0.0293 | 0.0012 | ..... | ..... | ..... | ..... | ..... | ..... |

*Member did not have full bearing across knife-edges due to initial wind of approximately 3½ degrees.

*8.—Oblique Loading Tests.*—As regards oblique loading, Fig. 14 shows that, if the column ends are hinged, the mid-section is rotated counter-clockwise by the bending moment and clockwise by the shear. Hence, the net angular rotation of the mid-section, $\alpha$, relative to the column axis is the difference of the shear and bending rotations, $\alpha_S - \alpha_B$. Also, owing to small rotations of the bearings and the machine during loading, the column as a whole suffers a small rotation, $\alpha'$. Hence, the entire rotation, $\alpha_1$, of the mid-section, $\alpha_1$, will be $\alpha + \alpha'$. By attaching a pair of spirit levels to a side-bar supported at the column ends (Fig. 10) the rotation of the column as a whole, $\alpha'$, was

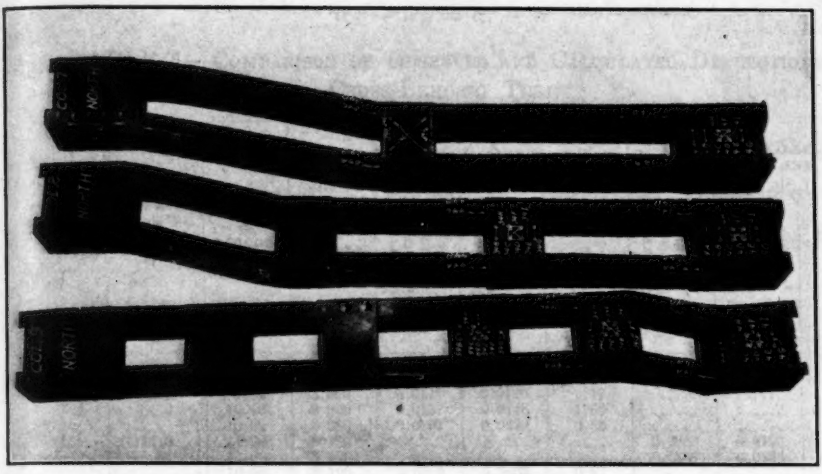


FIG. 13.—FAILURES OF COLUMNS NOS. 7, 8, AND 9.

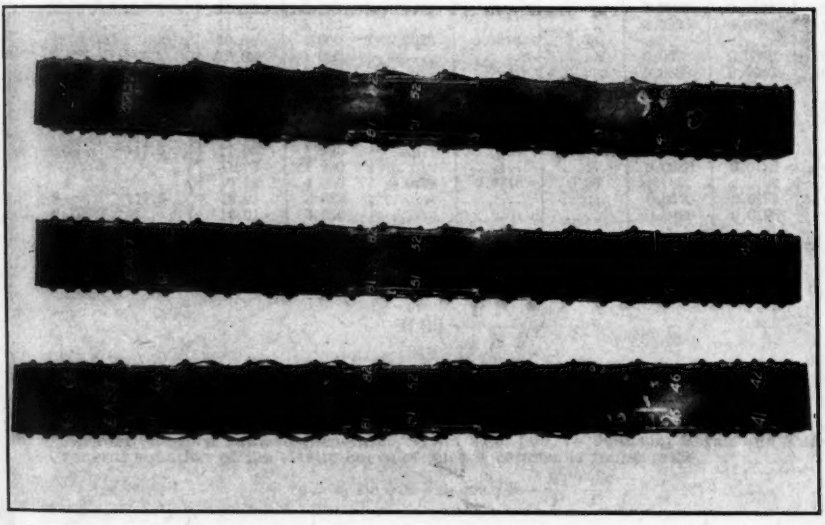
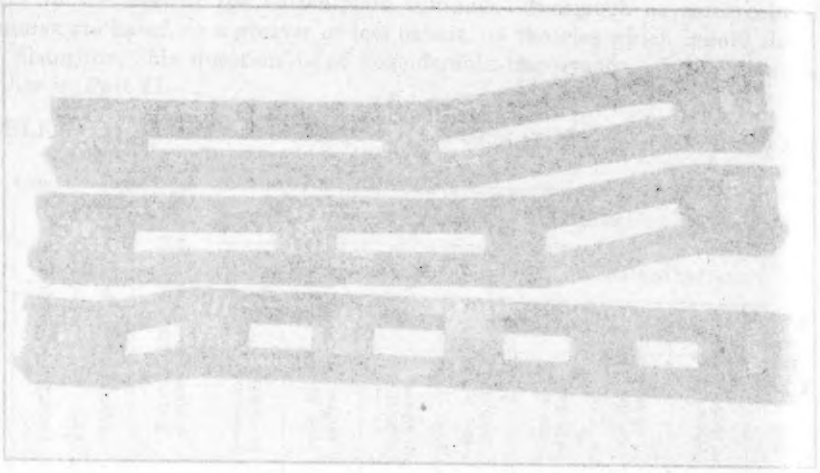
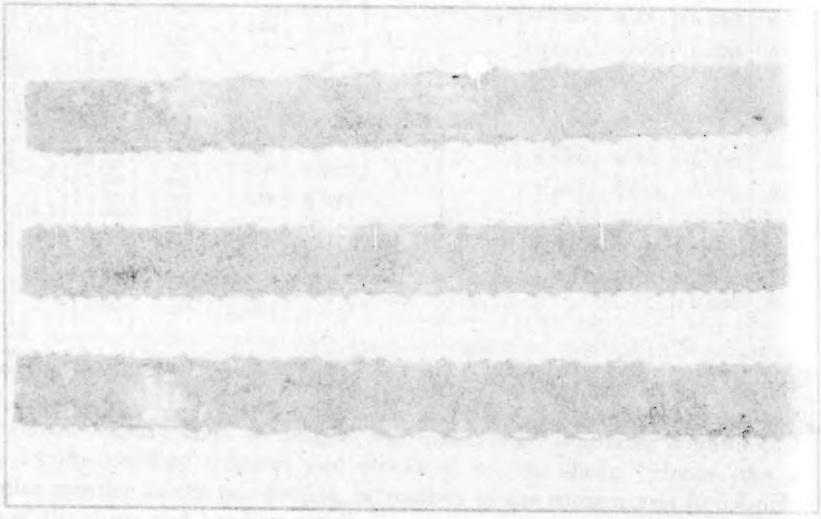


FIG. 12.—FAILURES OF COLUMNS NOS. 4, 5A, AND 6 TESTED UNDER OBLIQUE LOADS. NOTE BUCKLING OF LACING-BARS ON COLUMN NO. 4.





measured. Spirit levels, termed "shear-minus-moment" levels in Fig. 14, attached at the mid-section measured the entire rotation of that section, $\alpha_1$. Then,

$$\alpha_S - \alpha_B = \alpha_1 - \alpha'$$

TABLE 6.—COMPARISON OF OBSERVED AND CALCULATED DEFLECTIONS, CROSS-BENDING TESTS.

| Column No. | Length of span, in inches. | Depth of section, $h$, in inches. | Extreme fiber stress, $f$, in pounds per square inch. | Bending Parallel to Web. | | | Bending Parallel to Lacing Plane. | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Observed deflection, $A$, in inches. | Calculated deflection, $B$, in inches. | Ratio, $\frac{A}{B}$. | Observed deflection, $A$, in inches. | Calculated deflection, $B$, in inches. | Ratio, $\frac{A}{B}$. |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| 1 | 72.0 | 10.38 | 2 950 | 0.0118 | 0.0085 | 1.39 | ..... | ..... | ..... |
| 2 | 84.0 | 9.84 | 3 060 | 0.0147 | 0.0126 | 1.17 | ..... | ..... | ..... |
| 2 *A* | 84.0 | 10.46 | 3 820 | 0.0157 | 0.0148 | 1.06 | ..... | ..... | ..... |
| 4 | 114.0 | 10.00 | 3 060 | 0.0197 | 0.0229 | 0.86 | ..... | ..... | ..... |
| | | 11.90 | 3 120 | ..... | ..... | ..... | 0.0358 | 0.0196 | 1.83 |
| | | 11.90 | 3 120 | ..... | ..... | ..... | 0.0367 | 0.0196 | 1.87 |
| 5 | 114.0 | 10.00 | 3 120 | 0.0203 | 0.0233 | 0.87 | ..... | ..... | ..... |
| | | 12.00 | 3 130 | ..... | ..... | ..... | 0.0406 | 0.0195 | 2.08 |
| | | 12.00 | 3 130 | ..... | ..... | ..... | 0.0364 | 0.0195 | 1.87 |
| 5 *A* | 117.5 | 10.00 | 3 980 | 0.0294 | 0.0315 | 0.93 | ..... | ..... | ..... |
| | | 11.94 | 3 260 | ..... | ..... | ..... | 0.0402 | 0.0216 | 1.86 |
| | | 11.94 | 2 980 | ..... | ..... | ..... | 0.0387 | 0.0198 | 1.95 |
| 5 *B* | 117.5 | 9.99 | 5 940 | 0.0433 | 0.0472 | 0.92 | ..... | ..... | ..... |
| | | 12.03 | 4 920 | ..... | ..... | ..... | 0.0496 | 0.0324 | 1.53 |
| | | 12.03 | 4 920 | ..... | ..... | ..... | 0.0531 | 0.0324 | 1.64 |
| 6 | 114.0 | 10.00 | 3 140 | 0.0209 | 0.0234 | 0.89 | ..... | ..... | ..... |
| | | 11.82 | 3 180 | ..... | ..... | ..... | 0.0306 | 0.0201 | 1.52 |
| | | 11.82 | 3 180 | ..... | ..... | ..... | 0.0320 | 0.0201 | 1.59 |
| 7 | 117.5 | 10.04 | 4 000 | 0.0325 | 0.0316 | 1.03 | ..... | ..... | ..... |
| | | 12.02 | 1 000 | ..... | ..... | ..... | 0.0678 | 0.0066 | 10.28 |
| | | 12.02 | 1 000 | ..... | ..... | ..... | 0.0688 | 0.0066 | 10.41 |
| 8 | 117.5 | 10.04 | 4 000 | 0.0278 | 0.0316 | 0.88 | ..... | ..... | ..... |
| | | 12.00 | 2 010 | ..... | ..... | ..... | 0.0600 | 0.0133 | 4.51 |
| | | 12.00 | 2 020 | ..... | ..... | ..... | 0.0645 | 0.0133 | 4.85 |
| 8 *A* | 117.5 | 10.00 | 6 000 | 0.0468 | 0.0480 | 0.98 | ..... | ..... | ..... |
| | | 11.93 | 1 990 | ..... | ..... | ..... | 0.0628 | 0.0132 | 4.76 |
| | | 11.93 | 1 990 | ..... | ..... | ..... | 0.0628 | 0.0132 | 4.76 |
| 9 | 117.5 | 10.03 | 4 000 | 0.0293 | 0.0316 | 0.93 | ..... | ..... | ..... |
| | | 12.01 | 3 020 | ..... | ..... | ..... | 0.0505 | 0.0199 | 2.54 |
| | | 12.01 | 3 000 | ..... | ..... | ..... | 0.0496 | 0.0198 | 2.50 |

In most of the tests, $\alpha_B$ was calculated from the equation,

$$\alpha_B = \frac{0.05 \dfrac{P\,l^2}{24\,E\,I}}{1 - \dfrac{P\,l^2}{24\,E\,I}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

* From "Modern Framed Structures," Part 2, p. 508, by Johnson, Bryan and Turneaure, the general equation of the elastic curve of such a column is found to be,

$$y = c_1 \sin q\,x + c_2 \cos q\,x - \frac{M_1}{P} - \frac{V_1\,x}{P}$$

in which, $q = \sqrt{\dfrac{P}{E\,I}}$ and $M_1$ and $V_1$ are the moment and shear, respectively, at the end taken as origin. By differentiation the constants may be eliminated and the rotation of the mid-section can be determined from the first derivative of the equation for $y$. By substituting $e = 0.025l$ and $V_1 = 0.05$,

$$\alpha_B = 0.05 - \frac{0.05\,q\,\dfrac{l}{2}}{\sin q\,\dfrac{l}{2}}$$

Expansion of the sine term reduces this to Equation (1).

and the net angular shear distortion, $\alpha_S$, was determined from the equation,

$$\alpha_S = \alpha - \alpha' + \alpha_B$$

In tests on Columns Nos. 2*A* and 5*B* the shear distortion was measured more directly by adding rotations measured by "shear-minus-moment" levels to those measured by the "moment" levels (see Fig. 14).

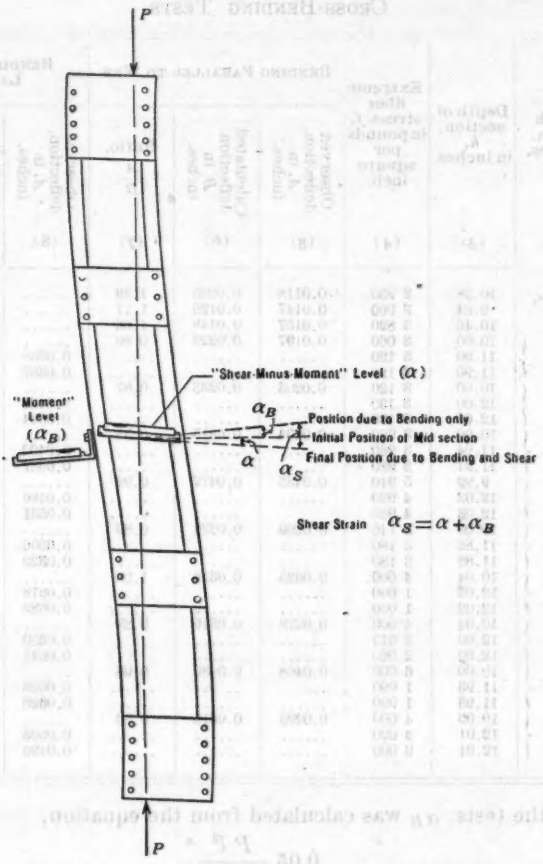



FIG. 14.—METHOD OF MEASURING SHEAR STRAINS IN COLUMNS OF SERIES (*A*).

Typical results of these shear distortion measurements are shown in Fig. 15 for unit loads, $P/A$, up to 15 000 lb. per sq. in. The diagram shows sets after certain loads as well as deformations under load. The solid rolled and riveted sections show little shear distortion and negligible set; the laced channel columns, considerable shear distortion and relatively large sets; and the batten-plate columns, very large distortions and sets.

In the case of the batten-plate columns the shear distortion is, in part, local bending of channels between plates, but the result is a measure of shear flexibility as a whole and of set. The superior rigidity of the solid and riveted sections is marked.

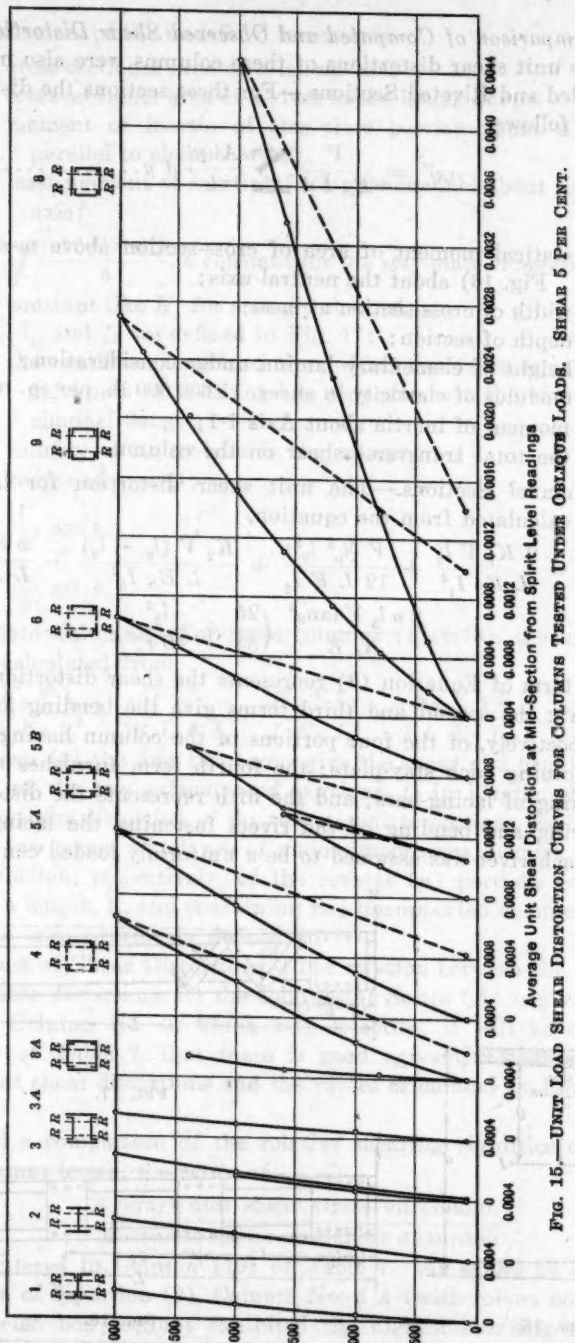



FIG. 15.—UNIT LOAD SHEAR DISTORTION CURVES FOR COLUMNS TESTED UNDER OBLIQUE LOADS. SHEAR 5 PER CENT.

*8(a).—Comparison of Computed and Observed Shear Distortions.*— Calculations of the unit shear distortions of these columns were also made.

Solid Rolled and Riveted Sections.—For these sections the distortions were computed as follows:

$$\alpha_S'' = \frac{V}{E_S I d} \sum \frac{A' y'}{b} d y \quad \ldots\ldots (2)$$

in which,

$A' y'$ = statical moment of area of cross-section above $m$-$m$ (shaded in Fig. 16) about the neutral axis;
$b$ = width of cross-section at $m$-$m$;
$d$ = depth of section;
$d y$ = height of elementary lamina under consideration;
$E_S$ = modulus of elasticity in shear, 11 600 000 lb. per sq. in.;
$I$ = moment of inertia about Axis 1-1;
$V$ = the total transverse shear on the column.

Laced Channel Sections.—The unit shear distortion for laced channel sections was calculated from the equation,

$$\alpha_S'' = \frac{2 K_1 V l_1}{L E_S I_1^2} + \frac{V (l_0^3 l_2^3)}{12 L E I_2} + \frac{K_2 V (l_0 + l_2)}{L E_S I_2^2} + \frac{n S u l}{L A_1 E} + \frac{2 n l_3 V \tan_\theta^2}{A_3 E} \left(\frac{25}{54} + \frac{l_3^2}{40 r_3^2}\right) \quad \ldots\ldots (3)$$

The first term of Equation (3) represents the shear distortion of the stay-plate portion*; the second and third terms give the bending and shear distortions, respectively, of the four portions of the column having unsupported channels adjoining each stay-plate; the fourth term furnishes the distortion due to straining of lacing-bars; and the fifth represents the distortion caused by the shearing and bending of the rivets fastening the lacing-bars to the channels. Each rivet was assumed to be a uniformly loaded cantilever.

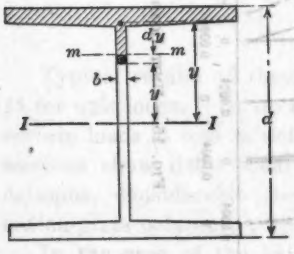



FIG. 16.

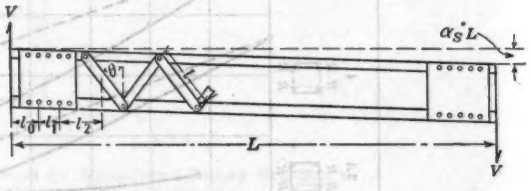



FIG. 17.

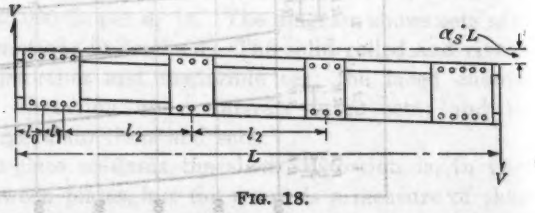



FIG. 18.

* The derivation of the general expression for shear deflection may be found in "Strength of Materials," by Maurer and Withey, p. 322.

The significances of the undefined terms are:

$A_1$ = cross-sectional area of a lacing-bar;
$A_3$ = cross-sectional area of a rivet in the lacing-bar;
$I_1$ = moment of inertia of stay-plate portion about a gravity axis parallel to channel webs;
$I_2$ = least moment of inertia of a single channel about its own gravity axis;
$K_1 = \int \frac{(A' y')^2 \, d y}{b}$, a constant for the stay-plate cross-section;
$K_2$ = constant like $K_1$ for a single channel;
$l$, $l_0$, $l_1$, $l_2$, and $L$ are defined in Fig. 17;
$l_3$ = the cantilever length of rivet which was taken equal to twice the thickness of a lacing-bar plus one-half the rivet grip in the channel flange;
$n$ = number of lacing-bars in the column;
$r_3$ = radius of a rivet;
$S = \frac{V}{2} \sec \theta$;
$u = \frac{1}{2} \sec \theta$.

Batten-Plate Columns.—For these columns (Fig. 18), the unit shear distortion was calculated from,

$$\alpha_S'' = \frac{K_2 V l_0}{L E_S I_2^2} + \frac{V l_1^3}{12 L E I_2} + \frac{K_1 V l_1}{L E_S I_1^2} + \frac{K_2 V n l_2}{2 L E_S I_2^2} + \frac{V n l_2^3}{24 L E I_2} \quad \ldots (4)$$

In Equation (4) the first two terms give the shear and bending distortion, respectively, of the two extreme end portions, $l_0$ in length; the third term furnishes the shear distortion of the two stay-plate portions, each $l_1$ in length; and the last two terms, which are of major importance, furnish the shear and bending distortion, respectively, of the several ($n$) portions of the column, each having a length, $l_2$, and containing two unsupported channels. The other symbols have been previously defined.

Comparison of Shear Distortions.—The relation between the measured and calculated shear distortions for the columns of Series ($A$) is given in Table 7. Aside from Column No. 3, which was defective, it will be observed from Column (9) of Table 7, that there is good agreement between the elastic measured unit shear distortions and the values calculated by Equations (1) to (4).

To afford a comparison of the relative shearing rigidities of the various types of columns tested, the ratios of,

$$\frac{\text{Average unit shear stress on column}}{\text{Measured elastic unit shear distortion}}$$

have been entered in Column (10) of Table 7. As would be expected from consideration of Equation (2), Column No. 8 $A$ (with rollers normal to webs) and the riveted box sections exhibited the highest shearing rigidity. The superior shearing rigidity of the laced-channel columns over the batten-plate columns is also noteworthy.

TABLE 7.—COMPARISON OF THE SHEARING RIGIDITIES OF THE COLUMNS OF SERIES (*A*):

(Obliquity of loading, 5%; $E_s$ = 11 600 000 lb. per sq. in. Columns Nos. 1, 2, 3, 4, and 6 had been subjected to similar loading prior to the runs herein recorded.)

| Column No. | Direction of rollers. | Applied load, in pounds. | Average unit shear on column, in pounds per square inch, ($S$). | Modulus of elasticity, 1 000 000 lb. per sq. in. | Average Unit Shear Distortion. | | | | Relative rigidity, $\left(\frac{S}{10\,000\ a_s'}\right)$. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Measured. | | Calculated, ($a_s''$). | Ratio, $\left(\frac{a_s'}{a_s''}\right)$. | |
| | | | | | Total, ($a_s$). | Elastic, ($a_s'$). | | | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| 1 | | 192 800 | 730 | 28.2 | 0.000188 | 0.000188 | 0.000155 | 1.21 | 388 |
| 2 | | 207 200 | 783 | 29.7 | 0.000241 | 0.000241 | 0.000253 | 0.95 | 304 |
| 2 *A* | | 220 000 | 748 | 29.0 | 0.000279* | 0.000271* | 0.000258 | 1.05 | 276 |
| 3 | | 215 000 | 734 | 28.6 | 0.000277† | 0.000277† | 0.000161 | 1.72 | 264 |
| 3 *A* | | 211 450 | 734 | 27.4 | 0.000188‡ | 0.000188‡ | 0.000190‡ | 0.99 | 390 |
| 3 *A* | | 211 450 | 734 | 27.4 | 0.000157 | 0.000157 | 0.000161 | 0.98 | 467 |
| 8 *A* | | 130 600 | 578 | 29.9 | 0.000072 | 0.000072 | 0.000072 | 1.00 | 803 |

* Measured with "moment" level apparatus.
† Column No. 8 had a wind of approximately 3½ degrees.
‡ Between roots of flanges.

‡ Between roots of flanges.



TABLE 7.—(*Continued.*)

| Column No. | Direction of rollers. | Applied load, in pounds. | Average unit shear on column, in pounds per square inch, ($S$). | Modulus of elasticity, 1 000 000 lb. per sq. in. | Average Unit Shear Distortion. | | | | Relative rigidity, $\left(\frac{S}{10\,000\ a_s'}\right)$. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Measured. | | Calculated, ($a_s''$). | Ratio, $\left(\frac{a_s'}{a_s''}\right)$. | |
| | | | | | Total, ($a_s$). | Elastic, ($a_s'$). | | | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| 4 | | 167 800 | 728 | 29.0 | 0.001046 | 0.000863 | 0.000848 | 1.02 | 84 |
| 5 *A* | | 169 300 | 728 | 29.0 | 0.001232 | 0.000784 | 0.000702 | 1.12 | 93 |
| 5 *B* | | 131 320 | 577 | 29.4 | 0.001022* | 0.000489* | 0.000555 | 0.88 | 118 |
| 5 *B* | | 131 320 | 577 | 29.4 | 0.000731 | 0.000478 | 0.000555 | 0.86 | 121 |
| 5 *B* | | 40 400 | 177 | 29.4 | 0.000161 | 0.000141 | 0.000171 | 0.82 | 125 |
| 6 | | 164 500 | 728 | 29.0 | 0.001044 | 0.000644 | 0.000509 | 1.26 | 113 |
| 7 | | 89 700 | 379 | 29.0 | 0.007899 | 0.005229 | 0.004740 | 1.10 | 7 |
| 8 | | 113 400 | 479 | 29.0 | 0.004448 | 0.003041 | 0.002770 | 1.10 | 16 |
| 8 *A* | | 96 700 | 428 | 29.9 | 0.003521 | 0.002611 | 0.002450 | 1.07 | 16 |
| 8 *A* | | 40 200 | 178 | 29.9 | 0.001483 | 0.001073 | 0.001020 | 1.02 | 17 |
| 9 | | 173 200 | 730 | 29.0 | 0.002711 | 0.001734 | 0.001610 | 1.08 | 42 |

* Measured with "moment" level apparatus.

*9.—Stresses in Lacing.*—The tests on laced-channel columns afforded a good opportunity to examine the stresses in the lacing-bars. These were determined both by calculation* and by strain measurement for the central and also for the oblique loading tests. At first, the strain measurements were made on the median lines of the outer faces of the bars using 6 or 8-in. gauge lengths. Later, more accurate measurements were made along the median lines of the upper and lower edges of the bars, using 2-in. gauge lengths. Unless otherwise indicated, the calculations herein reported are based on strains measured by the later method. Column (10) of Table 8 shows that for oblique loading and values of $P/A$ of 14 000 lb. per sq. in., or higher, the ratio,

$$\frac{\text{Average stress in lacing-bars calculated from strain}}{\text{Average stress in lacing-bars calculated from load}}$$

is 21 to 24% less than unity. It is of interest to note that the calculated maximum "load" stress, Column (8) of Table 8, at which the thin bars of Column No. 4 buckled, was practically the same as the average ultimate load for three similar lacing-bars tested singly as columns (Column (11) Table 8).*

TABLE 8.—AVERAGE UNIT STRESSES IN LACING-BARS OF COLUMNS NOS. 4, 5, 5A, AND 6, UNDER OBLIQUE LOADS.

(Strains measured on 28 to 32 bars in each column.)

| Column No. | Nominal Size of Bars, in Inches. Length. | Width. | Thickness. | $\frac{l}{r}$. | Load on column, in pounds. | Shear on column, in pounds. | Average Unit Stress in Lacing-Bars Calculated from Loads, in pounds per square inch, $A$. | Average Unit Stress in Lacing-Bars Calculated from Strains, in pounds per square inch, $B$. | Ratio $\frac{B}{A}$ | Average ultimate compressive strength of four bars, in pounds per square inch. |
|---|---|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) |
| 4 | 10¼ | 2¼ | 5/16 | 188 | 167 800 | 8 390 | 11 260 | 9 815 | 0.87 | ..... |
| | | | | | 202 200 | 10 110 | 13 550 | 11 920 | 0.88 | ..... |
| | | | | | *262 200 | 13 110 | **17 600** | † | .... | **17 200** |
| 5 | 10¼ | 2½ | ¼ | 141 | *258 700 | 12 940 | 12 100 | ..... | .... | 19 000 |
| 5A | 10¼ | 2¼ | ¼ | 138 | 169 300 | 3 390 | 3 370 | 2 810 | 0.83 | ..... |
| | | | | | 169 300 | 8 465 | 8 410 | 7 300 | 0.87 | ..... |
| | | | | | 227 400 | 11 370 | 11 280 | 9 825 | 0.87 | ..... |
| | | | | | *259 000 | 12 950 | 12 860 | ..... | .... | 19 800 |
| 6 | 10¼ | 2½ | ⅜ | 95 | 164 500 | 8 225 | 4 930 | 3 745 | 0.76 | ..... |
| | | | | | 221 000 | 11 050 | 6 620 | 5 250 | 0.79 | ..... |
| | | | | | *271 500 | 13 580 | 8 120 | ..... | .... | 20 800 |

* Load at failure of column.
† Bars buckled.

Owing to imperfections in the connections and irregularities in the lacing-bars, the observed average strain for the lacing-bars varied considerably. If one considers variation among the tension bars, or among compression bars, the data in Table 9 indicate that the variation was least among the thin bars of Column No. 4; also, that the stress variation between bars became less as the loads increased. In the test of Column No. 4 the bars carrying

* See Equation (32), p. 447.

compression were the outer bars on both sides of the column; in Columns Nos. 5*A* and 6, the bars carrying compression were the outer bars on one side and the inner bars on the other side of the columns. Comparing the tabulated information for Column No. 4 with that for the other columns, it appears from Column (7) of the lower part of Table 9 that a more equal distribution of stress between the inner and the outer bars was obtained for Columns Nos. 5*A* and 6 than for Column No. 4.

TABLE 9.—VARIATION IN OBSERVED STRESSES IN LACING-BARS OF COLUMNS NOS. 4, 5*A*, AND 6 UNDER OBLIQUE LOADS.

(Strains measured on 28 to 33 bars in each column.)

| Column No. | Load on column, in pounds. | Obliquity of load, percentage. | Unit Stresses in Bars from Strains. Kind. | Minimum. | Maximum. | Average. | Range from Average, Percentage. Below. | Above. |
|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| 4 | 167 800 | 5 | Tension | 9 080 | 15 130 | 11 280 | 20 | 34 |
| | | | Compression | 6 040 | 10 700 | 8 350 | 28 | 28 |
| | 202 200 | 5 | Tension | 10 000 | 16 270 | 12 640 | 21 | 29 |
| | | | Compression | 8 610 | 13 480 | 11 200 | 23 | 20 |
| | 202 200 | 5 | Both | 8 610 | 16 270 | 11 920 | 28 | 36 |
| 5*A* | 169 300 | 2 | Tension | 700 | 3 940 | 2 370 | 71 | 66 |
| | | | Compression | 1 160 | 5 100 | 3 250 | 64 | 57 |
| | 169 300 | 5 | Tension | 5 800 | 8 590 | 7 180 | 19 | 20 |
| | | | Compression | 5 840 | 9 060 | 7 420 | 28 | 22 |
| | 227 400 | 5 | Tension | 7 680 | 13 250 | 9 570 | 20 | 38 |
| | | | Compression | 8 820 | 12 090 | 10 080 | 12 | 20 |
| | 227 400 | 5 | Both | 7 680 | 13 250 | 9 825 | 22 | 35 |
| 6 | 164 500 | 5 | Tension | 3 250 | 8 150 | 4 790 | 32 | 70 |
| | | | Compression | 1 160 | 4 640 | 2 700 | 57 | 72 |
| | 221 000 | 5 | Tension | 3 710 | 8 850 | 5 400 | 31 | 64 |
| | | | Compression | 3 020 | 8 150 | 5 100 | 41 | 60 |
| | 221 000 | 5 | Both | 3 020 | 8 850 | 5 250 | 42 | 69 |
| | | | Position of bar | | | | | |
| 4 | 202 000 | 5 | Inner | 10 000 | 16 270 | 12 460 | 21 | 29 |
| | | | Outer | 8 610 | 13 480 | 11 200 | 23 | 20 |
| 5*A* | 169 300 | 2 | Inner | 700 | 5 100 | 2 600 | 73 | 96 |
| | | | Outer | 1 160 | 3 940 | 3 020 | 62 | 30 |
| | 227 400 | 5 | Inner | 7 680 | 11 850 | 9 720 | 21 | 22 |
| | | | Outer | 7 690 | 13 250 | 9 930 | 23 | 33 |
| 6 | 221 000 | 5 | Inner | 3 020 | 7 690 | 5 130 | 41 | 50 |
| | | | Outer | 3 710 | 8 850 | 5 370 | 31 | 65 |

Table 10, Column (5), shows that for the high column loads the ratio of:

$$\frac{\text{Average "strain" stress in the most strained lacing-bar}}{\text{Average stress calculated from the load}}$$

in a certain tension bar of Column No. 6* reached the high value of 1.65. For compression bars, however, the ratios in Column (6), Table 10, are considerably less. The average "strain" stress in lacing-bars was also calculated for the lacing-bar panel, in which the two most highly strained bars were found. The ratio:

$$\frac{\text{Average "strain" stress in the lacing of the most strained panel}}{\text{Average "load" stress in the lacing}}$$

* It should be noted that the strains in the heavy bars of Column No. 6 were considerably smaller than those in the bars of the other columns, and the errors in measurement were proportionately greater.

shown in Column (7), Table 10, indicates that in the worst case one panel of Column No. 6* exhibited a ratio of 1.3; for the other columns, the ratio was less than 1.10.

TABLE 10.—COMPARISON OF STRESSES IN LACING-BARS CALCULATED FROM LOAD WITH STRESSES CALCULATED FROM STRAIN COLUMNS NOS. 4, 5*A*, AND 6.

(Strains measured on 28 bars in each column.)

| Column No. | Load on column, in pounds. | Obliquity of load, percentage. | Average unit stress in lacing-bars from Load (*A*), in pounds per square inch. | Ratio, Strain Stress / Load Stress (*A*) for: Bar with Most Strain Considering: All bars. | Bar with Most Strain Considering: Compression bars only. | Panel with most strain. |
|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 4 | 167 800 | 5 | 11 260 | 1.34 | 0.95 | 1.02 |
| | 202 200 | 5 | 13 550 | 1.20 | 0.99 | 1.00 |
| 5*A* | 169 300 | 2 | 3 370 | 1.51 | 1.51 | 1.10 |
| | 169 300 | 5 | 8 410 | 1.08 | 1.08 | 0.94 |
| | 227 400 | 5 | 11 280 | 1.17 | 1.07 | 1.00 |
| 6 | 164 500 | 5 | 4 930 | 1.65 | 0.94 | 1.30 |
| | 221 000 | 5 | 6 620 | 1.34 | 1.23 | 1.28 |

*10.—Tests of Individual Lacing-Bars.*—Compression tests were made on twenty individual lacing-bars, 10½ in. long, center to center, loaded in the manner indicated in Fig. 19. Measurements of crookedness at mid-length were made on a number of these bars and the results are recorded in Columns (6) and (7), Table 11. In all cases save one, the bars which were appreciably bowed outward, away from the channels, showed lower strength.

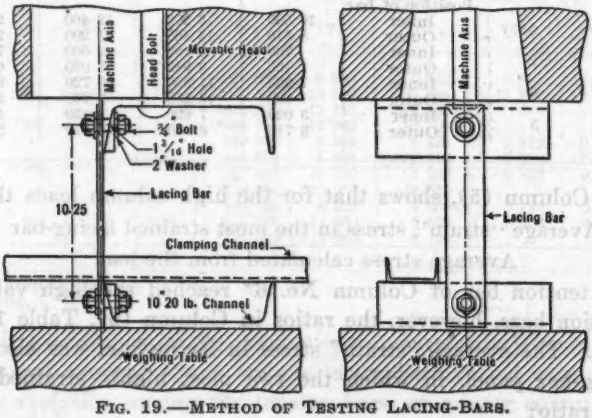



FIG. 19.—METHOD OF TESTING LACING-BARS.

To secure evidence concerning the magnitude of eccentricities in lacing, Bars 27*A*, 52*A*, and 61*A* were tested with Martens mirror extensometers

* It should be noted that the strains in the heavy bars of Column No. 6 were considerably smaller than those in the bars of the other columns, and the errors in measurement were proportionately greater.

attached to the middle 2 in. of each broad face. Also, in the column tests, measurements were made on the front faces as well as the top and bottom faces of fourteen bars in Column No. 4, and of ten bars each in Columns Nos. 5*A* and 6. From data obtained in these two types of test, Fig. 20 has been prepared. Apparently with the same diameter of bolt or rivet, the eccentricity is influenced markedly by the thickness of the bar. The eccentricities found in the bars tested separately under loads approximating 9 000 lb. per sq. in. were somewhat higher than the mean for the outer bars in the column tests. The lower eccentricities of the inner compression bars in the column tests are noteworthy. This evidence indicates that columns laced on two sides will have somewhat more uniform shearing strength and rigidity if the inner bars are sloped in opposite directions on the two sides.

TABLE 11.—RESULTS OF COMPRESSION TESTS ON LACING-BARS.

(Arrangement for testing shown in Fig. 18. Speed of movable head of machine was 0.06 in. per min. All specimens buckled outward at failure.)

| Bar No. | Dimensions, in Inches. | | Slenderness ratio, $\left(\frac{l}{r}\right)$. | Area, in square inches. | Bowed at Middle, in Inches. | | Maximum load, in pounds. | Maximum unit load, in pounds per square inch. |
|---|---|---|---|---|---|---|---|---|
| | Width. | Thickness. | | | In. | Out. | | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| 47*A* | 2.24 | 0.189 | 188 | 0.423 | * | ..... | 7 000 | 16 500 |
| 47*B* | 2.28 | 0.189 | 188 | 0.431 | ..... | 0.021 | 6 380 | 14 800 |
| 48*A* | 2.27 | 0.187 | 190 | 0.425 | * | ..... | 8 000 | 18 800 |
| 48*B* | 2.26 | 0.186 | 191 | 0.421 | * | ..... | 7 930 | 18 800 |
| Average... | .... | ..... | .... | ..... | ..... | ..... | ..... | 17 200 |
| 51*A* | 2.26 | 0 246 | 144 | 0.556 | * | ..... | 12 000 | 21 600 |
| 51*B* | 2.26 | 0.247 | 143 | 0.558 | * | ..... | 10 360 | 18 600 |
| 52*A* | 2.24 | 0.254 | 140 | 0.570 | * | ..... | 10 560 | 18 500 |
| 52*B* | 2.20 | 0.257 | 138 | 0.566 | 0.005 | ..... | 9 810 | 17 300 |
| 53*A* | 2.20 | 0.257 | 138 | 0.566 | ..... | 0.014 | 11 040 | 19 500 |
| 53*B* | 2.24 | 0.256 | 138 | 0.574 | ..... | 0.023 | 10 550 | 18 400 |
| 54*A* | 2.22 | 0.255 | 139 | 0.566 | 0.006 | ..... | 11 730 | 20 700 |
| 54*B* | 2.23 | 0.256 | 138 | 0.571 | ..... | 0.039 | 10 790 | 18 900 |
| 55*A* | 2.24 | 0.250 | 142 | 0.560 | 0.006 | ..... | 11 780 | 21 000 |
| 55*B* | 2.24 | 0.254 | 140 | 0.569 | ..... | 0.040 | 9 950 | 17 500 |
| 56*A* | 2.26 | 0.247 | 143 | 0.558 | ..... | 0.024 | 10 370 | 18 600 |
| 56*B* | 2.26 | 0.248 | 143 | 0.560 | 0.012 | ..... | 13 550 | 24 200 |
| Average... | .... | ..... | .... | ..... | ..... | ..... | ..... | 19 600 |
| 61*A* | 2.28 | 0.374 | 95 | 0.853 | * | ..... | 17 000 | 19 900 |
| 61*B* | 2.26 | 0.373 | 95 | 0.844 | ..... | 0.014 | 17 330 | 20 500 |
| 62*A* | 2.28 | 0.374 | 95 | 0.853 | * | ..... | 17 820 | 20 900 |
| 62*B* | 2.28 | 0.374 | 95 | 0.853 | * | ..... | 18 740 | 22 000 |
| Average... | .... | ..... | .... | ..... | ..... | ..... | ..... | 20 800 |

* No measurements made.

The maximum unit loads from the individual compression tests of lacing-bars have been plotted against corresponding slenderness ratios in Fig. 21. It will be observed that the plotted points lie well above the line representing

the Talbot and Moore equation, $P/A = 21\,400 - 45\,\frac{l}{r}$.* It is likely that the discrepancy is partly due to difference in size of fastening bolts. Bolts ½ in. in diameter were used by Talbot and Moore; in the Wisconsin tests they were ⅞ in. in size.

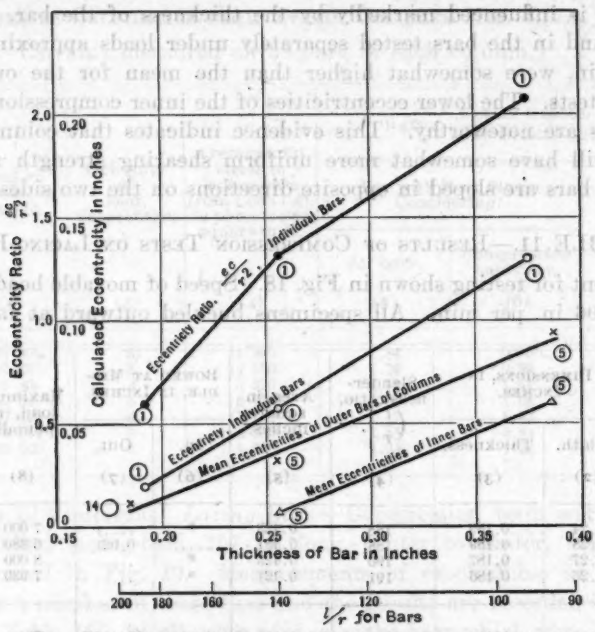



FIG. 20.—ECCENTRICITIES IN LACING-BARS UNDER COMPRESSION.

From the data now available, it appears that compression tests on individual lacing-bars yield safe results as a basis for design. More tests of this nature are much needed to supplement the meager fund of information.

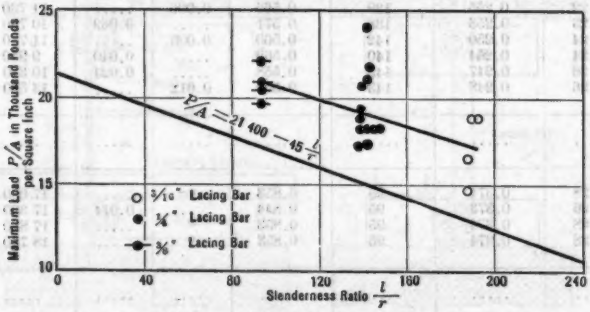

Maximum Load $P/A$ in Thousand Pounds per Square Inch
$P/A = 21\,400 - 45\,\frac{l}{r}$
5/16" Lacing Bar
¼" Lacing Bar
⅜" Lacing Bar
Slenderness Ratio $\frac{l}{r}$

FIG. 21.—COMPRESSIVE STRENGTHS OF INDIVIDUAL LACING-BARS.

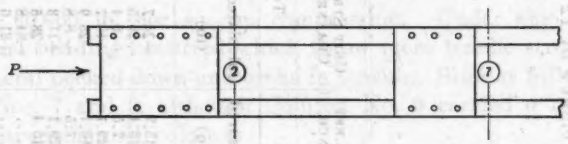
*11.—Stresses in Channels of Batten-Plate Columns.*—In the tests of the batten-plate columns, strains over 2-in. gauge lengths were measured on the

* See *Transactions*, Am. Soc. C. E., Vol. LXV (1909), p. 238.

outer fibers of the channel webs and toes at sections close to the lower stays and battens, and in Column No. 8*A*, in the middle of the unsupported channel lengths of the lower panel. The data from the tests made with central loading are assembled in Table 12. These results show that the fiber stress variations for this condition are not unduly large, but are comparable with variations found in the solid rolled, riveted, and laced columns.

TABLE 12.—STRAIN VARIATIONS IN CHANNELS OF BATTEN-PLATE COLUMNS UNDER CENTER LOAD.

$\left(\text{Increment in } \frac{P}{A} = 14\,580 \text{ lb. per sq. in.}\right)$




| Column No. | Section. | Channel. | Unit Strain in 0.00001. Heel. Front. | Heel. Rear. | Toe. Front. | Toe. Rear. | Axis of column. | Ratio: Bending Strain / Axial Strain × 100. Average. | Maximum.* |
|---|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| 7 | 2 | Left | 52.50 | 43.75 | 56.43* | 41.16 | 48.0 | 11.0 | 17.6 |
| | | Right | 46.25 | 45.00 | 54.11 | 55.48 | | | |
| | 7 | Left | 44.74 | 52.11 | 48.27 | 34.47 | 48.6 | 10.8 | 29.9 |
| | | Right | 50.74 | 46.34 | 63.12 | 49.88 | | | |
| 8 | 2 | Left | 47.87 | 55.12* | 53.14 | 42.74 | 50.8 | 6.3 | 8.6 |
| | | Right | 47.87 | 52.98 | 53.14 | 51.27 | | | |
| | 7 | Left | 46.13 | 50.00 | 54.90* | 44.66 | 49.5 | 7.6 | 11.0 |
| | | Right | 51.78 | 51.22 | 41.23 | 52.98 | | | |
| 9 | 2 | Left | 62.66* | 39.67 | 59.14 | 48.51 | 53.0 | 10.2 | 18.3 |
| | | Right | 57.86 | 51.30 | 54.34 | 51.30 | | | |
| | 7 | Left | 61.22 | 56.50 | 54.18 | 47.71 | 54.4 | 9.0 | 18.9 |
| | | Right | 50.66 | 49.73 | 64.74* | 49.73 | | | |

* Maximum stresses.

When the batten-plate columns were subjected to 5% shear loadings, the bending stresses in the unsupported lengths of channel became large. Table 13 shows some of the fiber stresses calculated from strains and from loads at the more important sections. In calculating the fiber stresses for the end panels, the results from two of the methods tried are presented. Each channel between plates was assumed to act as a fixed-end beam carrying axial end loads. From other calculations of fiber stress and from calculations for shear distortion, it appeared that the unsupported length of channel should be from the second rivet of the stay-plate to the center rivet of the batten-plate, as indicated by $l_c$ in the diagram of Table 13. In Method (*a*), Table 13, the inflection points of the channels were assumed to be at the middle of $l_c$. Strain measurements of Column No. 8*A* showed that the inflection point was nearer the first batten. Since the superior restraining influence of the stays over the battens was responsible for this condition, it was assumed in Method

TABLE 13.—COMPARISON OF CALCULATED AND MEASURED STRESSES IN CHANNEL FLANGES OF BATTEN-PLATE COLUMNS OF SERIES (*A*) UNDER 5 PER CENT. SHEAR.

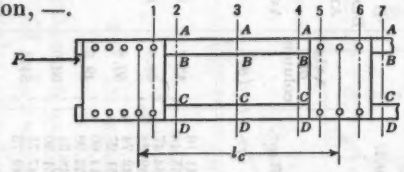
Column Data:

Length = 10 ft.

Material = two 10-in., 20-lb. channels.
four 12 by 13 by ¼-in. splice-plates.

Column No. 7, two 9 by 12 by ¼-in. batten-plates.
" Nos. 8 and 8*A*, four 9 by 12 by ¼-in. batten-plates.
" No. 9, eight 9 by 12 by ¼-in. batten-plates.

Gauge length, 2 in.; Tension, +; Compression, —.




| Column No. | Section considered and assumption.* | Unsupported length of channel, in inches. | Applied load, *P*, in 1 000 lb. | Moment on one channel at section, in inch-pounds. | Unit Stresses on Extreme Fibers of Left Channel in 1 000 Pounds per Square Inch. | | | | Unit Stresses on Extreme Fibers of Right Channel in 1 000 Pounds per Square Inch. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Heel *A*. | | Toe *B*. | | Toe *C*. | | Heel *D*. | |
| | | | | | Calculated. | Measured. | Calculated. | Measured. | Calculated. | Measured. | Calculated. | Measured. |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) |
| 7 | 2*b* | 49⅝ | 66.0 | −0.401 *P* | −12.4 | −13.2 | +13.5 | +15.7 | −24.7 | −26.0 | + 1.2 | + 1.3 |
| 8*A* | 2*a* | 32¾ | 96.7 | −0.269 *P* | −16.9 | −17.7 | + 9.4 | +13.1 | −26.5 | −30.2 | − 0.2 | + 1.9 |
| | 3*a* | 32¾ | 96.7 | −0.006 *P* | −11.3 | −12.6 | −10.7 | − 7.5 | − 6.4 | −10.2 | − 5.8 | − 5.4 |
| | 4*a* | 32¾ | 96.7 | +0.256 *P* | − 5.7 | − 4.2 | −30.8 | −29.1 | −13.7 | +10.2 | −11.4 | −12.1 |
| | 2*b* | 32¾ | 96.7 | −0.298 *P* | −17.4 | −17.7 | +11.8 | +13.1 | −28.9 | −30.2 | + 0.3 | + 1.9 |
| | 3*b* | 32¾ | 96.7 | −0.036 *P* | −11.8 | −12.6 | − 8.3 | − 7.5 | − 8.8 | −10.2 | − 5.3 | − 5.4 |
| | 4*b* | 32¾ | 96.7 | +0.227 *P* | − 6.3 | − 4.2 | −28.4 | −29.1 | +11.3 | +10.2 | −10.9 | −12.1 |
| | 7*c* | 33¼ | 96.7 | −0.260 *P* | −14.1 | −14.7 | +11.4 | +15.4 | −28.5 | −29.8 | − 3.0 | + 0.2 |
| | 1*b* | 32¾ | 96.7 | −0.376 *P* | −19.3 | ..... | −19.1 | ..... | −34.8 | ..... | + 2.0 | ..... |
| | 5*b* | 32¾ | 96.7 | +0.305 *P* | − 4.6 | ..... | −34.5 | ..... | +21.6 | ..... | −12.8 | ..... |
| 8 | 1*b* | 32¾ | 117.5† | −0.376 *P* | −34.0 | ..... | +31.3 | ..... | −60.0 | ..... | + 3.6 | ..... |
| 9 | 2*b* | 19½ | 137.6 | −0.124 *P* | −19.4 | −21.8 | − 2.5 | + 2.7 | −20.6 | −23.2 | − 3.8 | − 3.1 |

* (*a*) Point of inflection at middle of unsupported length ($l_c$).
(*b*) Point of inflection 0.586 of unsupported length ($l_c$) from second rivet of stay-plate.
(*c*) Point of inflection midway between center rivets of adjacent battens.

† Maximum load.

($b$), Table 13, that the segments of the unsupported length of the top and bottom panels were directly proportional to the number of rivets per inch of adjoining plate. Hence, the length of segment adjacent to the stay-plates $= \frac{\frac{5}{13}}{\frac{5}{13} + \frac{3}{9}} \cdot l_c = 0.536\ l_c$. If one compares the figures in bold-faced type, Table 13, it is evident that stresses calculated by Methods ($b$) and ($c$) check closely the strain stresses for these highly stressed fibers. To investigate points of greatest fiber stress, calculations at maximum loads by Method ($b$) were made for Sections 1 and 5 through rivets, also by Method ($c$) for Section 6. At the flange toes ($B$) of Section 1, the calculated fiber stress for Column No. 8 is 31 300 lb. per sq. in. tension and at Toes $C$ of the same section, 60 000 lb. per sq. in. compression. Under the latter stress, excessive lateral bending occurred which threw more tensile stress on Toe $B$. Finally, the metal necked down and broke in tension. Similar failures occurred in Columns Nos. 7 and 9, although Column No. 9 carried a load nearly as large as the laced-channel columns.

The detailed calculation for stresses at Points $A$, $B$, $C$, and $D$ in Section 2 of Column No. 8$A$ by Method ($b$) were made as follows: Each channel was assumed to carry one-half the transverse shear and one-half the bending moment (Fig. 22). Then, the distance from the inflection point to Section 2 is (32.75 × 0.536) — 5.625 = 11.92 in.

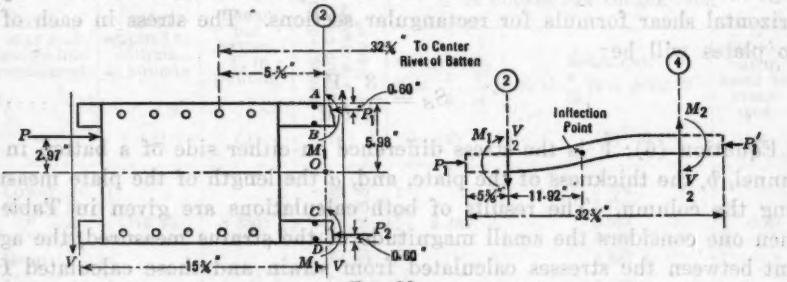



FIG. 22.

Then,

$$M_1 = 11.92\ \frac{V}{2} = 0.298\ P$$

$$\Sigma M_0 = 2.97\ P - (0.05\ P \times 15.75) - 5.38\ (P_1 - P_2) - 0.596\ P = 0$$

Since $P = P_1 + P_2$, $P_1 = 0.647\ P$ and $P_2 = 0.353\ P$.

Using $p = \frac{P}{A} + \frac{M\,c}{I}$ to find the unit fiber stress, $p$, $A = 5.65$ sq. in.; for the heel, $\frac{c}{I} = 0.220$; and for the toe, $\frac{c}{I} = 0.792$. Hence,

$$p_A = -\frac{0.647\ P}{5.65} - (0.298\ P \times 0.22) = -0.180\ P$$

$$p_B = -\frac{0.647\ P}{5.65} + (0.298\ P \times 0.792) = +0.122\ P$$

$$p_C = -\frac{0.353\,P}{5.65} - (0.298\,P \times 0.792) = -0.299\,P$$

$$p_D = -\frac{0.353\,P}{5.65} - (0.298\,P \times 0.22) = +0.003\,P$$

Substituting $P = 96\,700$ lb., the stresses given in Table 12 were obtained.

*12.—Shear Stress in Batten-Plates.*—Strain measurements were made on the stay and batten-plates of Columns Nos. 7 to 9; also on the stay-plates of Columns Nos. 4, 5*B*, and 6, as indicated in Figs. 4 to 7. It was found, however, that the strains on the outer faces of these plates were much larger than the average for the considered gauge line. Hence, in order to secure average strains, measurements on both sides of every plate had to be made. Although difficult, such measurements were made with the 2-in. Olsen gauge on the lower battens and stays of Columns Nos. 8 and 8*A*.

Plate measurements parallel and transverse to the axis of the column showed that these strains were insignificant. The strains on the 45° diagonals in these plates, however, were appreciable. From strains the shear stress in the plates was calculated, assuming Poisson's ratio to be 0.25, by,

$$S_S = \frac{4}{15}(4\,\varepsilon_1 + \varepsilon_2)\,E \qquad (5)$$

In this expression, $\varepsilon_1$ is the unit strain in one diagonal, and $\varepsilon_2$, that in the diagonal normal to it. The shear stress in the plates was also calculated by the horizontal shear formula for rectangular sections. The stress in each of the two plates will be:

$$S_S = \frac{3}{4}\frac{V}{b\,d} \qquad (6)$$

In Equation (6), $V$ is the stress difference on either side of a batten in one channel, $b$, the thickness of the plate, and, $d$ the length of the plate measured along the column. The results of both calculations are given in Table 14. When one considers the small magnitude of the strains measured, the agreement between the stresses calculated from strain and those calculated from shear is very satisfactory. Hence, the designer may safely use the horizontal shear formula (Equation (6)) to determine the shearing stresses in the battens of such columns.

## Tests of Series (*B*): Eccentric Loads

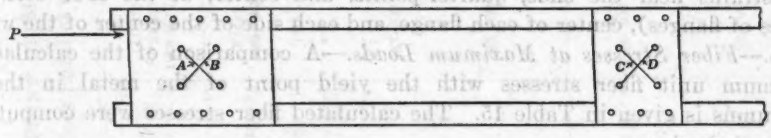
*13.—Method of Testing.*—These tests were made on solid rolled H-columns having values of $\frac{l}{r}$ from 20 to 158. The purpose of the tests was, in general, to compare the behavior of such columns furnished under ordinary commercial conditions, with that of the ideal column; that is, to ascertain the effect of crookedness, eccentricity or rolling, and other variations, on the strength. The tests also furnished some valuable information on the effect of form on the strength of columns.

All columns were first centered in the machine with reference to the calculated gravity axis of the end sections, taking also into account any

measured crookedness of the columns. Shims were used to secure even bearings in a direction parallel to the axis of the rollers of the bearing blocks. Further correction was made in the setting by applying loads up to about 10 000 lb. per sq. in., observing deflection at the center, and taking strain-gauge readings over the center section. After this careful centering, a center loading test up to 15 000 lb. per sq. in., or when possible up to the Euler load, was run. The columns were then shifted a calculated amount to give the desired eccentricity. Fig. 27 shows Column No. *H*7 ready to be tested.

TABLE 14.—COMPARISON OF CALCULATED AND MEASURED SHEAR STRESSES IN BATTEN-PLATES OF COLUMNS NOS. 8 AND 8*A* UNDER 5 PER CENT. SHEAR LOADS.

(Primed letters refer to gauge lines paralleling those shown on the opposite side of column. Nominal dimensions of stay-plates, 13 by 12 by ¼ in.; batten-plates, 9 by 12 by ¼ in.)




| Column No. | Batten or stay and gauge line considered. | Load applied to column, in pounds. | Shear per batten, $V$, in pounds. | Measured unit strain on 45° 2-in. gauge line, $\epsilon_1$. | Unit Shear Stress on Battens, in Pounds per Square Inch. | | Ratio, calculated to measured. |
|---|---|---|---|---|---|---|---|
| | | | | | Calculated, $S_s = \frac{3}{2}\frac{V}{A}$. | Measured, $S_s = \frac{4}{15}(4\epsilon_1 + \epsilon_2)E$. | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| 8 | Lower *A* | 113 400 | 8 330 | −24.85 | 5 460 | 5 270 | 0.88 |
| | Batten *B* | | | +31.25 | | 7 750 | |
| | " *A'* | | | −25.65 | | 5 960 | |
| | " *B'* | | | +25.65 | | 5 960 | |
| | Mean | | | 26.85 | | 6 225 | |
| 8*A* | Lower *A* | 96 700 | 7 100 | − 9.6 | 2 820 | 2 100 | 1.10 |
| | Stay *B* | | | +11.2 | | 2 720 | |
| | " *A'* | | | −11.2 | | 2 540 | |
| | " *B'* | | | +12.0 | | 2 850 | |
| | Mean | | | 11.0 | | 2 550 | |
| 8*A* | Lower *A* | 96 700 | 6 240 | −19.2 | 4 620 | 4 270 | 0.96 |
| | Batten *B* | | | +21.6 | | 5 200 | |
| | " *A'* | | | −19.2 | | 4 140 | |
| | " *B'* | | | +23.2 | | 5 690 | |
| | Mean | | | 20.8 | | 4 825 | |

* Elongation, +; compression, −.

Columns Nos. *H*2, *H*3, *H*4, and *H*5 were first loaded within the elastic limit with the eccentricity perpendicular to the web, then turned and tested to failure with the eccentricity in the plane of the web. Columns Nos. *H*1, *H*6, *H*7, *H*8, *H*9, and *H*10 were first loaded within the elastic limit with the eccentricity in the plane of the web, then turned and tested to failure with the eccentricity perpendicular to the web. Column No. 1 (the shortest) was then straightened and tested to failure with the eccentricity in the plane of

the web. Thus, all the columns were tested within the elastic limit with an eccentricity in each direction, four of the columns to failure by bending perpendicular to the web, six to failure by bending parallel to the web, and one (Column No. *H*1) also to failure by bending perpendicular to the web. To supplement the data for the shortest length tested, Column No. *H*10*B* was cut from the lower end of Column No. *H*10 and tested to failure with an eccentricity in the plane of the web.

Eccentricities were chosen so as to make the value of $\frac{e\,c}{r^2}$ (eccentric ratio) equal to 0.5, 1.0, or 2.0; in this expression, $e$ is eccentricity; $c$, distance to the extreme fiber; and $r$, the radius of gyration. In every case failure tests were made with $\frac{e\,c}{r^2} = 1.00$. With this value, the fiber stress on the concave side of the column (neglecting the effect of deflection) would be twice the average, and on the convex side, zero. Deflections at the center were measured, also strains near the ends, quarter-points, and center, at the four corners (edges of flanges), center of each flange, and each side of the center of the web.

*14.—Fiber Stresses at Maximum Loads.*—A comparison of the calculated maximum unit fiber stresses with the yield point of the metal in these H-columns is given in Table 15. The calculated fiber stresses were computed by the formula:

$$p = \frac{P}{A}\left[1 + \frac{(e + D)\,c}{r^2}\right]$$

in which, $D$ is measured deflection and $e$, eccentricity, including the effect of crookedness. The calculations assumed proportionality of stress and strain across the section and for values beyond the yield point give fictitious rather than real stresses. The results, however, serve for comparative purposes. The following significant results are noted:

(1) The maximum fiber stress is in every case considerably beyond the proportional limit of the material, but at "first scaling" the stress is considerably below the limit.

(2) For a column placed with its web perpendicular to the axis of the rollers (bending parallel to web), the maximum fiber stress averages somewhat greater than the yield point of the material, but is slightly less in two cases.

(3) For a column placed with its web parallel to the axis of the rollers (bending perpendicular to web), the maximum fiber stress is greater (generally very much greater) than the yield point of the material, and, therefore, greater than the maximum stress when tested in the other direction. The excess over the latter amounts to the following percentages: 41, 62, 42, 36, and 21, the last being for the longest column. These large differences are significant, and indicate a very considerable difference in the behavior of columns when the bending is in a plane parallel to an outstanding flange and when perpendicular to such a flange. This may be of importance when considering the application of formulas which take account of the form of section in terms of the ratio of $c$ to $r$. Of course, the effect of the difference in the ultimate fiber stress here shown does not indicate a correspondingly large difference in the ultimate strength of the column. This will be discussed later.

TABLE 15.—COMPARISON OF CALCULATED MAXIMUM UNIT STRESSES IN H-COLUMNS FOR LOADS AT OR NEAR ULTIMATE WITH YIELD POINT OF MATERIAL.

(With rollers normal to web, $e = 2.93$ in., $r = 3.40$ in.; with rollers parallel to web, $e = 0.98$ in., $r = 1.98$ in.)

| Column No. | Length, $l$, in inches. | $\frac{l}{r}$. | Direction of roller axes. | Ultimate Load, $\frac{P}{A}$, in Pounds per Square Inch. | | Deflections, $D$ in Inches, at: | | Calculated Unit Stresses, in Pounds per Square Inch, at: | | Properties of Material, in Pounds per Square Inch. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | At first scaling. | At ultimate. | First scaling. | Ultimate. | First scaling. | Ultimate. | Proportional limit in tension. | Average yield point of tension specimens. | Average ultimate, compression for whole prisms. |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) |
| *H* 1 | 39.1 | 11 | | 17 000 | 20 700 | .... | .... | 33 900 | 41 400 | 33 600 | 37 400 | 38 800 |
| | 39.1 | 20 | | 18 000 | 26 900 | .... | .... | 35 900 | 53 600 | 36 000 | 39 000 | 38 800 |
| *H*10 *B* | 39.6 | 12 | | 13 200 | 21 000 | 0.02 | 0.18 | 26 400 | 42 100 | 34 800 | 37 400 | 37 600 |
| *H* 6 | 39.6 | 20 | | 16 000 | 28 850 | 0.03 | 0.25 | 32 700 | 65 800 | 39 700 | 43 000 | 38 800 |

TABLE 15.—(*Continued.*)

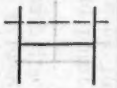
| Column No. | Length, $l$, in inches. | $\frac{l}{r}$. | Direction of roller axes. | Ultimate Load, $\frac{P}{A}$, in Pounds per Square Inch. | | Deflections, $D$, in Inches, at: | | Calculated Unit Stresses, in Pounds per Square Inch, at: | | Properties of Material, in Pounds per Square Inch. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | At first scaling. | At ultimate. | First scaling. | Ultimate. | First scaling. | Ultimate. | Proportional limit in tension. | Average yield point of tension specimens. | Average ultimate, compression for whole prisms. |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) |
| *H* 2 | 99.0 | 29 | | 16 000 | 19 950 | 0.17 | 0.41 | 32 900 | 42 600 | 33 600 | 37 400 | 38 800 |
| *H* 7 | 99.0 | 51 |  | 12 000 | 21 660 | 0.14 | 1.12 | 25 900 | 69 000 | 39 700 | 43 000 | 38 800 |
| *H* 3 | 168.3 | 50 | | 12 000 | 17 950 | 0.38 | 1.11 | 25 400 | 42 500 | 33 600 | 37 400 | 38 800 |
| *H* 8 | 168.3 | 86 | | 14 000 | 16 200 | 0.64 | 1.60 | 37 900 | 60 200 | 39 700 | 43 000 | 38 800 |
| *H* 4 | 237.8 | 70 | | 12 000 | 15 100 | 0.88 | 1.48 | 27 600 | 37 800 | 33 600 | 38 000 | 40 900 |

TABLE 15.—(*Continued.*)

| Column No. | Length, $l$, in inches. | $\frac{l}{r}$. | Direction of roller axes. | Ultimate Load, $\frac{P}{A}$, in Pounds per Square Inch. | | Deflections, $D$, in Inches, at: | | Calculated Unit Stresses, in Pounds per Square Inch, at: | | Properties of Material, in Pounds per Square Inch. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | At first scaling. | At ultimate. | First scaling. | Ultimate. | First scaling. | Ultimate. | Proportional limit in tension. | Average yield point of tension specimens. | Average ultimate compression for whole prisms. |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) |
| *H* 9 | 237.5 | 121 | | 10 200 | 10 620 | 1.30 | 2.80* | 33 900 | 51 600 | 42 600 | 45 800 | 39 400 |
| *H* 5 | 306.9 | 90 | | 11 500 | 12 600 | 1.71 | 2.55 | 29 700 | 36 200 | 33 100 | 36 400 | 35 300 |
| *H*10 | 306.9 | 158 | | 7 750 | 7 750 | 2.85 | 3.48 | 38 800 | 44 000 | 34 800 | 41 800 | 37 600 |

* Estimated.

Table 16 gives some of the stresses from strain measurements as compared to calculated stresses under loads within the elastic limit. Unit stresses were calculated from strains as follows: For Fiber A,

$$p_A = \frac{P}{A}\left(\frac{4\,e_A}{e_A + e_B + e_C + e_D}\right)$$

The top and bottom gauge lines were 4 in. and 12 in., respectively, from the end of the column. The center gauge distances were 8 in. for these readings.

Considering all H-columns and unit loads, $P/A$, sufficient to produce stresses of 16 000 to 27 000 lb. per sq. in. on the outer concave fibers at mid-section, the average variation of the average gauge stress was only 2.6% from the calculated stress, and the maximum variation was 11.7 per cent.

A variation between individual and average gauge readings (large only at end sections) indicated the effect of irregularity of end condition in a plane parallel to the axis of the roller (fixed direction), and shows the difficulty of flat end tests.

*15.—Measured and Calculated Deflections.*—Figs. 23, 24, and 25 show comparisons between observed and calculated central deflections, the calculated deflections being determined by the theoretical formula for eccentric load, as follows:

$$D = e\left(\sec\sqrt{\frac{P}{E\,I}}\frac{l}{2\,r} - 1\right)$$

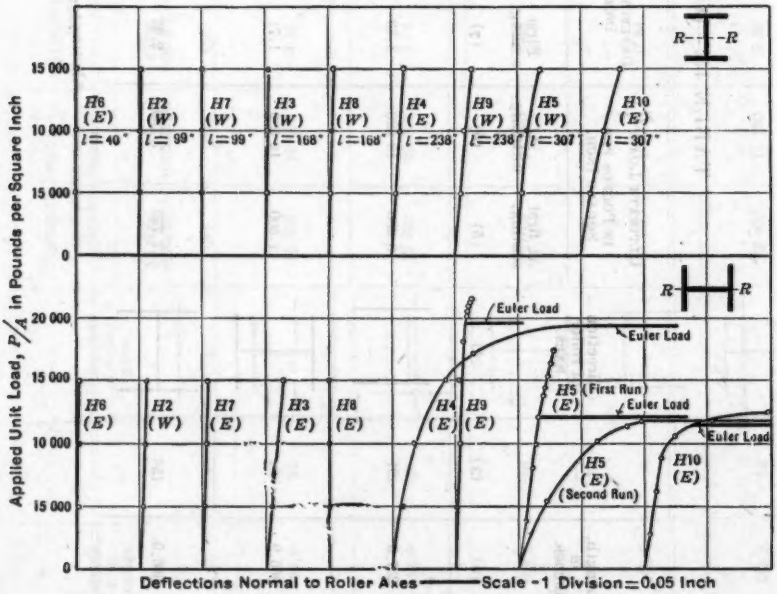



FIG. 23.—LOAD-DEFLECTION CURVES FOR 8-INCH, 32-POUND, H-COLUMNS, UNDER CENTER LOADINGS.

The value of $E$ used was that obtained from strain measurements on the center section, and $r$ was the measured value. Note the very close agreement

between observed and calculated deflections. Fig. 25 also shows the marked effect of the magnitude of the eccentricity of the load on the deflection of a long column like Column No. *H*4.

TABLE 16.—COMPARISON OF MAXIMUM FIBER STRESSES IN H-COLUMNS CALCULATED FROM STRAINS WITH STRESSES FROM LOAD AND DEFLECTION.

$\left(\text{Eccentric loading, } \frac{e\,c}{r^2} = 1.\right)$

| Column No. and Section.* | $\frac{l}{r}$. | Position of roller axes. | Load, $\frac{P}{A}$, in pounds per square inch. | Unit Stresses in Outer Fibers on Concave Side of Column. in Pounds per Square Inch. | | | | Percentage variation of gauge stress from average. |
|---|---|---|---|---|---|---|---|---|
| | | | | Gauge *A*. | Gauge *B*. | Average Gauges *A* and *B*. | Calculated, $S = \frac{P}{A}\left[1+\frac{(e+D)\,c}{r^2}\right]$. | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| *H*1 *T* | 20 | | 7 900 | 16 500 | 14 700 | 15 600 | 15 800 | 5.8 |
| *H*1 *M* | | | | 16 300 | 15 300 | 15 800 | 15 700 | 3.2 |
| *H*1 *B* | | | | 16 000 | 15 300 | 15 700 | 15 700 | 1.9 |
| *H*2 *T* | 51 | B A | 8 950 | 18 000 | 16 300 | 17 200 | 17 400 | 4.7 |
| *H*2 *M* | | | | 19 400 | 20 600 | 20 000 | 19 700 | 3.0 |
| *H*2 *B* | | | | 16 700 | 18 400 | 17 600 | 17 600 | 4.5 |
| *H*3 *T* | 87 | C D | 8 300 | 16 600 | 20 000 | 18 300 | 18 300 | 9.3 |
| *H*3 *M* | | | | 20 400 | 20 400 | 20 400 | 21 700 | 0.0 |
| *H*3 *B* | | $e = 0.88$ in. | | 19 700 | 15 500 | 17 600 | 18 200 | 11.9 |
| *H*10 *T* | 159 | | 5 910 | 13 600 | 13 600 | 13 600 | 12 800 | 0.0 |
| *H*10 *M* | | | | 19 600 | 18 900 | 19 300 | 19 800 | 1.6 |
| *H*10 *B* | | | | 15 400 | 12 300 | 13 900 | 13 300 | 10.8 |
| *H*1 *T* | 11 | | 8 900 | 16 700 | 17 400 | 17 100 | 17 500 | 1.8 |
| *H*1 *M* | | | | 15 300 | 18 800 | 17 100 | 17 500 | 10.0 |
| *H*1 *B* | | | | 14 000 | 18 000 | 16 000 | 17 500 | 12.5 |
| *H*2 *T* | 29 | B A | 11 900 | 21 800 | 24 000 | 22 900 | 23 400 | 4.8 |
| *H*2 *M* | | | | 26 200 | 20 400 | 23 300 | 23 900 | 12.4 |
| *H*2 *B* | | | | 24 000 | 22 100 | 23 100 | 23 300 | 3.9 |
| *H*3 *T* | 50 | C D | 8 900 | 13 900 | 20 900 | 17 400 | 17 400 | 29.1 |
| *H*3 *M* | | | | 18 200 | 18 600 | 18 400 | 18 200 | 1.1 |
| *H*3 *B* | | $e = 2.93$ in. | | 18 700 | 14 100 | 16 400 | 17 500 | 14.0 |
| *H*10 *T* | 90 | | 8 890 | 22 100 | 13 100 | 17 600 | 17 300 | 25.6 |
| *H*10 *M* | | | | 19 400 | 21 900 | 20 700 | 20 800 | 5.8 |
| *H*10 *B* | | | | 15 200 | 19 600 | 17 400 | 17 600 | 12.6 |

* *T* = top ; *M* = middle ; *B* = bottom.

There is also shown in these diagrams the theoretical ultimate strength of each column, indicated by the short horizontal line labeled "Calculated *P*/*A* from Yield Point." These values were calculated by the secant formula, using as the ultimate fiber stress the yield point strength of the flange material for the column in question and a value of *E* as determined from 20-in. strain-gauge measurements at the mid-section. The actual ultimate strengths are also shown.

*16.—Comparison of Ultimate Strength of* H*-Columns When Bent in Different Directions.*—Fig. 26 enables one to judge of the effect of direction of bending on the ultimate strength for equal values of $\frac{l}{r}$ and on the actual, as compared to the theoretical, strength. The full line represents the theoretical strength for a fiber stress equal to the average tensile yield point of the

FIG. 21.—LOAD-DEFLECTION CURVES FOR 8-INCH, 32-POUND, H-COLUMNS UNDER ECCENTRIC LOADINGS. $\frac{ec}{r^2} = 1.$

material when $\frac{e\,c}{r^2} = 1$. The dashed line shows the theoretical unit load $(P/A)$ at which the stress in the outer fibers would under perfect conditions reach the average proportional limit for the same eccentricity ratio. Comparing maximum load points with the upper curve, the excess strength beyond the yield point of the columns with $\frac{l}{r}$ less than 100, bent to failure under an eccentricity normal to the web, is of considerable value in considering the margin of safety in designing. Such excess strength is not pronounced in the columns

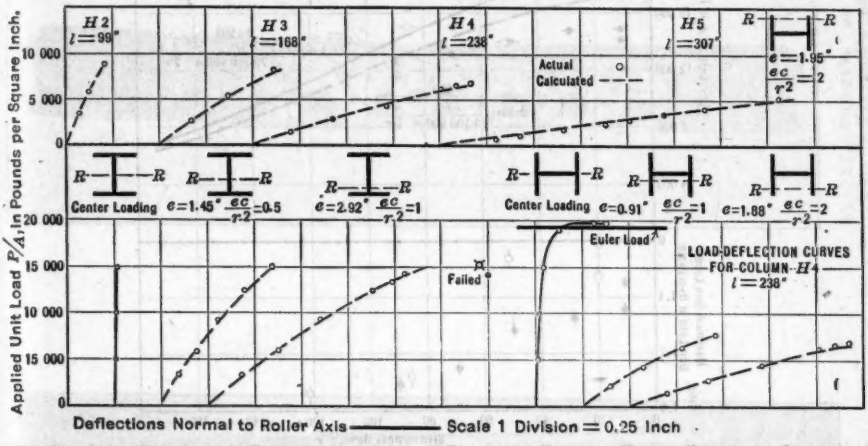



FIG. 25.—LOAD DEFLECTION CURVES FOR 8-INCH, 32-POUND, H-COLUMNS UNDER ECCENTRIC LOADINGS.

bent to failure under an eccentricity in the plane of the web. It should be noted that all these tests were made with the same $\frac{e\,c}{r^2}$, but that the value of $e$ in the plane of the web was three times the value of $e$ for the other position. Hence, the results do not show that a given H-column will be as strong as, or stronger against buckling normal to the web than, against buckling parallel thereto, but only that the calculated extreme fiber stress at failure is likely to be higher in the first case than in the second.

Comparing the plotted points representing the first observed over-strain at the quarter-points or mid-section with the dashed line, Fig. 26, one perceives that the variability of strain distribution produced over-strain in many of the shorter columns at loads somewhat less than those calculated. This condition was more common for bending in the plane of the web. The lower part of Fig. 26 affords the assurance that for short H-columns bent in either of the aforementioned ways there is a comfortable margin of safety between the first appearance of strain lines and failure.

These H-column tests furnish satisfactory confirmation of the reliability and accuracy of the secant formula when the conditions are known. There is need, however, of further tests of this character on sections of different form, to assist in answering the question of the effect of form in column design.

The arrangement of apparatus for the tests of Series (*B*) is shown in Fig. 27.

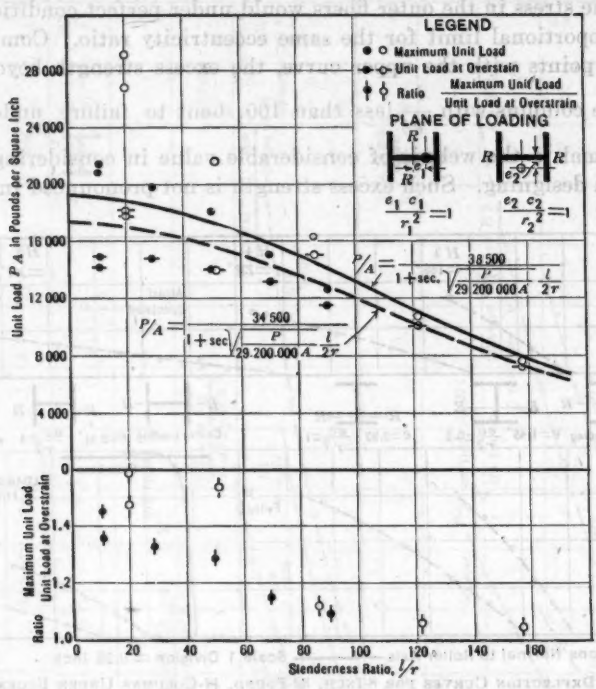

LEGEND
Maximum Unit Load
Unit Load at Overstrain
Ratio
Maximum Unit Load / Unit Load at Overstrain
PLANE OF LOADING
Unit Load P/A in Pounds per Square Inch
Maximum Unit Load / Unit Load at Overstrain
Ratio
Slenderness Ratio, l/r

FIG. 26.

## TESTS OF SERIES (*C*)

*17.—Procedure in Testing.*—In this series a laced channel column, No. 5*B*, and a batten-plate column, No. 8*A*, (Fig. 28) were tested both under eccentric loads and under oblique loads. The purpose of the eccentric load tests was to ascertain if there was sufficient shear strain present to increase materially the lateral deflection of these columns. The function of the oblique loading tests was to supplement the data on shear distortion and stresses in battens and channels of Series (*B*) with results as already presented.

In the eccentric load tests, each column was first tested with $\frac{e\,c}{r^2} = 0.4$ and bent normal to the web planes without over-strain. It was then shifted to a parallel position with an eccentricity equal in amount but opposite in sign. Both center deflections and strains over 8-in. gauge lengths on the corner fibers near the ends and at the center were read during these tests. Following the oblique loading tests, Column No. 5*B* was tested to destruction under eccentric loads with $\frac{e\,c}{r^2} = 0.6$, $e$ being normal to the webs; and Column No. 8*A* was crushed under eccentric loads with $\frac{e\,c}{r^2} = 1.0$, $e$ being parallel to the webs.

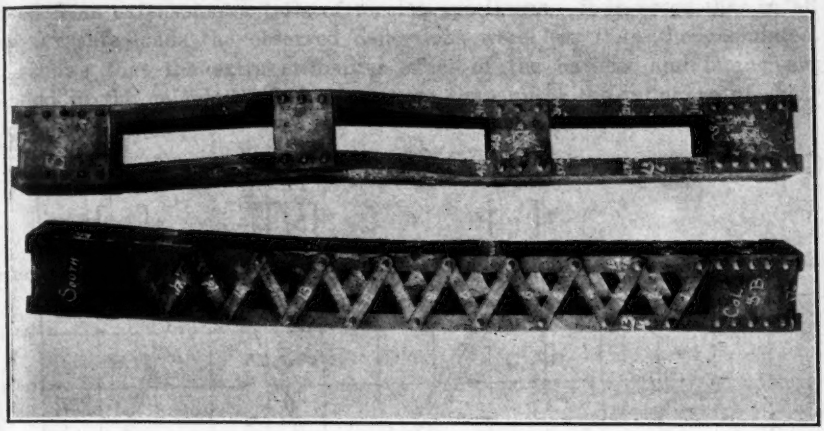



FIG. 28.—FAILURES OF COLUMNS NOS. 5B AND 8A TESTED UNDER ECCENTRIC LOADS.

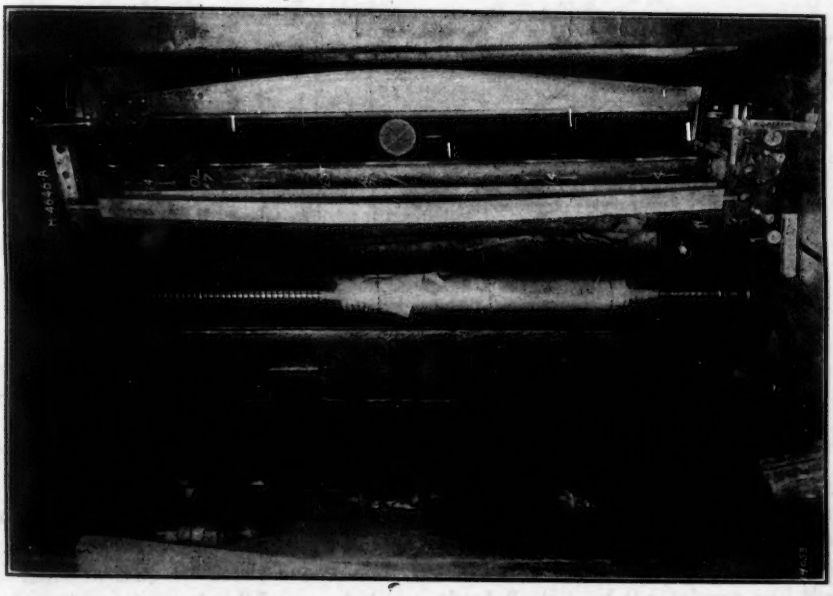
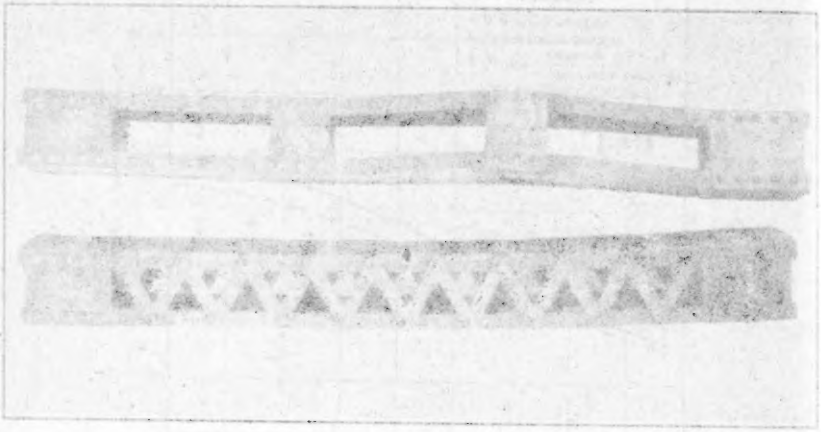
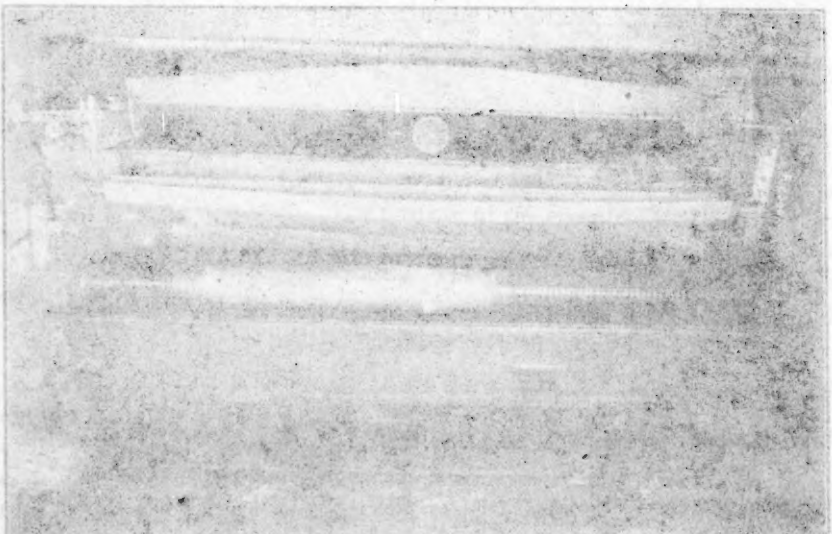


FIG. 27.—COLUMN NO. 8T READY FOR AN ECCENTRIC LOAD TEST.





*18.—Deflection Data.*—The load-deflection curves for these eccentric load tests and the deflection curves calculated by the secant formula (which neglects shear distortion) are shown in Fig. 29. For the calculated curves, $E$ was derived from extensometer tests of tension specimens. It is noted that in all cases for safe loads the observed deflections were less than the calculated. This shows that the extra stiffening effect of the battens and lacing not included in the calculated deflection more than offset the influence of shear distortions.

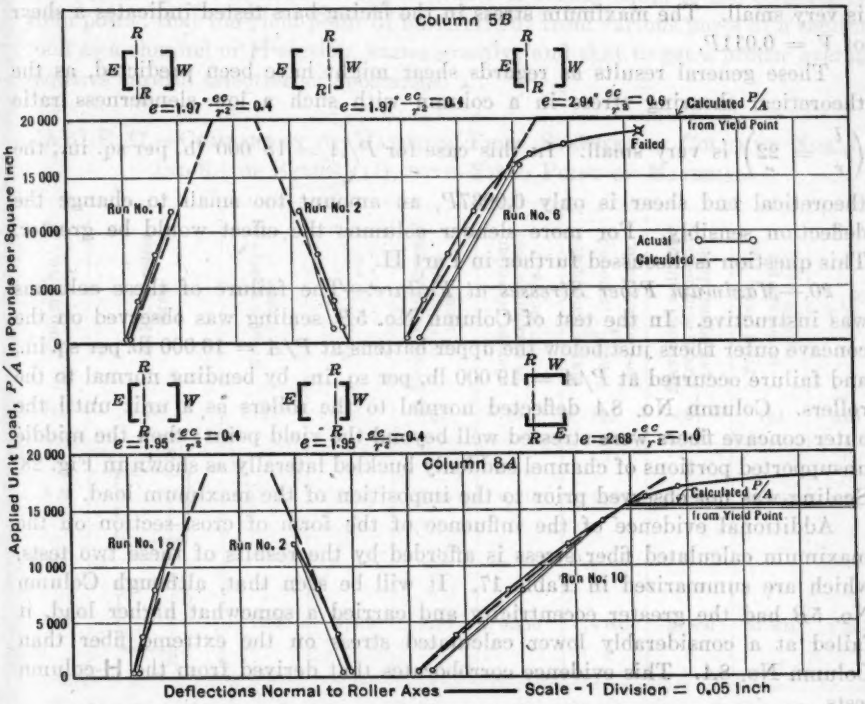



FIG. 29.—LOAD-DEFLECTION CURVES FOR COLUMNS NOS. 5B AND 8A OF SERIES (C).

At $P/A$ = 12 000 lb. per sq. in., the deflection of Column No. 5*B*, in Run No. 1, was 0.0448 in.; in Run No. 2, it was 0.0431 in. This discrepancy corresponds to an eccentricity of 0.036 in. east, or a westward bow at mid-length of 0.036 in. Measurements on the column showed an average westward bow of 0.023 in.

The marked difference in stiffness of the batten-plate column (No. 8*A*) in the first and second runs is attributed to the secondary bending of the channel to which the deflectometer dial was attached. In Run No. 1 this channel was the more heavily stressed; hence, if it bowed away from the column axis the dial would register the difference between the deflection of the column and the deflection of the unsupported length of channel. In the second run the channel, being less heavily loaded, would deflect less; hence, the recorded deflection would be greater.

*19.—Stresses in Lacing of Eccentrically Loaded Columns.*—To secure further evidence concerning shear in Column No. 5*B* under eccentric loading, strain measurements were made on the top and bottom edges of the second and third lacing-bars from each end on both sides during Run No. 6 to failure. From these strains the calculated average stress in the bars, at $P/A = 18\,000$ lb. per sq. in., was 1 140 lb. per sq. in. At this load stresses in the eight bars varied from 230 to 2 300 lb. per sq. in. These data, like the deflection-test evidence, indicate that the transverse shear in the eccentrically loaded column is very small. The maximum stress in the lacing-bars tested indicates a shear of $V = 0.011P$.

These general results as regards shear might have been predicted, as the theoretical shearing stress in a column with such a low slenderness ratio $\left(\frac{l}{r} = 22\right)$ is very small. In this case for $P/A = 18\,000$ lb. per sq. in., the theoretical end shear is only $0.0037P$, an amount too small to change the deflection sensibly. For more slender columns the effect would be greater. This question is discussed further in Part II.

*20.—Maximum Fiber Stresses at Failure.*—The failure of these columns was instructive. In the test of Column No. 5*B*, scaling was observed on the concave outer fibers just below the upper battens at $P/A = 16\,000$ lb. per sq. in., and failure occurred at $P/A = 19\,000$ lb. per sq. in., by bending normal to the rollers. Column No. 8*A* deflected normal to the rollers as a unit until the outer concave fibers were stressed well beyond the yield point; then the middle unsupported portions of channel suddenly buckled laterally as shown in Fig. 28. Scaling was not observed prior to the imposition of the maximum load.

Additional evidence of the influence of the form of cross-section on the maximum calculated fiber stress is afforded by the results of these two tests, which are summarized in Table 17. It will be seen that, although Column No. 5*B* had the greater eccentricity and carried a somewhat higher load, it failed at a considerably lower calculated stress on the extreme fiber than Column No. 8*A*. This evidence corroborates that derived from the H-column tests.

## PART II.—GENERAL REPORT

*21.—The Report of 1926.*—In the report of 1926,* the subject was divided into the following subdivisions:

(A) Strength as Related to Yield Point.—This topic was discussed with reference to results of tests on columns with slenderness ratios, $\frac{l}{r}$, not greater than 100 and with flat ends or flat-end behaviors. It was shown that for $\frac{l}{r} = 50$, the average ultimate strength was about 95% of the tensile yield point of the material composing the column, ranging in quality from nickel steel with a yield point of 56 000 lb. per sq. in. to low carbon steel with a

* *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1485.

yield point of 25 000 lb. per sq. in. For $\frac{l}{r} = 100$, the average strength was about 88% of the yield point. The great majority of the results was larger than would be calculated by assuming an eccentric ratio, $\frac{e\,c}{r^2} = 0.1$, and by taking due account of the slenderness ratio, $\frac{l}{r}$. It was also shown that the tensile yield point of steel of various grades is closely equal to the compressive yield point; that the yield point of material cut from various parts of a section, such as a channel or H-section, varies greatly; and that to get a proper average requires careful selection of specimens.

TABLE 17.—COMPARISON OF MAXIMUM FIBER STRESSES IN COLUMNS NOS. 5*B* AND 8*A* OF SERIES (*C*), WITH YIELD POINT OF MATERIAL.

| Column No. | Position of roller axes. | Area, in square inches. | Values of $\frac{c}{r^2}$. | Eccentricity normal to rollers, in inches. | Center deflection at maximum load, in inches. | Maximum applied load, $\frac{P}{A}$, in pounds per square inch. | Maximum unit fiber stress, $S$, in pounds per square inch. | Yield point, $Y_F$, in pounds per square inch. | $\frac{S}{Y_F}$ |
|---|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| 5 *B* | | 11.38 | 0.203 | 2.94 | 0.210 | 19 000 | 31 200 | 32 400 | 0.96 |
| 8 *A* | | 11.30 | 0.378 | 2.68 | 0.410 | 18 000 | 38 800 | 31 800 | 1.22 |

(B) Results of Tests on Columns of All Lengths Showing Integral Action.—All available tests were plotted and discussed. It was shown that tests on flat-end columns and those pin-end tests that showed flat-end behavior gave results fairly represented by a parabolic curve beginning at the yield point for $\frac{l}{r} = 0$, and tangent to the Euler curve, $P/A = \frac{4\,\pi^2\,E}{\left(\frac{1.12\,l}{r}\right)^2}$; and that tests on pin-end columns gave results fairly represented by a parabola beginning at the yield point and tangent to the Euler curve, $P/A = \frac{\pi^2\,E}{\left(\frac{0.78\,l}{r}\right)^2}$.

(C) Effect of Various Details on Strength of Columns.—Under this head was discussed the data relative to eccentricities, it being shown that few tests gave values greater than those represented by the eccentric ratio, $\frac{e\,c}{r^2} = 0.1$,

corresponding to a bending stress for short lengths of 10% of the direct stress. The unsupported lengths of column components were discussed as regards thickness of outside plates, and results were given to indicate that a ratio of rivet pitch to thickness equal to 20 was sufficiently low to develop the full strength of the column, and that current practice was conservative in this respect.

(D) Column Theories; The Column in a Truss; Working Formulas.—This proposed subdivision was left for a subsequent report.

*22.—The Present Report.*—In the general discussion of this report, it is proposed to give first some further data under Part (A), relating to the yield point of the material and to column strength; then to take up some general theories under Part (D) and finally to present some data and discussion under Part (C). The column tests described in Part I are of value mainly as throwing light on various matters coming under Part (C).

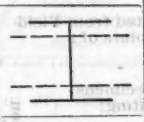
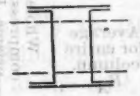
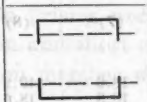
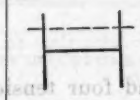
(A) Column Strength as Related to Properties of the Material

*23.—Column Strength and Yield Point.*—In Tables 18 and 19 are brought together the results of the tests described in Part I, arranged to show in a convenient form the relation of strength to yield point. Table 18 gives a comparison between calculated unit fiber stresses and yield points; and Table 19, a comparison between actual and calculated loads for the eccentric load tests. In both tables, three different values are used for the yield point: (1) The yield point from tension specimens taken from that part of the section which includes the most strained fibers; (2) the average yield point for the entire section; and (3) the yield point as given by the ultimate compressive strength of short prisms of the columns.

Referring to Table 18, it will be seen from the values given in Columns (7), (8), and (9), that the yield point values do not differ greatly except in the last group, where the most strained fibers are the edges of the flanges, the yield point of which is considerably higher than the average. The oblique loading tests showed a calculated maximum fiber stress 10 to 20% greater than the yield point. In these tests the fiber stress is a maximum at the end of the column, and diminishes toward the center so that there is little or no buckling action. The behavior is similar in nature to that of a column in a structure with heavy secondary bending moments at each end, bending the column into an S-shaped curve. All these columns were relatively short.

The eccentric load tests show two general results corresponding to the two conditions discussed in Part I, Section 16. Those tests where the deflection was in a direction such that a large part of the cross-section was stressed to the maximum, or nearly so, showed stresses from 96 to 115% of the yield point (Column (10), Table 18); those tests where but a small part of the section was stressed to the maximum, showed calculated stresses from 105 to 160% of the yield point, the small value of 105% being for a very long column. As stated in Part I, these excessive fiber stresses are fictitious in the sense that they are not actual stresses, but calculated values based on the elastic behavior of the material up to failure.

TABLE 18.—Comparison of Maximum Unit Fiber Stresses Calculated from Maximum Loads and Deflections with Yield Points of Material, Wisconsin Tests.

| Kind of loading. | Column No. | Type and position of roller axes. | $\frac{l}{r}$. | $\frac{c}{r^2}$. | Maximum unit fiber stress in 1 000 lb. per sq. in., $S$. | Yield Point of Material in 1 000 Lb. per Sq. In. | | | Ratios. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Tension. | | Compression. | | | |
| | | | | | | Average for most strained fibers, $Y_F$ | Average for entire column, $Y_A$. | Ultimate of whole prisms, $Y_C$. | $\frac{S}{Y_F}$. | $\frac{S}{Y_A}$. | $\frac{S}{Y_C}$. |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| Oblique | 1 | | 18 | 0.295 | 43.9 | 38.3 | 38.8 | 37.9 | 1.14 | 1.13 | 1.16 |
| | 2 |  | 21 | 0.271 | 42.9 | 36.1 | 36.1 | 36.5 | 1.19 | 1.19 | 1.18 |
| | 3 |  | 20 | 0.270 | 41.5 | 35.0 | 35.8 | 34.4 | 1.19 | 1.16 | 1.21 |
| | 3A | | 20 | 0.268 | 39.9 | 36.5 | 35.9 | 35.5 | 1.09 | 1.11 | 1.12 |
| | 4 |  | 22 | 0.206 | 36.8 | 32.0 | 32.6 | 32.2 | 1.15 | 1.13 | 1.14 |
| | 5 | | 22 | 0.203 | 36.8 | 32.0 | 32.6 | 32.2 | 1.15 | 1.13 | 1.14 |
| | 5A | | 22 | 0.205 | 35.9 | 32.3 | 32.6 | 32.0 | 1.11 | 1.10 | 1.12 |
| | 6 | | 22 | 0.206 | 38.7 | 32.0 | 32.6 | 32.2 | 1.21 | 1.19 | 1.20 |
| Eccentric | 5B | | 22 | 0.203 | 31.2 | 32.4 | 32.2 | 31.4 | 0.96 | 0.97 | 0.99 |
| | 8A | | 33 | 0.373 | 38.8 | 31.8 | 32.8 | 30.9 | 1.22 | 1.18 | 1.25 |
| Eccentric | H 1 | | 11 | 0.342 | 41.4 | 37.4 | 37.6 | 38.8 | 1.11 | 1.10 | 1.07 |
| | H10B | | 12 | 0.340 | 43.1 | 37.4 | 38.0 | 37.6 | 1.15 | 1.13 | 1.15 |
| | H 2 | | 29 | 0.341 | 42.6 | 37.4 | 37.6 | 38.8 | 1.14 | 1.13 | 1.10 |
| | H 3 | | 50 | 0.342 | 42.5 | 37.4 | 37.6 | 38.8 | 1.14 | 1.13 | 1.09 |
| | H 4 | | 70 | 0.341 | 37.8 | 38.0 | 38.6 | 40.9 | 0.99 | 0.98 | 0.92 |
| | H 5 | | 90 | 0.339 | 36.2 | 36.4 | 36.7 | 35.3 | 0.99 | 0.99 | 1.02 |
| Eccentric | H 1 |  | 20 | 1.06 | 53.6 | 39.0 | 37.6 | 38.8 | 1.37 | 1.42 | 1.38 |
| | H 6 | | 20 | 1.05 | 65.8 | 43.0 | 39.6 | 38.8 | 1.53 | 1.66 | 1.69 |
| | H 7 | | 51 | 1.04 | 69.0 | 43.0 | 39.6 | 38.8 | 1.60 | 1.74 | 1.78 |
| | H 8 | | 86 | 1.05 | 60.2 | 43.0 | 39.6 | 38.8 | 1.40 | 1.52 | 1.55 |
| | H 9 | | 121 | 1.02 | 51.6 | 45.3 | 41.8 | 39.4 | 1.14 | 1.23 | 1.31 |
| | H10 | | 158 | 1.06 | 44.0 | 41.8 | 38.0 | 37.6 | 1.05 | 1.16 | 1.17 |

In Table 19 are given values of observed and calculated ultimate strengths of columns tested eccentrically, the calculated values being based on three different values of yield point as given in Columns (6), (7), and (8) of the table. Columns (9), (10), and (11) give ratios of observed to calculated stresses. The results are relatively about the same as given in Table 18, except that the ratios of observed to calculated loads of Table 19 are somewhat less than the ratios of fiber stresses to yield points of Table 18; that is, the ultimate loads do not show the same relative excess as the ultimate fiber stresses. This is due in part to the fact that in a column the fiber stress increases at a faster rate than the load.

TABLE 19.—COMPARISON OF MAXIMUM UNIT LOADS CARRIED BY COLUMNS WITH UNIT LOADS CALCULATED FROM YIELD POINT OF MATERIAL BY THE SECANT FORMULA, WISCONSIN ECCENTRIC LOAD TESTS.

| Column No. | Type and position of roller axes. | $\frac{l}{r}$. | $\frac{c}{r^2}$. | Maximum Unit Load, $\frac{P}{A}$, in 1000 Pounds per Square Inch. | | | | Ratios. | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Observed, $P$. | Calculated from Yield Points of: | | | $\frac{P}{P_F}$. | $\frac{P}{P_A}$. | $\frac{P}{P_C}$. |
| | | | | | Tension specimens representing: | | Whole compression prisms, $P_C$. | | | |
| | | | | | Average of most strained fibers, $P_F$. | Average for entire column, $P_A$. | | | | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) |
| *H* 1 | | 11 | 0.342 | 20.7 | 18.5 | 18.6 | 19.2 | 1.12 | 1.11 | 1.08 |
| *H* 10 *B* | | 12 | 0.340 | 21.0 | 18.5 | 18.8 | 18.6 | 1.14 | 1.12 | 1.13 |
| *H* 2 | | 29 | 0.341 | 20.0 | 18.1 | 18.2 | 18.7 | 1.10 | 1.10 | 1.07 |
| *H* 3 | | 50 | 0.342 | 18.0 | 17.0 | 17.1 | 17.6 | 1.06 | 1.05 | 1.02 |
| *H* 4 | | 70 | 0.341 | 15.1 | 15.6 | 15.8 | 16.6 | 0.97 | 0.96 | 0.91 |
| *H* 5 | | 90 | 0.339 | 12.6 | 13.4 | 13.5 | 13.1 | 0.94 | 0.93 | 0.96 |
| *H* 1 | | 20 | 1.06 | 26.9 | 18.1 | 18.2 | 18.8 | 1.48 | 1.48 | 1.43 |
| *H* 6 | | 20 | 1.05 | 28.9 | 20.9 | 19.3 | 18.9 | 1.38 | 1.50 | 1.53 |
| *H* 7 | | 51 | 1.04 | 21.7 | 18.9 | 17.6 | 17.3 | 1.16 | 1.24 | 1.27 |
| *H* 8 | | 86 | 1.05 | 16.2 | 15.2 | 14.4 | 14.2 | 1.07 | 1.12 | 1.14 |
| *H* 9 | | 121 | 1.02 | 10.6 | 11.8 | 11.4 | 11.0 | 0.90 | 0.93 | 0.96 |
| *H* 10 | | 158 | 1.06 | 7.8 | 8.2 | 7.9 | 7.8 | 0.95 | 0.99 | 1.00 |
| 5 *B* | | 22 | 0.203 | 19.0 | 20.0 | 19.8 | 19.4 | 0.95 | 0.96 | 0.98 |
| 8 *A* | | 33 | 0.373 | 18.0 | 15.3 | 15.6 | 14.9 | 1.18 | 1.15 | 1.21 |

*24.—Procedure in Testing.*—Two hundred and four tension tests and one hundred and eighty-two compression tests were made on specimens cut from the shapes used in the Wisconsin tests. Extra lengths of the stock from

which the columns were fabricated were furnished by the manufacturers with the columns. From these, test specimens were cut in accordance with the diagrams in Figs. 30 and 31. In these tests the proportional limit, yield point, ultimate strength, percentage elongation, percentage reduction in area, and modulus of elasticity were found. The speed of the movable head of the machine in the tension tests was 0.14 in. per min. as far as the yield point, and 0.023 to 0.068 per min. in the compression tests. Data on strengths as presented herein are confined to tests on coupons.

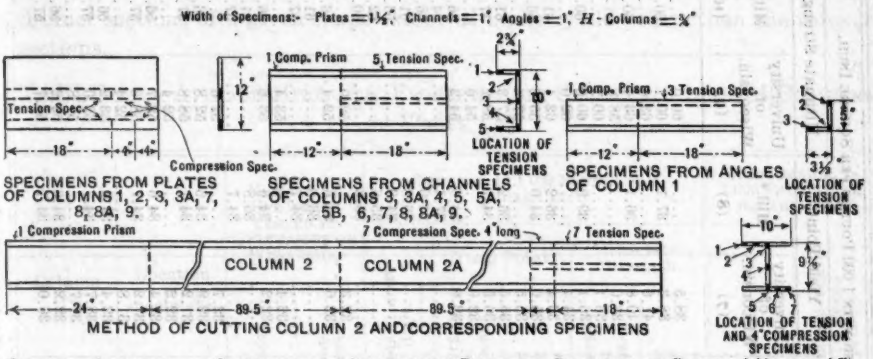

Width of Specimens:- Plates = 1½", Channels = 1", Angles = 1", H - Columns = ¾"
Tension Spec.
Compression Spec.
SPECIMENS FROM PLATES OF COLUMNS 1, 2, 3, 3A, 7, 8, 8A, 9.
1 Comp. Prism
5 Tension Spec.
SPECIMENS FROM CHANNELS OF COLUMNS 3, 3A, 4, 5, 5A, 5B, 6, 7, 8, 8A, 9.
LOCATION OF TENSION SPECIMENS
1 Comp. Prism
3 Tension Spec.
SPECIMENS FROM ANGLES OF COLUMN 1
LOCATION OF TENSION SPECIMENS
1 Compression Prism
7 Compression Spec. 4" long
7 Tension Spec.
COLUMN 2
COLUMN 2A
METHOD OF CUTTING COLUMN 2 AND CORRESPONDING SPECIMENS
LOCATION OF TENSION AND 4" COMPRESSION SPECIMENS

FIG. 30.—LOCATION OF SPECIMENS IN COMPONENT PARTS OF COLUMNS OF SERIES (*A*) AND (*C*).

Columns Nos. *H*1 to *H*5 (Fig. 31) were cut at the University Shops from lengths of 8-in., 32-lb., H-sections, Heat No. 52*E*465. Columns Nos. *H*6 to *H*10 were cut at the mill, in similar manner, from Heat No. 39*F*273. No 4-in. compression prisms were made corresponding to Columns Nos. *H*9 and *H*10. All tension and short compression specimens were ¾ in. wide. The ends of all compression member were milled.

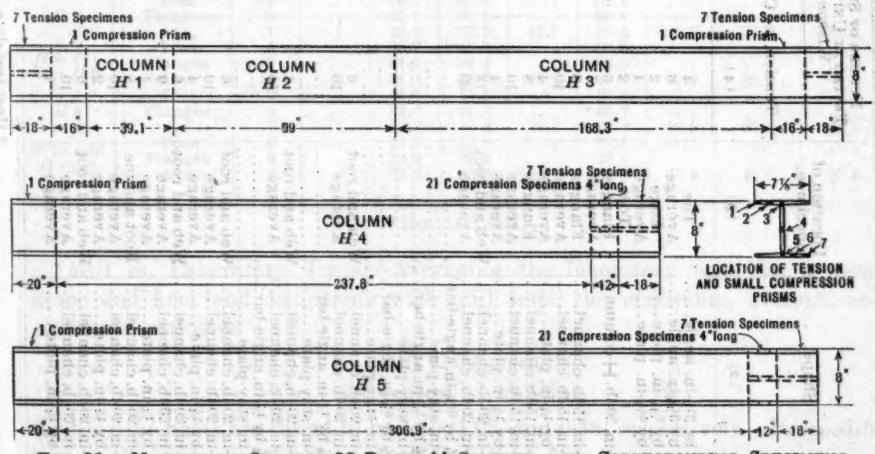



FIG. 31.—METHOD OF CUTTING 32-POUND H-COLUMNS AND CORRESPONDING SPECIMENS.

*25.—Quality of Steel.*—Tables 20 and 21 give the tension and compression test data for the specimens of Series (*A*), (*B*), and (*C*). In compiling

TABLE 20.—AVERAGE STRENGTH PROPERTIES OF MATERIAL IN COLUMNS OF SERIES (*A*) AND (*C*).

| Column No. | Shape. | Location of specimen. | Number of Specimens Tested at University of Wisconsin. | | Tension, in 1 000 Pounds per Square Inch. | | | | | Compression, in 1 000 Pounds per Square Inch. | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Proportional limit. | Yield Point. | | Ultimate Strength. | | | |
| | | | Tension. | Compression. | | University of Wisconsin. | Mill.* | University of Wisconsin. | Mill.* | Proportional limit. | Ultimate strength or yield point. |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| 1 | 5 by 3½-in. angle | Toe | 4 | ..... | 33.9 | 38.5 | ..... | 60.0 | ..... | ..... | ..... |
| | 5 by 3½-in. angle | Average | 6 | 2 | 33.5 | 38.3 | 37.1 | 60.1 | 60.5 | 34.0 | 37.9† |
| | 10 by 5/16-in. plate | Edge | 2 | 5 | 37.8 | 41.6 | ..... | 63.4 | ..... | 39.0 | 41.8 |
| | 10 by 5/16-in. plate | Average | 4 | 10 | 37.9 | 40.8 | 36.1 | 63.4 | 60.1 | 37.6 | 39.8 |
| 2 | 10-in. 50-lb. H-column | Toes | 2 | ..... | 36.0 | 37.0 | ..... | 58.8 | ..... | 32.6 | 36.6 |
| | | Flanges | 6 | ..... | 33.7 | 36.1 | ..... | 60.1 | ..... | 32.2 | 35.7 |
| | | Average | 7 | 1 | 33.7 | 36.1 | 39.9 | 60.0 | 60.0 | 25.0 | 36.5† |
| 3 | 10-in. 15-lb. channel | Flanges | 8 | ..... | 26.6 | 34.4 | ..... | 62.0 | ..... | ..... | ..... |
| | 10-in. 15-lb. channel | Average | 10 | 2 | 29.4 | 36.4 | 36.2 | 62.4 | 61.5 | 22.5 | 34.8† |
| | 12 by ¼-in. plate | Average | 4 | 8 | 33.2 | 35.0 | 34.0 | 55.2 | 56.2 | 29.6 | 34.4 |
| 3A | 10-in. 15-lb. channel | Flanges | 8 | ..... | 31.2 | 35.5 | ..... | 57.4 | ..... | ..... | ..... |
| | 10-in. 15-lb. channel | Average | 10 | 2 | 30.5 | 35.5 | 36.2 | 57.3 | 61.5 | 25.0 | 33.4† |
| | 12 by ¼-in. plate | Average | 4 | 8 | 33.4 | 36.5 | 34.7 | 58.2 | 57.4 | 31.0 | 35.5 |
| 4, 5, 6, | 10-in. 20-lb. channel | Web and root | 18 | ..... | 26.4 | 32.0 | ..... | 54.0 | ..... | ..... | ..... |
| | 10-in. 20-lb. channel | Average | 30 | 5 | 27.1 | 32.6 | 34.6 | 54.2 | 56.8 | 23.0 | 32.2† |
| 4 | 2¼ by 5/16-in. angle-bar | ..... | ..... | ..... | ..... | ..... | 37.4 | ..... | 64.1 | ..... | ..... |
| | ¼-in. stay-plate | ..... | ..... | ..... | ..... | ..... | 34.0 | ..... | 56.2 | ..... | ..... |
| 5 | 2½ by ¼-in. angle-bar | ..... | ..... | ..... | ..... | ..... | 36.8 | ..... | 61.3 | ..... | ..... |
| | ¼-in. stay-plate | ..... | ..... | ..... | ..... | ..... | 34.0 | ..... | 56.2 | ..... | ..... |
| 6 | 2½ by ⅜-in. angle-bar | ..... | ..... | ..... | ..... | ..... | 35.6 | ..... | 62.4 | ..... | ..... |
| | ⅜-in. stay-plate | ..... | ..... | ..... | ..... | ..... | 36.3 | ..... | 60.4 | ..... | ..... |
| 5A | 10-in. 20-lb. channel | Web and root | 6 | ..... | 30.0 | 32.3 | ..... | 52.0 | ..... | ..... | ..... |
| | 10-in. 20-lb. channel | Average | 10 | 2 | 30.0 | 32.6 | 34.6 | 52.4 | 56.8 | 26.0 | 32.0† |
| | 2¼ by ¼-in. angle-bar | ..... | ..... | ..... | ..... | ..... | 36.8 | ..... | 61.3 | ..... | ..... |
| | ¼-in. stay-plate | ..... | ..... | ..... | ..... | ..... | 34.7 | ..... | 57.4 | ..... | ..... |
| 5B | 10-in. 20-lb. channel | Web and root | 6 | ..... | 25.3 | 32.4 | ..... | 53.4 | ..... | ..... | ..... |
| | 10 in. 20-lb. channel | Average | 10 | ..... | 25.1 | 32.2 | 34.6 | 53.6 | 56.8 | 24.5 | 31.4† |
| | 2¼ by ¼-in. angle-bar | ..... | ..... | ..... | ..... | ..... | 36.8 | ..... | 61.3 | ..... | ..... |
| | ¼-in. stay-plate | ..... | ..... | ..... | ..... | ..... | 34.7 | ..... | 57.4 | ..... | ..... |
| 7 | 10-in. 20-lb. channel | Web and root | 6 | ..... | 27.4 | 32.7 | ..... | 52.6 | ..... | ..... | ..... |
| | 10-in. 20-lb. channel | Average | 10 | 2 | 28.1 | 32.9 | 34.6 | 52.8 | 56.8 | 26.2 | 33.6† |
| | 12 by ¼-in. plate | Average | 4 | 8 | 35.6 | 37.2 | 34.7 | 58.8 | 57.4 | 31.2 | 35.1 |
| 8 | 10-in. 20-lb. channel | Web and root | 6 | ..... | 28.4 | 32.2 | ..... | 52.4 | ..... | ..... | ..... |
| | 10 in. 20-lb. channel | Average | 9 | 2 | 27.9 | 32.4 | 34.6 | 52.7 | 56.8 | 25.0 | 33.6† |
| | 12 by ¼-in. plate | Average | 4 | 8 | 31.8 | 37.2 | 34.7 | 58.3 | 57.4 | 31.3 | 36.0 |
| 8A | 10-in. 20-lb. channel | Root and toe | 8 | ..... | 29.1 | 31.8 | ..... | 52.4 | ..... | ..... | ..... |
| | 10-in. 20-lb. channel | Average | 10 | ..... | 28.6 | 32.4 | 34.6 | 53.2 | 56.8 | 24.8 | 30.9† |
| | 12 by ¼-in. plate | Average | 2 | 4 | 34.8 | 38.7 | 34.7 | 62.0 | 57.4 | ..... | 38.2 |
| 9 | 10-in. 20-lb. channel | Web and root | 6 | ..... | 27.8 | 31.7 | ..... | 51.8 | ..... | ..... | ..... |
| | 10-in. 20-lb. channel | Average | 10 | 2 | 27.9 | 32.2 | 34.6 | 52.2 | 56.8 | 25.0 | 31.4† |
| | 12 by ¼-in. plate | Average | 4 | 8 | 33.1 | 36.0 | 34.7 | 58.7 | 57.4 | 31.6 | 35.8 |

* Heat test data. †Ultimate strength of a full-sized prism.

the averages for the different shapes, the results were weighted in proportion to the volume of the shape tributary to the specimen.

Study of the tabulated data obtained at Wisconsin shows that all shapes passed the specification requirements for yield point, but the average tensile strength for all the 10-in., 20-lb. channels was below requirements. The variations in the different properties are given in Table 22. Attention is called to the similarity in range for tensile and compressive yield point, also to the wide range of proportional limit values. It will also be noted that the thinner sections of a given shape exhibited higher yield points than the thicker sections.

TABLE 21.—AVERAGE STRENGTH PROPERTIES OF STEEL IN 8-INCH H-COLUMNS OF SERIES (*B*).

| Column No. | Location of specimens. | Number of Specimens Tested at University of Wisconsin. | | Tension, in 1 000 Pounds per Square Inch. | | | | | Compression, in 1 000 Pounds per Square Inch. | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Proportional limit. | Yield Point. | | Ultimate Strength. | | Proportional limit. | Ultimate strength of full-sized prisms. |
| | | Tension. | Compression. | | University of Wisconsin. | Mill.* | University of Wisconsin. | Mill.* | | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) |
| *H* 1, *H* 2 and *H* 3 | Toes | 4 | .. | 36.0 | 39.0 | .... | 61.1 | .... | .... | .... |
| | Flanges | 12 | .. | 33.6 | 37.4 | .... | 60.8 | .... | .... | .... |
| | Ave. | 14 | 2 | 34.2 | 37.6 | 43.7 | 61.0 | 64.3 | 27.0 | 38.8 |
| *H* 4 | Toes | 2 | .. | 34.8 | 39.6 | .... | 60.4 | .... | .... | .... |
| | Flanges | 6 | .. | 33.6 | 38.0 | .... | 61.2 | .... | .... | .... |
| | Ave. | 7 | 1 | 35.2 | 38.6 | 43.7 | 61.7 | 64.3 | 26.0 | 40.9 |
| *H* 5 | Toes | 2 | .. | 34.0 | 37.6 | .... | 58.2 | .... | .... | .... |
| | Flanges | 6 | .. | 33.1 | 36.4 | .... | 58.3 | .... | .... | .... |
| | Ave. | 7 | 1 | 33.9 | 36.7 | 43.7 | 58.4 | 64.3 | 28.5 | 35.3 |
| *H* 6, *H* 7 and *H* 8 | Toes | 4 | .. | 39.7 | 43.0 | .... | 62.9 | .... | .... | .... |
| | Flanges | 12 | .. | 34.6 | 39.4 | .... | 60.5 | .... | .... | .... |
| | Ave. | 14 | 2 | 34.4 | 39.6 | 42.2 | 60.6 | 60.9 | 26.3 | 38.8 |
| *H* 9 | Toes | 2 | .. | 42.6 | 45.3 | .... | 65.8 | .... | .... | .... |
| | Flanges | 6 | .. | 39.0 | 41.3 | .... | 64.7 | .... | .... | .... |
| | | 7 | 2 | 39.7 | 41.8 | 42.2 | 64.2 | 60.9 | 25.1 | 39.4 |
| *H* 10 and *H* 10 *B* | Toes | 2 | .. | 34.8 | 41.8 | .... | 61.1 | .... | .... | .... |
| | Flanges | 6 | .. | 30.9 | 37.4 | .... | 59.0 | .... | .... | .... |
| | Ave. | 7 | 2 | 32.4 | 38.0 | 42.2 | 58.9 | 60.9 | 25.6 | 37.6 |

* Heat test data.

Mill *vs.* Laboratory Tests.—Averaging the laboratory tests for a given shape and heat and comparing with mill tests, the maximum, average, and minimum ratios of

$$\frac{\text{Mill yield point}}{\text{Laboratory yield point}}$$

for the nine heats are found to be 1.16, 1.02, and 0.88, respectively. Proceeding in a similar manner, the corresponding ratios of

$$\frac{\text{Mill ultimate strength}}{\text{Laboratory ultimate strength}}$$

are, respectively, 1.07, 1.00, and 0.95.

* Heat test data. †Ultimate strength of a full-sized prism.

Comparison of Yield Points in Tension and Compression.—Although the results in Table 22 indicate a considerable range in yield points of the material in the different shapes, the data of Table 23 shows that the ratio,

$$\frac{\text{Average yield point in tension}}{\text{Average yield point in compression}}$$

was nearly constant. For the nine heats represented, this ratio varied from 0.99 to 1.05, and averaged 1.01. These data supplement and confirm the statement previously advanced,* that, in general, the tensile yield point is a close indication of the compressive yield point, and, for, the purpose in question, probably as satisfactory an indication of the column strength.

TABLE 22.—VARIATIONS IN STRENGTH PROPERTIES OF SPECIMENS IN WISCONSIN TESTS.

| Property tested. (1) | Kind of stress. (2) | Highest average stress, in 1 000 lb. per sq. in. (3) | Shape having highest stress. (4) | Lowest average stress, in 1 000 lb. per sq. in. (5) | Shape having lowest stress. (6) | Ratio, Highest-lowest / Specified minimum in percentage. (7) |
|---|---|---|---|---|---|---|
| Proportional limit.. | Tension | 39.7 | 8-in. 32-lb. H-column | 25.1 | 10-in. 20-lb. channel | 49* |
| Yield point..... | Tension | 41.8 | 8-in. 32-lb. H-column | 31.7 | 10-in. 20-lb. channel | 33 |
| Ultimate strength.. | Tension | 64.2 | 8-in. 32-lb. H-column | 52.2 | 10-in. 20-lb. channel | 22 |
| Proportional limit.. | Compression | 37.6 | 10 by 5/16-in. plate | 22.5 | 10-in. 15-lb. channel | 50* |
| Yield point..... | Compression | 40.9 | 8-in. 32-lb. H-column | 30.9 | 10-in. 20 lb. channel | 33* |

* No minimum specified—based on 30 000 lb. per sq. in.

*26.—Effect of Location on Strength of Specimens.*—Table 24 furnishes information regarding the effect of the position of the test piece in the shape on the yield point and ultimate strength. For the H-columns the specimens cut from the intermediate portions of the flanges† appear to represent best the entire shape. They also were the least variable. For the channels the web specimens were most representative. For the plates there was not much difference (2%) between specimens cut from the edge and from the center, although the former were consistently the stronger. The number of specimens from angles is too small to warrant definite conclusions. The root specimens were the best representatives for the angles tested. In most cases the specimens cut from the toes or edges of shapes exhibited higher yield points than other specimens cut from the same shapes.

*27.—The Uniformity of the Modulus of Elasticity of the Wisconsin Test Columns.*—By plotting the average strains for gauge lines which were symmetrically disposed about the axis of a given column, the modulus of elasticity was computed for the mid-segments of all Wisconsin test columns and in most cases for the segments at or near the quarter-points. Owing to the effect of

* *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1524.

† See similar results in *Technologic Paper No. 328*, U. S. Bureau of Standards.

irregularities in milling of the ends of columns and in the bearing blocks, the average moduli for the gauge lines nearest the column ends are omitted from this discussion. The results of these calculations appear in Tables 25 and 26. In calculating $E$ for the segments of the built-up columns, the strains for the four corner gauge lines were averaged; but, in similar calculations for the H-columns, strains along the eight gauge lines shown in Fig. 8 were usually averaged. Values of mean $E$ for the column ("column-$E$") were obtained by averaging the values of $E$ for the segments.

TABLE 23.—Relationship Between Yield Points in Tension and Compression for Specimens Cut from Columns of Series (*A*), (*B*), and (*C*).

| Shape. | Number of | | | | Average Yield Point in 1 000 Pounds per Square Inch. | | Ratio, $\frac{T}{C}$. |
|---|---|---|---|---|---|---|---|
| | Heat. | Pieces sampled. | Specimens Tested in. | | | | |
| | | | Tension. | Compression. | Tension, $T$. | Compression, $C$. | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| 8-in. 32-lb. H-column | 52E465 and 39F273 | 6 | 56 | 10* | 38.8 | 38.5 | 1.00 |
| 8-in. 32-lb. H-column | 52E465 | 2 | 14 | 42 | 37.6 | 38.0 | 0.99 |
| 10-in. 50-lb. H-column | 33E287 | 1 | 7 | 7 | 36.1 | 36.0 | 1.00 |
| | 33E287 | .. | .. | 1* | .... | 36.5 | 0.99 |
| 10-in. 15-lb. channel | 13291 | 2 | .. | 10 | 35.5† | 33.8 | 1.05 |
| | 13291 | 4 | 20 | 4* | 36.0 | 34.1 | 1.05 |
| 10-in. 20-lb. channel | 21244 | 8 | .. | 39 | 32.5† | 31.6 | 1.03 |
| | 21244 | 18 | 80 | 17* | 32.5 | 32.2 | 1.01 |
| 12 by ¼-in. plate | 11010 and 904905 | 11 | 22 | 44 | 36.6 | 35.8 | 1.02 |
| 10 by 5/16-in. plate | 925696 | 2 | 4 | 10 | 40.8 | 39.8 | 1.02 |
| 5 by 3½ by 5/16-in. angle | 48182 | 2 | 6 | 2* | 38.3 | 37.9 | 1.01 |

* Full-sized prisms.

† Averages corresponding to those in Column (7).

It should be observed that practically all the columns were subjected to several trial loadings prior to the determination of the strains used in these calculations. This condition contributed to the obliteration of initial sets and to somewhat higher moduli of elasticity than would have been obtained had it been possible to have made all measurements during an initial loading.

The data in Tables 25 and 26 show that the average value of "column-$E$" for the 8-in. gauge lines of both the built-up and H-columns averaged 29 800 000 lb. per sq. in.; also, that "column-$E$" for the 20-in. gauge lines averaged 29 400 000 and 29 800 000 lb. per sq. in. for the respective types of columns. Considering 8-in. gauge lines, Column No. 1 had the lowest "column-$E$", 27 800 000 lb. per sq. in., and Columns Nos. 7 and $H5$ the highest, 30 900 000 lb. per sq. in. Hence, the range in "column-$E$" for the built-up columns is 10%; whereas, for the H-columns, it is 7 per cent.

Since marked variation in values of $E$ within a column materially affects its strength, the percentage range in $E$ for the three segments of each

column were inserted in Columns (8) of Tables 25 and 26 to secure some data covering this point. These values were calculated from,

$$\frac{\text{maximum } E - \text{minimum } E}{\text{"column-}E\text{"}}$$

The average range of $E$ for the built-up columns was 5.8% and for the H-columns the range was 3.9 per cent. The greatest range, 9.8%, occurred in batten-plate column (No. 9).

TABLE 24.—RELATIONSHIP BETWEEN TENSILE STRENGTH PROPERTIES OF SPECIMENS CUT FROM DIFFERENT PARTS OF STRUCTURAL SHAPES, SERIES (*A*) TO (*C*).

| Shape. | Location of specimen. | Number of specimens tested. | Yield Point. | | | | Ultimate Strength. | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | Mean in 1 000 lb. per sq. in. | Ratio to whole. | Average Percentage Variation from | | Mean in 1 000 lb. per sq. in. | Average Percentage Variation from | |
| | | | | | Mean. ‡ | Grand mean. † | | Mean. ‡ | Grand mean. † |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| 8-in., 32-lb. H-column... | Toe | 16 | 41.0 | 1.06 | 5.7 | 6.9 | 61.7 | 3.5 | 3.1 |
| | Flange | 16 | 38.2 | 0.99 | 5.0 | 5.0 | 60.5 | 2.9 | 3.0 |
| | Root | 16 | 36.2 | 0.94 | 4.7 | 7.0 | 60.0 | 4.2 | 4.0 |
| | Web | 8 | 40.5 | 1.04 | 3.9 | 5.1 | 61.6 | 3.9 | 4.0 |
| | Whole | 8 | 38.8* | 1.00 | .... | 6.2 | 60.8* | .... | 3.5 |
| 10-in., 15-lb. channel.... | Toe | 8 | 34.7 | 0.98 | 4.0 | 3.9 | 59.5 | 5.1 | 3.1 |
| | Root | 8 | 35.1 | 0.99 | 4.1 | 3.8 | 59.8 | 3.6 | 2.1 |
| | Web | 4 | 37.4 | 1.06 | 5.6 | 5.9 | 60.2 | 5.0 | 2.5 |
| | Whole | 4 | 36.0* | 1.00 | .... | 4.3 | 59.9* | .... | 2.4 |
| 10-in., 20-lb. channel.... | Toe | 35 | 33.9 | 1.04 | 4.4 | 5.7 | 54.5 | 1.7 | 3.1 |
| | Root | 36 | 31.4 | 0.97 | 3.6 | 4.8 | 52.5 | 0.9 | 2.1 |
| | Web | 18 | 32.8 | 1.01 | 1.9 | 1.9 | 53.4 | 2.5 | 2.5 |
| | Whole | 18 | 32.5* | 1.00 | .... | 4.6 | 53.3* | .... | 2.4 |
| 12 by ¼-in. plate..... | Edge | 11 | 36.9 | 1.01 | 3.0 | 3.1 | 59.1 | 2.5 | 2.5 |
| | Center | 11 | 36.3 | 0.99 | 2.4 | 2.4 | 59.0 | 2.2 | 2.2 |
| | Whole | 11 | 36.6* | 1.00 | .... | 2.7 | 59.0* | .... | 2.3 |
| 5 by 3 by 5/16-in. angle.. | 5-in. toe | 2 | 36.4 | 0.95 | 0.3 | 5.5 | 59.7 | 0.5 | 0.7 |
| | 3-in. toe | 2 | 41.3 | 1.08 | 2 1 | 6.8 | 60.4 | 0.5 | 0.5 |
| | Root | 2 | 38.0 | 0.99 | 0.7 | 1.4 | 60.3 | 0.5 | 0.5 |
| | Whole | 2 | 38.3* | 1.00 | .... | 4.6 | 60.2* | .... | 0.6 |

*Calculated by weighting specimens in proportion to tributary area.
† Mean of all specimens from the type of shape indicated in Column (1).
‡ Mean of specimens from the location indicated in Column (2).

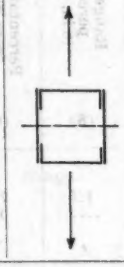
If one assumes $E$ equal to 30 000 000 lb. per sq. in. for calculations in connection with these columns, the results entered in Columns (9) of Tables 25 and 26 show the average errors in percentage for the different segments and gauges of these columns. For the built-up columns the average percentage variation of $E$ from 30 000 000 lb. per sq. in. is only 3.2; for the H-columns, it is only 2.1. The maximum variation, 8.7%, is for Column No. 3*A*.

When one takes into account the nature of strain-gauge measurements and the variability of the cross-sectional area of such members, these data

TABLE 25.—VALUES OF MODULUS OF ELASTICITY FOR DIFFERENT PORTIONS OF BUILT-UP COLUMNS AS FOUND BY STRAIN GAUGE MEASUREMENTS IN WISCONSIN CENTER LOAD TESTS.

| Column No. | Type and position of roller axes. | Position of gauged portion. | Number of gauge lines averaged. | Length of gauge, in inches. | Average Modulus of Elasticity, $E$, in 1 000 000 Pounds per Square Inch, for | | Range in $E$ in percentage (8-in. gauges). | Percentage variation from 30 000 000 lb. per sq. in. |
|---|---|---|---|---|---|---|---|---|
| | | | | | Gauged portion. | Column (8-in. gauges). | | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| 1 | | Middle | 4 | 8 | 27.8 | 27.8 | 1.5 | 7.7 |
| | | Middle | 4 | 20 | 28.2 | .... | .... | 6.0 |
| 2 | | Top quarter | 4 | 8 | 30.0 | 29.7 | 2.7 | 0.0 |
| | | Middle | 4 | 8 | 29.2 | .... | .... | 2.7 |
| | | Bottom quarter | 4 | 8 | 29.9 | .... | .... | 0.3 |
| | | Middle | 4 | 20 | 31.5 | .... | .... | 5.0 |
| 3 | | Middle | 4 | 8 | 30.4 | 30.4 | .... | 1.3 |
| | | Middle | 4 | 20 | 28.6 | .... | .... | 4.7 |
| 3A | | Middle | 4 | 8 | 27.9 | 27.5 | .... | 7.0 |
| | | Middle | 4 | 20 | 27.4 | .... | .... | 8.7 |
| 4 | | Top quarter | 4 | 8 | 30.2 | 30.0 | 7.0 | 0.7 |
| | | Middle | 4 | 8 | 28.9 | .... | .... | 3.7 |
| | | Bottom quarter | 4 | 8 | 31.0 | .... | .... | 3.3 |
| | | Middle | 4 | 20 | 29.9 | .... | .... | 0.3 |
| 5 | | Top quarter | 4 | 8 | 31.6 | 30.6 | 6.2 | 5.3 |
| | | Middle | 4 | 8 | 30.6 | .... | .... | 2.0 |
| | | Bottom quarter | 4 | 8 | 29.7 | .... | .... | 1.0 |
| 5A | | Top quarter | 4 | 8 | 31.8 | 30.0 | 9.7 | 6.0 |
| | | Middle | 4 | 8 | 28.9 | .... | .... | 3.7 |
| | | Bottom quarter | 4 | 8 | 29.4 | .... | .... | 2.0 |
| | | Middle | 4 | 20 | 28.9 | .... | .... | 3.7 |
| 5B | | Top quarter | 4 | 8 | 31.2 | 30.5 | 4.9 | 4.0 |
| | | Middle | 4 | 8 | 29.7 | .... | .... | 1.0 |
| | | Bottom quarter | 4 | 8 | 30.6 | .... | .... | 2.0 |
| 6 | | Top quarter | 4 | 8 | 30.1 | 30.6 | 2.6 | 0.3 |
| | | Middle | 4 | 8 | 30.9 | .... | .... | 3.0 |
| | | Bottom quarter | 4 | 8 | 30.7 | .... | .... | 2.3 |
| | | Middle | 4 | 20 | 30.1 | .... | .... | 0.3 |

TABLE 25.—(*Continued.*)

| Column No. | Type and position of roller axes. | Position of gauged portion. | Number of gauge lines averaged. | Length of gauge, in inches. | Average Modulus of Elasticity, $E$, in 1 000 000 Pounds per Square Inch, for | | Range in $E$, in percentage (8-in. gauges). | Percentage variation from 30 000 000 lb. per sq. in. |
|---|---|---|---|---|---|---|---|---|
| | | | | | Gauged portion. | Column (8-in. gauges). | | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| 7 |  | Top quarter | 4 | 8 | 29.6 | 30.9 | 7.4 | 1.3 |
| | | Middle | 4 | 8 | 31.9 | .... | .... | 6.3 |
| | | Bottom quarter | 4 | 8 | 31.2 | .... | .... | 4.0 |
| | | Middle | 4 | 20 | 29.4 | .... | .... | 2.0 |
| 8 | | Top quarter | 4 | 8 | 28.7 | 28.6 | 0.7 | 4.3 |
| | | Middle | 4 | 8 | 28.5 | .... | .... | 5.0 |
| | | Bottom quarter | 4 | 8 | 28.7 | .... | .... | 4.3 |
| | | Middle | 4 | 20 | 29.7 | .... | .... | 1.0 |
| 8A | | Top quarter | 4 | 8 | 30.2 | 29.9 | 6.7 | 0.7 |
| | | Middle | 4 | 8 | 28.8 | .... | .... | 4.0 |
| | | Bottom quarter | 4 | 8 | 30.8 | .... | .... | 2.7 |
| 9 | | Top quarter | 4 | 8 | 30.8 | 30.7 | 9.8 | 2.7 |
| | | Middle | 4 | 8 | 32.2 | .... | .... | 7.3 |
| | | Bottom quarter | 4 | 8 | 29.2 | .... | .... | 2.7 |
| | | Middle | 4 | 20 | 30.2 | .... | .... | 0.7 |
| Average for 8-in. gauge lines | | | | | | 29.8 | 5.8 | 3.2* |
| Average for 20-in. gauge lines | | | | | | 29.4 | .... | 3.2* |
| Average for all gauge lines | | | | | | 29.6 | .... | 3.2* |

* Calculated by averaging the appropriate results in Column (9).

furnish convincing proof of the uniformity of the modulus of elasticity of steel within a given shape and as between shapes.

Values of $E$ from tests on the coupons cut out of these different shapes exhibit similar uniformity, but the average values of the tension and compression moduli from the H-column coupon tests, approximately 29 200 000 and 29 000 000 lb. per sq. in., respectively, are somewhat less than the values found in the column tests.

### (D) Column Theories: The Column in a Truss

It is not intended to give herein any considerable discussion of the various column formulas, but only a brief review of the various types and some points relative to their derivation and applicability.

*28.—The Euler Formula.*—This well-known formula for various end conditions has been shown by many tests conducted with carefully designed end conditions to be correct and reliable under the conditions assumed. It is based, as is well known, on the elastic modulus of the material and not on its strength. It is, therefore, applicable, if at all, only to lengths in which buckling can occur within the elastic limit of the material. It has proven very serviceable for such slender columns as airplane struts, but in the general field of structural engineering, it serves mainly as a guide for the limiting value, for the longer lengths, in the study of tests of columns and of column formulas, where conditions may approach those assumed in the Euler formula.

TABLE 26.—VALUES OF MODULUS OF ELASTICITY FOR DIFFERENT PORTIONS OF H-COLUMNS AS FOUND BY STRAIN GAUGE MEASUREMENTS IN WISCONSIN CENTER LOAD TESTS.

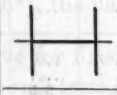
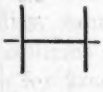
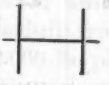
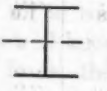
| Column No. | Position of roller axes. | Position of gauged portion. | Number of gauge lines, average. | Length of gauge, in inches. | Average Modulus of Elasticity, $E$, in 1 000 000 Pounds per Square Inch for: | | Range in $E$, in percentage (8-in. gauges). | Percentage variation from 30 000 000 lb. per sq. in. |
|---|---|---|---|---|---|---|---|---|
| | | | | | Gauged portion. | Column (8-in. gauges). | | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| H 1 | | Middle | 8 | 8 | 30.4 | 30.4 | .... | 1.3 |
| |  | Middle | 8 | 8 | 29.7 | 29.7 | .... | 1.0 |
| H 6 | | Middle | 4 | 8 | 29.8 | 29.8 | .... | 0.7 |
| | | Middle | 4 | 8 | 29.2 | 29.2 | .... | 2.7 |
| H 2 |  | Top quarter | 8 | 8 | 29.8 | .... | .... | 0.7 |
| | | Middle | 6 | 8 | 30.2 | .... | .... | 0.7 |
| | | Bottom quarter | 8 | 8 | 28.7 | 29.6 | 5.1 | 4.3 |
| | | Top quarter | 8 | 8 | 29 3 | .... | .... | 2.3 |
| | | Middle | 6 | 8 | 29.2 | .... | .... | 2.7 |
| | | Bottom quarter | 8 | 8 | 28.8 | 29.1 | 1.7 | 4.0 |
| H 7 | | Top quarter | 8 | 8 | 29.4 | .... | .... | 2.0 |
| | | Middle | 6 | 8 | 29.9 | .... | .... | 0.3 |
| | | Bottom quarter | 8 | 8 | 29.4 | 29.6 | 1.7 | 2.0 |
| | | Top quarter | 8 | 8 | 28.9 | .... | .... | 3.7 |
| | | Middle | 6 | 8 | 28.9 | .... | .... | 3.7 |
| | | Bottom quarter | 8 | 8 | 28.6 | 28.8 | 1.0 | 4.8 |
| H 3 | | Top quarter | .. | .. | .... | .... | .... | .... |
| | | Middle | 6 | 8 | 29.4 | .... | .... | 2.0 |
| | | Bottom quarter | 8 | 8 | 31.4 | 30.4 | 6.6 | 4.7 |
| | | Middle | 4 | 20 | 29.9 | .... | .... | 0.3 |
| | | Top quarter | 7 | 8 | .... | .... | .... | .... |
| | | Middle | 6 | 8 | 29.0 | .... | .... | 3.3 |
| | | Bottom quarter | 7 | 8 | 30.2 | 29.6 | 4.1 | 0.7 |
| | | Middle | 4 | 20 | 30.0 | .... | .... | 0.0 |

TABLE 26.—(*Continued.*)

| Column No. (1) | Position of roller axes. (2) | Position of gauged portion. (3) | Number of gauge lines, average. (4) | Length of gauge, in inches. (5) | Average Modulus of Elasticity, $E$, in 1 000 000 Pounds per Square Inch for: Gauged portion. (6) | Column (8-in. gauges). (7) | Range in $E$, in percentage (8-in. gauges). (8) | Percentage variation from 30 000 000 lb. per sq. in. (9) |
|---|---|---|---|---|---|---|---|---|
| H 8 | | Top quarter | 8 | 8 | 29.1 | .... | .... | 3.0 |
| | | Middle | 6 | 8 | 29.2 | .... | .... | 2.7 |
| | | Bottom quarter | 8 | 8 | 30.0 | 29.4 | 3.1 | 0.0 |
| | | Middle | 4 | 20 | 29.8 | .... | .... | 0.7 |
| |  | Top quarter | 8 | 8 | 28.7 | .... | .... | 4.3 |
| | | Middle | 6 | 8 | 29.7 | .... | .... | 1.0 |
| | | Bottom quarter | 8 | 8 | 30.7 | 29.7 | 6.7 | 2.3 |
| | | Middle | 4 | 20 | 30.2 | .... | .... | 0.7 |
| H 4 | | Top quarter | 8 | 8 | 31.4 | .... | .... | 4.7 |
| | | Middle | 6 | 8 | 31.4 | .... | .... | 4.7 |
| | | Bottom quarter | 8 | 8 | 29.7 | 30.8 | 5.5 | 1.0 |
| | | Middle | 4 | 20 | 29.8 | .... | .... | 0.7 |
| | | Top quarter | 8 | 8 | 29.0 | .... | .... | 3.3 |
| | | Middle | 6 | 8 | 29.2 | .... | .... | 2.7 |
| | | Bottom quarter | 8 | 8 | 29.5 | 29.2 | 1.7 | 1.7 |
| | | Middle | 4 | 20 | 30.0 | .... | .... | 0.0 |
| H 9 | | Top quarter | 8 | 8 | 30.4 | .... | .... | 1.3 |
| | | Middle | 6 | 8 | 29.7 | .... | .... | 1.0 |
| | | Bottom quarter | 8 | 8 | 29.7 | 29.9 | 2.3 | 1.0 |
| | | Middle | 4 | 20 | 29.5 | .... | .... | 1.7 |
| |  | Top quarter | 8 | 8 | 30.4 | .... | .... | 1.3 |
| | | Middle | 6 | 8 | 29.2 | .... | .... | 2.7 |
| | | Bottom quarter | 8 | 8 | 29.2 | 29.6 | 4.1 | 2.7 |
| | | Middle | 4 | 20 | 29.0 | .... | .... | 3.3 |
| H 5 | | Top quarter | .. | .. | .... | .... | .... | .... |
| | | Middle | 6 | 8 | 29.5 | .... | .... | 1.7 |
| | | Bottom quarter | 8 | 8 | 32.3 | 30.9 | 9.1 | 7.7 |
| | | Middle | 4 | 20 | 30.1 | .... | .... | 0.3 |
| | | Top quarter | 8 | 8 | 30.5 | .... | .... | 1.7 |
| | | Middle | 6 | 8 | 30.0 | .... | .... | 0.0 |
| | | Bottom quarter | 8 | 8 | 30.9 | 30.5 | 3.0 | 3.0 |
| | | Middle | 4 | 20 | 30.2 | .... | .... | 0.7 |
| H 10 |  | Top quarter | 8 | 8 | 30.4 | .... | .... | 1.3 |
| | | Middle | 6 | 8 | 30.6 | .... | .... | 2.0 |
| | | Bottom quarter | 8 | 8 | 29.8 | 30.3 | 2.6 | 0.7 |
| | | Middle | 4 | 20 | 29.3 | .... | .... | 2.3 |
| | | Top quarter | 8 | 8 | 28.3 | .... | .... | 5.7 |
| | | Middle | 6 | 8 | 29.4 | .... | .... | 2.0 |
| | | Bottom quarter | 8 | 8 | 29.3 | 29.0 | 3.8 | 2.3 |
| | | Middle | 4 | 20 | 30.3 | .... | .... | 1.0 |
| Average for 8-in. gauge lines | | | | | | 29.8 | 3.9 | 2.4* |
| Average for 20-in. gauge lines | | | | | | 29.8 | .... | 1.0* |
| Average for all gauge lines | | | | | | 29.8 | .... | 2.1* |

* Calculated by averaging the appropriate results in Column (9).

For pivoted ends, the formula is,

$$P/A = \frac{\pi^2}{\left(\frac{l}{r}\right)^2} \quad \ldots\ldots (7)$$

For rigidly fixed ends, the formula becomes,

$$P/A = \frac{4\,\pi^2\,\varepsilon}{\left(\frac{l}{r}\right)^2} \quad \ldots\ldots (8)$$

or the same result as by using the half length in Equation (7).

In some respects, it is helpful, in applying a column formula to a particular case, to estimate, from a knowledge of end conditions, the probable points of inflection and then to use the resulting "free length" in a formula for pivoted ends. Thus, in the tests plotted in Figs. 7, 8, and 9 (report of 1926*), the flat-end tests appear to agree, for high values of $\frac{l}{r}$, with the Euler curve for fixed ends for a value of 1.12 $l$, or for a free length of 0.56 $l$; and the pin-end tests, for a value of 0.78 $l$; that is, in the case of the flat-end tests, the points of inflection were about 0.56 $l$ apart, and for the pin-end tests about 0.78 $l$ apart; the flat-end columns were not quite fixed; and the pin-end columns were about midway between theoretically fixed and pivoted conditions.

Whatever the column formula adopted, it is obvious that for large values of $\frac{l}{r}$, the formula should give results somewhere between those of the Euler formulas for pivoted and for fixed ends; for columns with very little end rigidity, values nearly as low as the Euler formula for pivoted ends; and for columns well fixed, values somewhat less than those of the Euler formula for fixed ends.

*29.—Karman's and Engesser's Variations.*—These formulas attempt to use the Euler method and apply it to the actual stress-strain diagram of the material beyond its elastic limit. They also take account of the effect of form of section. This is a rational method, and fair agreement is found between theory and test results; but to use it in practice requires the assumption of a standard stress-strain diagram of the material. The actual variations from the average in this respect and the variations due to other causes leads to the conclusion that the application of these refinements is not warranted in practice.

*30.—The Theoretical Formula for Eccentric Loads.*—A very useful formula for many purposes is the theoretical formula for pivoted-end columns with loads applied with a known eccentricity, the same at both ends. Inasmuch as considerable use will be made of formulas of this type in subsequent discussions, it will be convenient to list herein the notation used, repeating part of that already used:

$p = P/A$ = average compressive stress per unit area of column;

$e$ = eccentricity;

* *Transactions*, Am. Soc. C. E., Vol. 89 (1926), pp. 1527–1529.

$c$ = distance from gravity axis to extreme fiber;

$\frac{e\,c}{r^2}$ = eccentric ratio;

$p_e = \frac{\pi^2 E}{\left(\frac{l}{r}\right)^2}$ = Euler load per square inch for centrally loaded pivoted-end columns;

$\phi = \frac{\pi}{2}\sqrt{\frac{p}{p_e}}$, a function of frequent occurrence;

$q = \sqrt{\frac{P}{E\,I}}$;

$f_b$ = intensity of bending stress on extreme fiber at center;

$f$ = total unit stress on extreme fiber at center on concave side.

Referring to Fig. 32, the equation of the elastic line is,

$$y = e\,(\tan\phi \sin q\,x + \cos q\,x - 1)$$

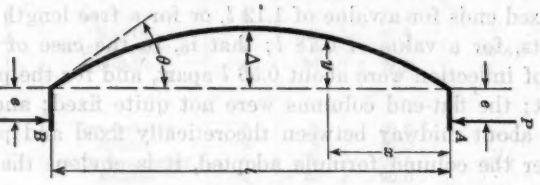



FIG. 32.

The inclination at the end is,

$$\frac{d\,y}{d\,x}(x = 0) = \theta = e\,q \tan\phi \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$$f_b = p\,\frac{e\,c}{r^2}\sec\phi$$

The total stress is,

$$f = p + f_b = p\left(1 + \frac{e\,c}{r^2}\sec\phi\right)$$

and,

$$p = \frac{f}{1 + \frac{e\,c}{r^2}\sec\phi} \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Placing $f$ = yield-point strength of the material, the value of $p$ will approximate to the ultimate strength of the column.

If the loads are applied centrally, but the column is initially bent to an amount equal to $e$ at the center, the resulting stresses are somewhat less than for an eccentricity of loading of the same amount, but the difference is not great enough to warrant special treatment of this case. Where there is both eccentricity of load and crookedness of column, both eccentricities may be combined with little error.*

* For full discussion, see work on "Columns," by E. H. Salmon, Oxford Technical Publications, pp. 29, 42.

The foregoing formula, frequently known as the "secant" formula, is satisfactory from a theoretical standpoint, and for analytical purposes has many advantages over others; but for practical use, the eccentricity, $e$, must be assumed or determined by tests or observations, in much the same manner as the constants in other formulas, and when this is done, the formula is not readily solved for particular values of $\frac{l}{r}$, as the value of $p$ is included also in the quantity, $\phi$. This objection can be removed by the use of tables, but there is something to be said in favor of a formula that can be readily applied to any particular case. Various approximations to the secant formula have been proposed, the most elaborate study known to the Committee being that by Mr. R. W. Hawken,* in which he analyzes about seven different approximations and proposes one of his own.†

The accuracy of the secant formula when the eccentricity is known is well indicated by the results of the eccentric load tests of Series (*C*). The value of $E$ used in each case was that for the column in question, but as these values varied not more than 3% from the average, an average value for this quantity might well have been used. So far as these tests are concerned, therefore, the irregularities are of much less importance than Salmon predicated in his treatment of this question.

*31.—Fixed Ends.*—A column with ends fixed in direction at all loads can have no eccentricity of loading; that is, if the column is held securely in alignment by the end supports, there can be no bending effect at the ends due to eccentricity of loading. As a matter of fact, most columns rigidly connected in a framework are not really fixed-end columns, but are subjected to a certain amount of end moment and rotation of axis at the end, due to the action of the framework as a whole. This action will be discussed subsequently; but such a column is not a fixed-end column. The only kind of eccentricity which can exist in a fixed-end column is that (1) due to crookedness or curvature of

* "Column Analysis and Design," Univ. of Queensland, Sydney, 1918; see, also, "A Simplified Column Formula of the Secant Type," by J. B. Hunley, *Bulletin No. 300*, Am. Ry. Eng. Assoc., 1927.

† The approximate formula proposed by Hawken is,

$$p = \frac{f}{1 + \frac{e\,c}{r^2} \cdot \frac{1 + 0.25\,\frac{p}{p_e}}{1 - \frac{p}{p_e}}}$$

in which, $\sec \frac{\pi}{2}\sqrt{\frac{p}{p_e}}$ of Equation (10) is replaced by the quantity, $\frac{1 + 0.25\,\frac{p}{p_e}}{1 - \frac{p}{p_e}}$. This is remarkably close to the true value, giving results for $p$ in error by less than 1% for values of $\frac{p}{p_e}$ as great as 0.9. This equation is readily solvable for $p_e$, giving,

$$p_e = p + \frac{\frac{f}{p} - \left(1 - 0.25\,\frac{e\,c}{r^2}\right)}{\frac{f}{p} - \left(1 + \frac{e\,c}{r^2}\right)}$$

and from the relation $\left(\frac{l}{r}\right)^2 = \frac{\pi^2 E}{p_e}$, various values of $\frac{l}{r}$ can be directly calculated for various values of $p$, and curves or tables prepared for various values of $\frac{e\,c}{r^2}$.

the axis of resistance; (2) due to actual crookedness of the section; or (3) due to variation in the section or in the modulus of elasticity. The effect of a given amount of initial curvature can be determined by the same general process as applied to the eccentrically loaded, pivoted-end column. The mathematical process is somewhat complicated, and it will be sufficient to give here the general results as contained in Salmon's analysis. He shows that the bending stresses in such a case, assuming parabolic initial curvature, are, for small values of $\frac{p}{p_e}$ $\left(\text{low loads, or low values of } \frac{l}{r}\right)$, about twice as great at the end as the center, like the bending stresses in an ordinary beam fixed at the ends. The sum of these end and center stresses is somewhat less than the center fiber stress in a pivoted end column of half the length, with the same initial eccentricity. For higher loads, the center and end stresses approach equality, and the sum of the two differs by an increasing amount from the fiber stress in a pivoted-end column of half length. Considering all the conditions, it may be concluded that the effect of eccentricity due to initial bends in a truly fixed-end column, on the bending stress in the column, is not greatly different from the effect of an eccentricity of one-half as much on a pivoted-end column of one-half the length. This will be very small.

*32.—The Rankine Formula.*—This well-known equation is of the form,

$$p = \frac{f}{1 + K\left(\frac{l}{r}\right)^2} \qquad (11)$$

in which, $K$ is a constant depending on the end conditions and rests on experimental determination. For the ultimate strength, $f$ is approximately the yield point of the material.

A rather serious objection to this formula is that, in the nature of the case, $K$ can hardly be a constant, as may readily be shown. The formula can be written, $f = p + p\,K\left(\frac{l}{r}\right)^2$, in which, the term, $p\,K\left(\frac{l}{r}\right)^2$, represents the bending fiber stress, $f_b$, at the center. Also, necessarily, $f_b = p\,\frac{\Delta\, c}{r^2}$, in which, $\Delta$ equals the total eccentricity at the center of the column. If this eccentricity is all due to bending, for any beam, $\Delta = \frac{K' f_b l^2}{c}$, in which, $K'$ is a constant depending on the manner of loading. $\left(\text{For a uniformly loaded beam, } \Delta = \frac{5}{48}\,\frac{f_b\, l^2}{c\, E}, \text{ and } K' = \frac{5}{48\, E}\right)$. Then,

$$K\left(\frac{l}{r}\right)^2 = \frac{\Delta\, c}{r^2} = \frac{K' f_b\, l^2\, c}{c\, r^2} = K' f_b \frac{l^2}{r^2}$$

or $K = K' f_b$. That is, the "constant", $K$, is proportional to $f_b$. If the eccentricity, $\Delta$, is due largely to initial bends and eccentricities, then $K$ is more nearly a constant, but, in any case, at failure, there will be a considerable degree

of bending and hence $K$ will not be a constant, but will increase with an increase in the bending stress, $f_b$. The fact that $K$ should increase with $f_b$, or with $\frac{l}{r}$, is indicated in the comparison of the formula with results of tests in the 1926 report of the Committee,* in which, the Rankine formula is seen to descend too rapidly from the horizontal for low values of $\frac{l}{r}$. An increasing value of $K$ with proper constants would give a flatter curvature near the origin and larger values of $p$.

While the Rankine formula, making $K$ a constant, is irrational, it has been widely used, and by selecting proper constants, it can be made to fit results of tests fairly well over the working range. A common method of adjustment is to use a relatively large value of $f$ so as to raise the curve near the $Y$-axis and then to slice off the top by adopting a maximum limit for $p$ somewhat less than $f$.

*33.—Empirical Formulas.*—Many straight-line formulas have been, and still are, in use. These are usually determined with reference to the yield point strength for low values of $\frac{l}{r}$; to the Euler formula for very high values; and to results of tests between these limits. The Johnson parabolic formula is perhaps the best of the empirical formulas so far as it goes. It is a parabola having its vertex on the $Y$-axis, and drawn tangent to a suitable Euler curve. It is not applicable beyond its point of tangency, which is at a point the $y$-ordinate of which equals one-half the ordinate for $x = 0$, or $p = \frac{1}{2} f$.

*34.—Shear in Columns.*—Two cases need to be considered:

(*a*) The shear in a column bent in single curvature: This will be a maximum at the ends. For the theoretical case of the pivoted-end column with eccentric loads, the shear is, from Equation (9),

$$V = P \tan \theta = P e q \tan \phi = P e \sqrt{\frac{P}{E I}} \tan \frac{l}{2} \sqrt{\frac{P}{E I}} \quad \ldots\ldots (12)$$

For practical calculations it may be assumed that the column at failure is bent in a parabolic curve with an inclination at the end of $\tan \theta = \frac{4 e'}{l}$, in which, $e'$ is the central ordinate. The bending stress is, $f_b = \frac{p e' c}{r^2}$, or $e' = \frac{f_b r^2}{p c}$; hence, $\tan \theta = \frac{4 f_b r^2}{p c l}$, and the shear is,

$$V = P \frac{4 f_b r^2}{p c l} \quad \ldots\ldots (13)$$

At failure the bending stress, $f_b$, is the stress, $f - p$, from any rational column formula, $f$ being the yield point and $p$, the ultimate unit load on the column. Equation (13) gives the shear corresponding to the ultimate load, $P$.

(*b*) The shear in a column of a truss with rigid joints where the joint deformation or twisting is such as to cause the column to bend in an

* *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1485.

S-shaped curve. The moments at the ends are the moments causing secondary stresses in the members. Columns subject to such loads are in effect acted on by direct forces applied eccentrically at each end, but of opposite direction, resulting in an obliquity of the applied load. If the eccentricity of each end equals $e$, the obliquity is $\frac{2\,e}{l}$, and the shear is,

$$V = P\,\frac{2\,e}{l} \qquad (14)$$

The quantity, $e$, may be expressed in terms of secondary stress or of ratio of secondary to axial stress. The secondary stress (bending fiber stress) due to an eccentricity, $e$, is $f_b = p\,\frac{e\,c}{r^2}$ and $\frac{f_b}{p} = \frac{e\,c}{r^2}$ = stress ratio = $\alpha$. Then, $e = \frac{\alpha\,r^2}{c}$, and,

$$V = P\,\frac{2\,\alpha\,r^2}{c\,l} \qquad (15)$$

This will be large only for small values of $\frac{l}{r}$. The values of $\alpha$ will vary greatly, being high for short members and for bending in the plane of the truss. In this plane the value of $\alpha$ may easily reach 0.40 or 0.50, but in a plane perpendicular to the truss, which is generally the plane of the lacing, the secondary stresses due to vertical loads will be small, and a value of 0.20 is probably a sufficiently high estimate. This value gives a shear of $V = P\,\frac{0.4\,r^2}{c\,l}$. Comparing this result with Equation (13) it will be seen that it gives higher values of $V$ whenever $\frac{f_b}{p}$ is less than 0.1; that is, for relatively short columns.

*35.—Factor of Safety Method in Column Design.*—Whatever formula may be used, the logical process in column design is to select a formula which will as nearly as practicable give values of ultimate strength, or some point which represents the practical ultimate value of the column; then to divide the resulting value of average stress, $P/A$, by the factor of safety corresponding to that used in other members, the object being to produce a structure in which all parts will be stressed to failure, or to their useful limits, simultaneously as the loads are increased.

Columns, unlike other members, are stressed by an amount (maximum fiber stress) which increases at a faster rate than the applied load, and hence if they should be designed so that under working loads the actual maximum fiber stress is, say, one-half the yield point of the material, an increase of load of 50% would produce more than a 50% increase in fiber stress, and the factor of safety for the column would be considerably less than 2.0. In most column formulas (where the load, $p$, is expressed as an explicit function of $f$), the working formula is obtained by applying the factor of safety to the entire expression for $p$ (the numerator in the Rankine formula), but in the case of the secant formula this cannot be done. If the factor of safety be applied to the numerator, $f$, the result will be a value of $p$ which will produce the assumed fiber stress, $f$. In

this case, ultimate values of $p$ must be tabulated, and the factor of safety applied to them. The same comments may be made with respect to shearing stresses. The logical method is to estimate shearing stresses at or near failure and apply a consistent factor of safety to such values for design purposes.

*36.—The Action of a Column in a Truss.*—The study of the strength of columns has necessarily been based almost wholly on results of tests in a testing machine, where the end conditions have been made as definite as possible, either flat-ended (corresponding generally to a fixed-end condition), or pin-ended, or some approximation to pivot ends. It has been difficult to arrange end conditions of any kind so as to meet precisely the corresponding theoretical conditions. In the case of flat-end tests much shimming has been required to adjust for inaccurately cut ends or non-parallel testing-machine plates. To simulate pivoted ends requires either knife-edge bearings, suitable for very small columns, or some other frictionless bearing. The pin bearing commonly used is far from frictionless, in fact, a pin-ended column in a testing machine acts at first like a fixed-end column, and if reasonably straight, continues so to act nearly or quite up to the maximum load. For pivoted ends, the roller device used in the tests described in Part I is probably the best practicable arrangement. On account of its very small frictional resistance, it gives a close approximation to the truly pivoted end; and because of its definiteness it would seem the most satisfactory arrangement to use in studies for the correlation of tests with theory and for the relative effects of the various elements of column construction. When this is all done, however, there remains the important task of relating the results of these tests and analyses on pivoted-end columns to the behavior of columns in a structure.

To arrive at some judgment in the matter, it is desirable first to consider the action of a compression member in a truss with riveted joints. Consider, first, the action in the plane of the truss. The effect of loading such a structure is to develop certain bending moments in all the members at a joint, giving rise to what are usually known as secondary stresses. These deformations and stresses are caused by the changes in the length of the various members, and are subject to reasonably close calculation. Some of the members will be bent in opposite directions at the two ends, giving rise to an S-shaped curvature; others will be bent in single curvature. The top chord members in a Pratt truss in all panels except the end ones are generally examples of single curvature. The verticals and end posts are likely to be bent in reversed curves. In any case, the angle of bend at each end is a fairly definite and calculable quantity, depending almost wholly on the axial strains in the various members. So far as this bending affects the strength of the compression member, it is obvious that the worst case is where the bending is of single curvature, thereby forcing the member into a form giving the maximum deflection near the center and thus reducing the strength of the member as a column. This bending action always occurs in a truss, but it is an effect not easily produced in a testing machine. This action may be investigated theoretically as follows.

Assume $A\,B$ (Fig. 33) to be a column in a truss, subjected to an axial load, $P$, and moments, $M$, at each end. These moments are due to the deforma-

tion of the truss as a whole and, within certain limits, are proportional to the load on the structure and the load, $P$.

The bending moment, $M$, at any point, $C$, a distance, $x$, from $A$, is,

$$M = M_1 + P\,y$$

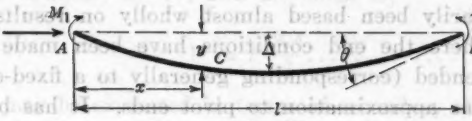



FIG. 33.

Then,

$$\frac{d^2 y}{d x^2} = -\frac{M}{E I} = -\frac{M_1}{E I} - \frac{P y}{E I}$$

From this is derived, as in the secant formula:

$$y = \frac{M_1}{P}\,[\sec q\,x - 1] \quad \ldots\ldots (16)$$

$$\tan \theta = \frac{q\,M_1}{P} \tan \phi \quad \ldots\ldots (17)$$

and,

$$\text{Center deflection} = \Delta = \frac{M_1}{P}\,[\sec \phi - 1] \quad \ldots\ldots (18)$$

Now, $\theta$ is the deflection angle fixed by the twist of the joints, $A$ and $B$, due to the distortion of the truss, and is, up to loads approaching the ultimate, closely proportional to the load or to the stress, $P$. These angles, $\theta$, are functions from which the secondary stresses may be calculated, and their magnitudes may be estimated by expressing them in terms of the corresponding bending or secondary stresses. For a member bent in single curvature with equal deflection angles, $\theta$, at each end, the corresponding secondary or bending stress at the end is,

$$f_s = \frac{2\,c\,E}{l} \tan \theta$$

Expressing the secondary stress as a fraction, $\alpha$, of the axial unit stress, $p$,

$$f_s = \alpha\,p = \frac{2\,c\,E \tan \theta}{l}$$

or,

$$\tan \theta = \frac{\alpha\,p\,l}{2\,c\,E} \quad \ldots\ldots (19)$$

in which, $\alpha$ has a numerical value amounting ordinarily, in the cases considered, to 0.1 to 0.2; and in any particular case $\tan \theta$ will increase directly with the load or the value of $p$. This value of $\tan \theta$ may then be used in Equation (17); then solving,

$$M_1 = \frac{\alpha\,p\,l\,P}{2\,c\,E\,q \tan \phi} \quad \ldots\ldots (20)$$

Substituting $p_e$ (Euler's value) for $\frac{\pi^2 E}{\left(\frac{l}{r}\right)^2}$, and $\phi\sqrt{2}$ for $\frac{\pi}{2}\sqrt{\frac{p}{p_e}}$,

$$M_1 = \frac{\alpha P r^2 \phi}{c \tan \phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

and, at the center,

$$M_c = M_1 + P y = M_1 [1 + \sec \phi - 1] = \frac{\alpha P r^2 \phi}{c \sin \phi}$$

The bending stress due to $M_c$ is,

$$f_b = \frac{M_c c}{I} = \alpha p \frac{\phi}{\sin \phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

This represents the bending stress at the center of the column the ends of which are gradually turned through an angle, $\theta$, proportional to the stress, $p$, but which are otherwise securely held. If $f$ = total fiber stress (yield point at failure) then, as before,

$$f = p\left(1 + \frac{\alpha \phi}{\sin \phi}\right)$$

and,

$$p = \frac{f}{1 + \frac{\alpha \phi}{\sin \phi}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

as the corresponding column formula.

Comparing this with the secant formula for pivoted ends, and assuming $\alpha = \frac{e c}{r^2}$, it will be noted that sec $\phi$ has been replaced by $\frac{\phi}{\sin \phi}$. In Fig. 34, Curve $A$ is the secant formula of Equation (10) and Curve $B$ is the formula of Equation (23). In these calculations, $f = 36\ 000$, $\frac{e c}{r^2} = \alpha = 0.25$, and $E = 29\ 000\ 000$. There is also a plotted curve, $C$, from Equation (10) for pivoted ends, but using in the formula the half length, corresponding to Euler's formula for fixed ends. This is,

$$p = \frac{f}{1 + \frac{e c}{r^2} \sec \frac{\phi}{2}}$$

It is noted that Curve $B$ lies somewhat below Curve $C$, showing that the end conditions assumed are not much different from fixed-end conditions with initial eccentricity due to curvature.

It may be concluded, therefore, that the strength of a column in a truss under the conditions assumed is not much less than that of a pivoted-end column of one-half the length.

In the case of columns bent in a reversed curve by the secondary stress moments, the bending stresses will be a maximum at the ends and small near the center, so that buckling action will be very small. The strength of such columns will be closely determined by the sum of the direct and bending stresses at the ends, as in the case of oblique loading tests given in Part I.

The strength of columns against buckling at right angles to the plane of the truss is not so readily estimated as in the case already considered. The

secondary stress moments and distortions in this direction are certainly quite small in top chord members of through bridges; but on the other hand, the lateral members do not offer very great resistance to the lateral buckling of chord members, in alternate directions in successive panels, giving a much nearer approach to pin-end conditions than the action in the plane of the truss. This apparent relative weakness in a lateral direction can be equalized by increasing the slenderness ratio, $\frac{l}{r}$, in this plane, a variation corresponding to common practice. Vertical posts with floor-beams and sway-bracing attached are thoroughly supported laterally, and act very definitely in the manner assumed in the foregoing analysis. The buckling tendency is small, and the maximum stress is likely to be at the top of the floor-beam. Compression web members in triangular trusses and the like are apt to be much less securely held against lateral bending than against bending in a vertical plane, although when riveted into heavy chord members they are quite well fixed. Here, again, equalization of buckling strength may be given by increasing the value of $\frac{l}{r}$ in a lateral direction.

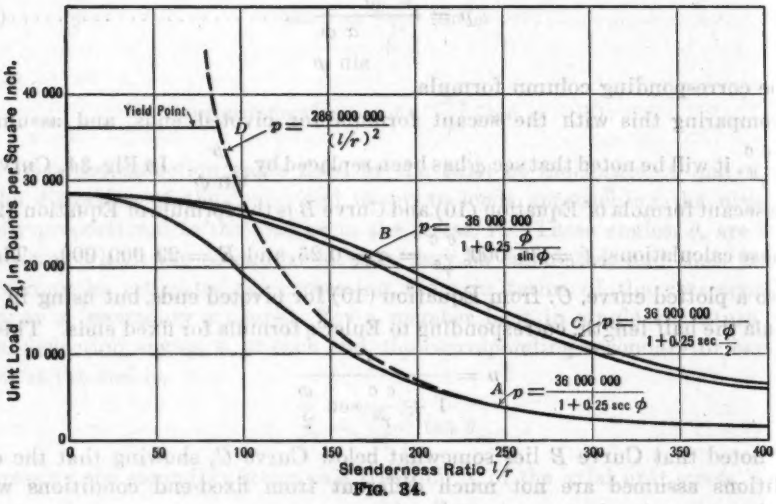



FIG. 34.

*37.—Effect of Dead Weight.*—The effect of the weight of the member on the center moment is of considerable interest. An exact analysis is quite complex, but an approximate estimate may be made by comparing the center deflection which may occur under the conditions already considered with the center deflection of the column due to its own weight, assuming fixed ends and the column to be horizontal.

The center deflection, not considering weight, is given by Equation (18). Substituting the value of $M_1$ from Equation (21),

$$\Delta = \frac{\alpha r^2 \phi}{c} \frac{(1 - \cos \phi)}{\sin \phi}$$

The deflection of a beam fixed at the ends is $D = \frac{w\,l^4}{384\,E\,I}$, in which, $w$ = the weight per unit length. This is approximately equal to $0.35A$ lb. per in. of length, whence,

$$D = \frac{0.35\,l^2}{384\,E}\left(\frac{l}{r}\right)^2$$

For purposes of comparison, consider, for example, a slender member with $\phi = \frac{\pi}{2}$; $\frac{l}{r}$ will equal about 108. Assume $l = 648$ in., $r = 6$ in., $c = 8$ in. For these values, $\Delta = 2.16$ in. and $D = 0.15$ in., or only 7% of $\Delta$. Again, take $\phi = \frac{\pi}{4}$, for which $\frac{l}{r}$ is about 75. With the same values of $r$ and $c$, the deflections will be, respectively, 0.42 in. and 0.032 in., the latter being 7.5% of the former. It appears, therefore, that the effect of weight is of little relative importance under the conditions assumed.

*38.—Effect of Obliquity of Flat Ends.*—Where dependence is placed mainly on butted ends, with relatively few rivets in splice-plates, the effect of variation from the true angle can readily be determined from the foregoing analysis. Assuming the worst condition, that the obliquity is the same at each end, so that the member is too short along one edge and too long along the other, the effect of any angle of obliquity is to cause a bending stress, found from Equation (19) by solving for $\alpha\,p$ ($\tan\theta$ will equal $\theta$), giving,

$$\alpha\,p = \frac{2\,c\,E}{l}\,\theta \qquad \ldots\ldots(24)$$

in which, $\theta$ = the angle of obliquity at each end and $\alpha\,p$ is the resulting bending stress.

This bending stress is to be added to that already found. In addition, the resulting deflection will have the same effect as initial curvature, the center deflection corresponding to the obliquity, $\theta$, being,

$$\Delta = \frac{\theta\,l}{4} \qquad \ldots\ldots(25)$$

Obviously, it is quite important in the case of large structures, where butted ends are depended upon, that the facing be accurately done. The magnitude of the allowable obliquity for any assumed value of stress, $\alpha\,p$, is, from Equation (24), $\theta = \frac{\alpha\,p}{2\,c\,E}$. For example, assume $\alpha\,p = 2\,500$ lb. per sq. in. = about 10% of the ultimate strength. Also, let, $\frac{l}{2\,c} = 15$. Then,

$$\theta = \frac{2\,500 \times 15}{30\,000\,000} = 0.00125$$

which is the permissible error in angle of the end face, or an error of 0.032 in. in a width of 24 in.

*39.—The Effect of Initial Bends.*—In the case here considered, this effect is of the same nature as in the fixed-end column, namely, less than the effect of one-half the same amount of deflection in a pivoted-end column of one-

half the length, and can be approximately allowed for by adding to $\alpha$ in the column formula one-half the quantity, $\frac{e' c}{r^2}$, in which, $e' =$ the deflection due to initial bends.

*40.—Effect of Deflection Due to Shear.*—In the tests of Part I, the relative rigidities of various columns in the plane of the web and at right angles thereto were determined, and it was shown that in the case of the laced channel columns the shearing distortion in the plane of the lacing was considerable under oblique loading. The relative deflections under cross-bending loads were also found, and in the case of the laced columns, the observed deflections in the plane of the lacing were nearly twice as great as the calculated deflections in which the effect of shearing strains was omitted. This raises the question of the effect of shearing distortion on the bending of columns in the plane of lacing as compared to bending in the plane of solid webs. To answer this in part, tests of Series (*C*) were made under eccentric loads, and so far as these tests were concerned, the effect of shearing strain on deflection in the plane of lacing was negligible (see Fig. 28). However, this question is worthy of further analysis.

It will be helpful to compare the shearing and bending stresses in a column with those in a uniformly loaded beam.

From the secant formula, the total shearing force, $V$, at the end of a column is,

$$V = P\, eq \tan \phi$$

and that per unit of sectional area is,

$$v = p\, eq \tan \phi \qquad (26)$$

The bending stress at the center is,

$$f_b = p \frac{e c}{r^2} \sec \phi$$

and, therefore, the shear in terms of the bending stress, $f_b$, is,

$$v = f_b \frac{q r^2}{c} \sin \phi$$

Writing

$$q = \sqrt{\frac{P}{E I}} = \frac{\pi}{l} \sqrt{\frac{p}{p_e}}$$

and taking $r = 0.8c$, approximately,

$$v = f_b \frac{2.52 \sqrt{\frac{p}{p_e}}}{\frac{l}{r}} \sin \phi \qquad (27)$$

For a beam uniformly loaded, the bending stress is,

$$f_b = \frac{w l^2 c}{8 A r^2}$$

The end shearing stress per square inch is,

$$v = \frac{w\,l}{2\,A}$$

and hence, with $r = 0.8c$,

$$v = \frac{3.2\,f_b}{\frac{l}{r}} \qquad (28)$$

Comparing Equations (27) and (28) for the same bending stress, $f_b$, the ratio,

$$\frac{v\text{ (column)}}{v\text{ (beam)}} = \frac{2.52\sqrt{\frac{p}{p_e}}}{3.2}\sin\phi = 0.79\sqrt{\frac{p}{p_e}}\sin\phi$$

Calculating this quantity, the values given in Table 27 are found, based on the secant formula for ultimate strength, $p = \frac{36\,000}{1 + 0.25\sec\phi}$. The ratio of shear in column to direct stress $\left(\frac{V}{P}\right)$ is also given.

TABLE 27.—Relation of Shears in Columns and Beams for Equal Bending Stresses.

| $p$. | $\frac{p}{p_e}$. | $\frac{l}{r}$. | $\frac{v\text{ (column)}}{v\text{ (beam)}}$, for equal bending stress. | $\frac{V}{P}$. |
|---|---|---|---|---|
| 28 500 | 0.047 | 22 | 0.058 | 0.0022 |
| 28 000 | 0.1 | 32 | 0.12 | 0.0034 |
| 27 100 | 0.2 | 45 | 0.23 | 0.0052 |
| 26 000 | 0.3 | 57 | 0.33 | 0.0072 |
| 24 700 | 0.4 | 69 | 0.41 | 0.0090 |
| 23 000 | 0.5 | 79 | 0.51 | 0.011 |
| 21 000 | 0.6 | 91 | 0.57 | 0.014 |
| 18 200 | 0.7 | 105 | 0.64 | 0.019 |
| 14 500 | 0.8 | 125 | 0.70 | 0.026 |
| 8 900 | 0.9 | 170 | 0.75 | 0.045 |

For ordinary values of $\frac{l}{r} = 50$ to 100, the ratio of shears for the same bending stress ranges from about 0.25 to 0.6.

In the cross-bending tests of Part I, the deflection due to shear in the laced channel columns averaged about 0.8 of that due to bending, and the value of $\frac{l}{r}$ was about 20. These bending tests were made with center loads. If they had been made with uniform loads, giving a shear distribution more nearly like that of a column, it can be shown that the average shearing stress for like deflection would have been 0.8 as much and hence the shearing distortions would have been less than those shown. Hence, $0.8 \times 0.8$, or about $\frac{2}{3}$ may be taken as representing the ratio of deflection due to shear if tested with uniform loads, to that due to bending under a central load. For the columns tested

in Series (*C*), $\frac{l}{r} = 21.9$, for which the ratio of column shear to beam shear, as shown in Table 27, is about 0.06, and, hence, the relative shear distortion for this column, as a column, would be about $0.06 \times \frac{2}{3} = 0.04$ of that due to bending. Cross-bending tests of columns with $\frac{l}{r} = 50$, and with similar proportions to these tested, would, in accordance with Equation (28), have shown a relative shear deflection of about $\frac{2}{3} \times \frac{20}{50} = 0.26$; and for a column of $\frac{l}{r} = 50$, the shearing stress for a like bending stress (see Table 27) would be about 0.3 as much as in a beam, and hence the deflection due to shear would be expected to be about $0.3 \times 0.26 = 0.08$ of that due to pure bending as assumed in the column analysis. For $\frac{l}{r} = 100$, the result would be about the same.

This gives an approximate measure of the relative rigidity in the plane of the lacing as compared to that in the plane of the webs. While other proportions of lacing and other tests would doubtless vary these results somewhat, this analysis is sufficient to indicate that shearing distortion is not of great relative effect upon the strength of columns. In this case, the actual effect upon the theoretical ultimate strength could be found by using a value of $E$ in the formula about 10% less than the value for direct stress, or, say, 26 000 000 lb. per sq. in. Any difference in strength in the two directions can be overcome by increasing the value of $r$, that is, by spreading the channels farther apart.

### (C) Strength of Columns as Affected by Various Details

#### 1.—Eccentricities in the Wisconsin Test Columns

*41.—Method of Measuring Eccentricity and Crookedness.*—In order to secure data for determining the gravity axis of each H-column, micrometer measurements of both end cross-sections were made and recorded on separate carbon prints. From these data the location of the column axis with respect to the center of the web was calculated. This information was used in centering the columns on the bearing blocks for testing.

In order to secure some information concerning the straightness of these test columns, two methods of measurement were used. In the more accurate method, which was conveniently used on the short built-up columns, two transits were set up so that the lines of sight described north and south planes about 18 in. apart; and two other transits were arranged to describe similarly spaced east and west planes. Within these imaginary planes the column was set vertically on a base plate which was accurately leveled with a very sensitive spirit-level. By holding steel rules normal to the column it was possible to determine the distance from each of the four imaginary planes to the column. Twelve such measurements, three per plane, were made at each end, at the quarter-points, and at the middle. They were recorded on suitable sketches. Then the column was turned upside down to determine the obliquity of the upper end. From these observations, crookedness, obliquity of ends, and the amount of wind were calculated.

The straightness of an H-column was determined by supporting it in a horizontal position and making measurements from planes formed by four fine wires. These wires were spaced fixed distances apart and ran along the corners of the column.

The obliquity of the ends of the H-columns was determined by measuring the thickness of the shims required under the lower bearing block to produce parallelism between the top of the column and the upper bearing.

By averaging all measurements in either direction for each section of the column, and comparing these mean measurements with the end means, a measure of crookedness was obtained. Herein, only the crookedness as referred to the mid-section will be reported.

*42.—Eccentricities in H-Columns Due to Rolling.*—The results of the center of gravity determinations on the 8-in. columns are given in Columns (3) and (4) of Table 28. For these columns the values of $\frac{c}{r^2}$ for the central axis normal to the web is approximately 0.3, whereas for the other gravity axis it is about 1.0. Hence, in a short column, an eccentricity of 0.06 in. in the plane of the web causes a bending stress of only 2%, whereas in the plane normal to the web it amounts to 6 per cent. Such eccentricity normal to the web would correspond to a displacement of the web of about 0.09 in. In a long column with a slenderness ratio of 150, the reduction in strength due to an eccentricity of 0.06 in. normal to the web would amount to about 5 per cent. In most cases the tabulated eccentricities are well below 0.06 in. Hence, aside from inconvenience in making connections to adjoining members, the observed rolling eccentricities are of importance in these columns.

*43.—Crookedness.*—The data in Column (5) of Table 29, show that crookedness is a far more important defect than rolling eccentricity in these small H-columns. The bowing in a plane normal to the web in Columns Nos. *H*9 and *H*10 would have caused an appreciable reduction in the strength of these columns had they been tested to destruction with rollers parallel to the webs. In Fig. 35 the crookedness in these H-columns has been plotted against the slenderness ratio. The crookedness at mid-section for the built-up columns has also been plotted against the slenderness ratio in Fig. 36, and the crookedness of both types has been plotted against the length of column in Fig. 37. These data show that, for columns of similar length, the built-up columns were the more crooked.

To enable comparisons to be made, the curves suggested by Salmon* have been incorporated in these diagrams. In Fig. 37 Salmon's curve lies well above the field of plotted points. Hence, the crookedness ($e'$) for one of these columns estimated by $e' = \frac{l}{750}$, would in most cases be considerably higher than the actual value. For these tests the crookedness was in practically all cases less than that given by $e' = \frac{l}{1\,500}$. In Figs. 35 and 36, plotted with

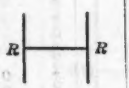
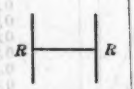
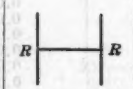
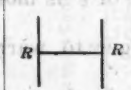
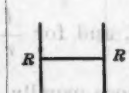
* "Columns," by E. H. Salmon, Oxford Technical Publications, p. 152.

## TABLE 28.—Eccentricities and Crookednesses of 8-Inch H-Columns, University of Wisconsin Tests.

| Column No. and length. | Position of rollers. | Eccentricity of Center of Web Due to Rolling, in Inches. | | Crookedness, $e'$, at mid-section, in inches. | Center Loading Test Eccentricity as Calculated from Measurements, in Inches. | | |
|---|---|---|---|---|---|---|---|
| | | Top. | Bottom. | | Top. | Bottom. | Mid-section. |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| $H$ 1 | | +0.02 | 0.00 | No measurements made, but column had good alignment. | 0.00 | 0.00 | 0.00± |
| $l$ = 39 in. | | +0.02 | +0.03 | | +0.03 | +0.04 | +0.04± |
| $H$ 2 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $l$ = 99 in. | | +0.05 | +0.03 | +0.12 | +0.08 | +0.06 | −0.05 |
| $H$ 3 | | +0.02 | 0.00 | +0.02 | +0.02 | 0.00 | +0.03 |
| $l$ = 168 in. | | +0.04 | +0.05 | −0.01 | −0.03 | 0.00 | −0.02 |
| $H$ 4 | | +0.02 | +0.01 | +0.09 | −0.07 | −0.09 | +0.01 |
| $l$ = 238 in. | | −0.02 | +0.04 | +0.03 | +0.01 | +0.06 | +0.07 |
| $H$ 5 | | +0.01 | 0.00 | −0.01 | −0.04 | −0.01 | −0.04 |
| $l$ = 307 in. | | −0.07 | +0.01 | −0.05<br>−0.10 | +0.04<br>+0.10 | +0.02<br>+0.08 | *−0.02<br>†−0.01 |

* Before over-strain. † After straightening.

TABLE 28.—(*Continued.*)

| Column No. and length. | Position of rollers. | Eccentricity of Center of Web Due to Rolling, in Inches. | | Crookedness, $e'$, at mid-section, in inches. | Center Loading Test Eccentricity as Calculated from Measurements, in Inches. | | |
|---|---|---|---|---|---|---|---|
| | | Top. | Bottom. | | Top. | Bottom. | Mid-section. |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| *H* 6 | | 0.00 | −0.01 | −0.01 | 0.00 | −0.01 | −0.01 |
| *l* = 40 in. |  | −0.02 | −0.01 | +0.01 | 0.00 | 0.00 | +0.01 |
| *H* 7 | | −0.01 | −0.03 | +0.01 | −0.01 | −0.03 | −0.01 |
| *l* = 99 in. |  | −0.01 | −0.02 | 0.00 | 0.00 | +0.01 | 0.00 |
| *H* 8 | | 0.00 | 0.00 | +0.02 | 0.00 | 0.00 | +0.02 |
| *l* = 168 in. |  | −0.01 | −0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| *H* 9 | | −0.04 | −0.04 | +0.08 | −0.01 | +0.01 | +0.08 |
| *l* = 238 in. |  | −0.02 | −0.04 | +0.16 | −0.15 | −0.16 | 0.00 |
| *H* 10 | | +0.06 | 0.00 | +0.04 | 0.00 | −0.02 | +0.03 |
| *l* = 307 in. |  | 0.00 | 0.00 | +0.23 | −0.14 | −0.16 | +0.08 |

respect to the slenderness ratio, Salmon's equation, $e' = 0.0023 \frac{l}{r}\left(\frac{l}{435\ r}\right)$, gives values too high for the H-columns and is more nearly a mean line than a limit for the built-up columns. For the H-columns practically all values lie below the boundary line representing $e' = \frac{l}{750\ r}$.

TABLE 29.—OBLIQUITIES IN WISCONSIN TEST COLUMNS.

| Column No. | Plane of obliquity referred to web. | Nominal width of column in plane of obliquity, in inches. | Obliquity. | |
|---|---|---|---|---|
| | | | In the width of the column, in inches. | Slope, in inches per inch. |
| (1) | (2) | (3) | (4) | (5) |
| *H* 4 | Normal | 8 | 0.005 | 0.0006 |
| *H* 6 | Normal | 8 | 0.024 | 0.0030 |
| *H* 7 | Parallel | 8 | 0.003 | 0.0004 |
| | Normal | 8 | 0.026 | 0.0033 |
| *H* 8 | Parallel | 8 | 0.026 | 0.0033 |
| | Normal | 8 | 0.023 | 0.0029 |
| *H* 9 | Parallel | 8 | 0.024 | 0.0030 |
| *H* 10 | Parallel | 8 | 0.032 | 0.0040 |
| | Normal | 8 | 0.045 | 0.0056 |
| 1 | Parallel | 10½ | 0.001 | 0.0001 |
| | Normal | 10 5/16 | 0.033 | 0.0032 |
| 2 | Parallel | 10 | 0.015 | 0.0015 |
| | Normal | 10 | 0.010 | 0.0010 |
| 2 *A* | Parallel | 10 | 0.053 | 0.0053 |
| | Normal | 10 | 0.008 | 0.0008 |
| 3 | Parallel | 10½ | 0.006 | 0.0006 |
| | Normal | 12 | 0.002 | 0.0002 |
| 3 *A* | Parallel | 10½ | 0.013 | 0.0012 |
| | Normal | 12 | 0.010 | 0.0008 |
| 4 | Parallel | 10 | 0.030 | 0.0030 |
| | Normal | 12 | 0.002 | 0.0002 |
| 5 *A* | Parallel | 10 | 0.003 | 0.0003 |
| | Normal | 12 | 0.036 | 0.0036 |
| 6 | Parallel | 10 | 0.028 | 0.0028 |
| | Normal | 12 | 0.004 | 0.0003 |
| 7 | Parallel | 10 | 0.006 | 0.0006 |
| | Normal | 12 | 0.002 | 0.0002 |
| 8 | Parallel | 10 | 0.013 | 0.0013 |
| | Normal | 12 | 0.020 | 0.0016 |
| 9 | Parallel | 10 | 0.019 | 0.0019 |
| | Normal | 12 | 0.007 | 0.0006 |

Considering all the results it would seem that the value of $e'$ is more nearly proportional to $l$ than to $\frac{l}{r}$, but the data are too meager to warrant any general conclusion. The relative influence of crookedness in these particular columns can be estimated to some advantage. Assume that $e' = \frac{l}{1\ 200}$ and $\frac{c}{r} = 1.2$; then $\frac{e'\ c}{r^2} = \frac{\frac{l}{r}}{1\ 000}$. For $\frac{l}{r} = 100$, $\frac{e'\ c}{r^2} = 0.1$, and for $\frac{l}{r} = 150$, $\frac{e'\ c}{r^2} = 0.15$. These values are considerably smaller than those usually assumed in the application of the theoretical formula for eccentric loads.

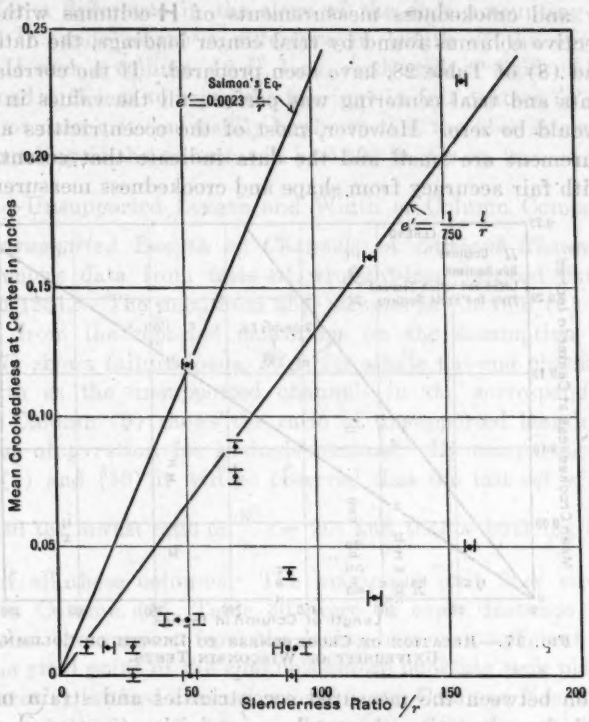



FIG. 35.—RELATION OF CROOKEDNESS TO SLENDERNESS RATIO, H-COLUMNS, UNIVERSITY OF WISCONSIN TESTS.

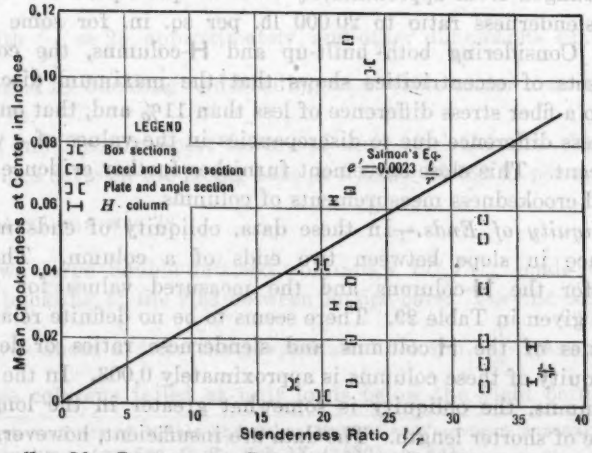



FIG. 36.—RELATION OF CROOKEDNESS TO SLENDERNESS RATIO, UNIVERSITY OF WISCONSIN TESTS.

*44.—Eccentricities in Center Loading Tests.*—In order to correlate rolling-eccentricity and crookedness measurements of H-columns with the position of the respective columns found by trial center loadings, the data in Columns (6), (7), and (8) of Table 28, have been prepared. If the correlation between measurements and trial centering was perfect, all the values in Column (8), Table 28, would be zero. However, most of the eccentricities at mid-section from measurement are small and the data indicate that eccentricity can be predicted with fair accuracy from shape and crookedness measurements.

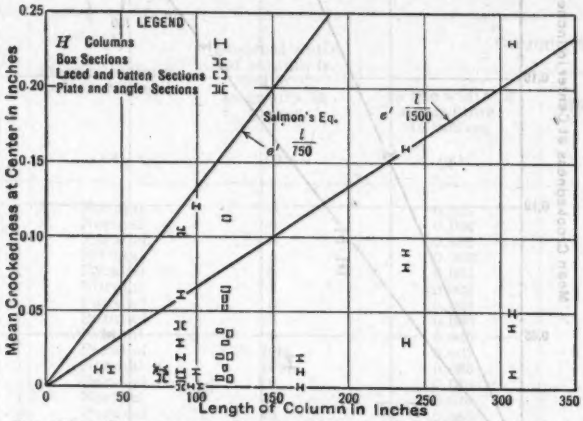



FIG. 37.—RELATION OF CROOKEDNESS TO LENGTH OF COLUMN, UNIVERSITY OF WISCONSIN TESTS.

Correlation between the measured eccentricities and strain measurements was also made by comparing the total eccentricities (measured eccentricities plus corresponding deflection) with the corresponding eccentricities computed by strain measurements.* In making this comparison the unit load ($P/A$) considered ranged from approximately 9 000 lb. per sq. in. for the column of maximum slenderness ratio to 20 000 lb. per sq. in. for some of the short H-columns. Considering both built-up and H-columns, the comparison of these two sets of eccentricities shows that the maximum discrepancy corresponded to a fiber stress difference of less than 11% and, that on the average, the fiber stress difference due to discrepancies in the values of $e$ would be less than 4 per cent. This close agreement furnishes further evidence of the value of shape and crookedness measurements of columns.

*45.—Obliquity of Ends.*—In these data, obliquity of ends means simply the difference in slope between two ends of a column. The calculated obliquities for the H-columns and the measured values for the built-up columns are given in Table 29. There seems to be no definite relation between the obliquities of the H-columns and slenderness ratios or lengths. The average obliquity of these columns is approximately 0.003. In the data for the built-up columns, the obliquity is somewhat greater in the longer columns than in those of shorter length. The data are insufficient, however, to establish definite conclusions regarding such relationship.

* For the method, see the 1926 report, *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1537.

Obliquities should be made as small as practicable, especially on short columns, for a difference in the slope of the ends amounting to 0.003 in a column 12 in. wide means a difference in length of the opposite sides of 0.036 in. If such a column, 10 ft. long, rests against rigid end bearings and carries a load sufficient to bring the bearings in full contact ($P/A$ = 4 500 lb. per sq. in., or more), it can readily be shown that the fiber stress caused by the resulting bending of the column will be 4 500 lb. per sq. in.

## 2.—Unsupported Length and Width of Column Components

*46.—Unsupported Length of Channels in Latticed Channel Columns.*—Table 30 shows data from tests of wrought-iron latticed channel columns reported in 1881.* The maximum fiber stresses in Column (6) Table 30, were calculated from the reported deflections on the assumption of fixed ends. Column (7) shows failure loads, $P/A$, for single flat-end channels having the same length as the unsupported channels in the corresponding fabricated columns. Column (9) shows the ratio of unsupported length of channel to least radius of gyration for a single channel. By comparing the results in Columns (9) and (10) it will be observed that the last set of 12-in. channel columns had the lowest ratio of $\frac{p'}{r_c}$ (= 26) and that a buckling failure occurred in tests of all these columns. The maximum unit fiber stresses for these columns in Column (6), Table 30, were in every instance high and it is probable that the support from the lacing-bars was sufficient in all cases to develop the yield point of the channels before buckling took place.

Bouscaren's† latticed channel wrought-iron Columns Nos. 37 and 38 failed by buckling of the channels between the points of attachment of the lacing-bars (single lacing). The maximum unit loads were 29 600 and 32 300 lb. per sq. in., respectively. The ratio, $\frac{p'}{r_c}$, for these columns was 28. Bouscaren's Column No. 41, with $\frac{p'}{r_c} = 25$, approximately, and other dimensions similar to Column No. 37, failed by buckling and bending as a whole under a unit load of 32 400 lb. per sq. in.

Talbot and Moore‡ report failure of their steel Column No. 1 by buckling of the flanges between the lacing-bars at $P/A$ = 23 450 lb. per sq. in. The ratio, $\frac{p'}{r_c}$, for that column was 38.

The two large double latticed columns,§ $LC_0LC_1$, made with four ribs, failed by buckling of the ribs between lattice-bars. For the column of carbon steel, $\frac{p'}{r_c}$ was about 27; for the Mayari steel column, $\frac{p'}{r_c}$ was approximately 23. Both these columns failed at unit loads below the yield point of the metal.

* Report on Tests of Metals and Other Materials, Watertown Arsenal, 1881.

† *Transactions*, Am. Soc. C. E., Vol. IX (1880), p. 447.

‡ *Bulletin No. 44*, Univ. of Illinois Eng. Experiment Station; also, *Transactions*, Am. Soc. C. E., Vol. LXV (1909), p. 202.

§ *Technologic Paper No. 101*, U. S. Bureau of Standards.

TABLE 30.—THE EFFECT OF UNSUPPORTED LENGTH OF CHANNEL ON THE STRENGTH OF LATTICED CHANNEL COLUMNS. SINGLE LACING-BARS WITH ALTERNATE SPACING OF RIVETS IN THE FLANGES OF A SINGLE CHANNEL.*

(Maximum unit load carried by 6-in. channel, 6 in. long, was 42 490 lb. per sq. in. Maximum unit load carried by 8-in. channel, 8 in. long, was 43 300 lb. per sq. in. Maximum unit load carried by 10-in. channel, 10 in. long, was 35 080 lb. per sq. in. Maximum unit load carried by 12-in. channel, 12 in. long, was 37 240 lb. per sq. in.)

| Column No. | Length, in inches. | Slenderness ratio, $\frac{l}{r}$ min. | End condition. | Maximum unit load, $\frac{P}{A}$, in pounds per square inch. | Calculated maximum unit fiber stress, in pounds per square inch. | Maximum unit load carried by one channel of length, $P'$, in pounds per square inch. | Rivet spacing on lacing-bars, $p'$, in inches. | Slenderness ratio of one unsupported channel, $\frac{p'}{r_c}$ | Kind of failure. |
|---|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| **Two 6-Inch Channels with 8-Inch Separation.** | | | | | | | | | |
| 1059 | 121 | 52 | Flat | 36 700 | 37 200 | 36 800 | 18 | 38 | Buckling |
| 1060 | 121 | 52 | Flat | 35 300 | 36 000 | ..... | .... | ..... | Buckling |
| 1095 | 125 | 53 | Flat and Pinned | 33 700 | 33 800 | ..... | .... | ..... | Buckling |
| 1096 | 125 | 53 | Flat and Pinned | 33 800 | 34 300 | ..... | .... | ..... | Buckling |
| 1107 | 144 | 62 | Pinned | 34 700 | 39 500 | ..... | .... | ..... | Deflection |
| 1108 | 144 | 62 | Pinned | 34 200 | 39 100 | ..... | .... | ..... | Deflection |
| 1 | 150 | 64 | Pinned | 35 900 | 37 900 | ..... | .... | ..... | Deflection |
| **Two 8-Inch Channels with 8-Inch Separation.** | | | | | | | | | |
| 1111 | 160 | 53 | Pinned | 34 800 | 35 100 | 35 300 | 18 | 34 | Deflection and buckling |
| 1112 | 160 | 53 | Pinned | 35 200 | 35 600 | ..... | .... | ..... | Deflection and buckling |
| 1113 | 200 | 67 | Pinned | 33 900 | 34 700 | ..... | .... | ..... | Deflection |
| 1114 | 200 | 67 | Pinned | 33 600 | 34 600 | ..... | .... | ..... | Deflection |
| 1115 | 240 | 80 | Pinned | 32 600 | 33 600 | ..... | .... | ..... | Deflection |
| 1116 | 240 | 80 | Pinned | 32 100 | 35 400 | ..... | .... | ..... | Deflection |
| 1123 | 280 | 93 | Pinned | 32 200 | 36 400 | ..... | .... | ..... | Deflection |
| **Two 10-Inch Channels with 8-Inch Separation.** | | | | | | | | | |
| 13 | 150 | 42 | Pinned | 35 600 | 35 900 | 34 400 | 22 | 33 | Buckling |
| 14 | 150 | 42 | Pinned | 35 400 | 35 900 | ..... | .... | ..... | Buckling |
| 11 | 200 | 55 | Pinned | 33 800 | 35 700 | ..... | .... | ..... | Buckling |
| 12 | 200 | 55 | Pinned | 34 000 | 37 000 | ..... | .... | ..... | Buckling |
| 3 | 250 | 69 | Pinned | 33 900 | 36 600 | ..... | .... | ..... | Buckling |
| 4 | 250 | 69 | Pinned | 33 700 | 36 700 | ..... | .... | ..... | Deflection |
| 5 | 300 | 83 | Pinned | 34 100 | 36 800 | ..... | .... | ..... | Deflection and buckling |
| 6 | 300 | 83 | Pinned | 31 900 | 36 400 | ..... | .... | ..... | Deflection and buckling |
| 26 | 350 | 97 | Pinned | 32 200 | 35 300 | ..... | .... | ..... | Deflection |
| 27 | 350 | 97 | Pinned | 29 400 | 33 900 | ..... | .... | ..... | Deflection |

TABLE 30.—(*Continued.*)

| Column No. | Length, in inches. | Slenderness ratio, $\frac{l}{r}$ min. | End condition. | Maximum unit load, $\frac{P}{A}$, in pounds per square inch. | Calculated maximum unit fiber stress, in pounds per square inch. | Maximum unit load carried by one channel of length, $P'$, in pounds per square inch. | Rivet spacing on lacing-bars, $p'$, in inches. | Slenderness ratio of one unsupported channel, $\frac{p'}{r_c}$. | Kind of failure. |
|---|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| TWO 12-INCH CHANNELS WITH 8-INCH SEPARATION. | | | | | | | | | |
| 1109 | 120 | 27 | Pinned | 33 400 | 33 700 | 36 700 | 22 | 26 | Buckling |
| 17 | 120 | 27 | Pinned | 35 100 | 35 100 | ..... | ..... | ..... | Buckling |
| 15 | 180 | 40 | Pinned | 33 800 | 34 000 | ..... | ..... | ..... | Buckling |
| 16 | 180 | 40 | Pinned | 35 500 | 35 800 | ..... | ..... | ..... | Buckling |
| 9 | 240 | 53 | Flat | 34 400 | 34 600 | ..... | ..... | ..... | Buckling |
| 10 | 240 | 53 | Flat | 33 600 | 34 700 | ..... | ..... | ..... | Buckling |
| 7 | 300 | 67 | Flat | 32 900 | 35 100 | ..... | ..... | ..... | Buckling |
| 8 | 300 | 67 | Flat | 34 200 | 35 100 | ..... | ..... | ..... | Buckling |
| 28 | 360 | 80 | Pinned | 31 600 | 34 000 | ..... | ..... | ..... | Buckling |
| 29 | 360 | 80 | Pinned | 31 300 | 35 100 | ..... | ..... | ..... | Deflection |

* From Report on Tests of Metals and Other Materials, 1881.

These failures were undoubtedly influenced by the character and arrangement of the lattice.

Of the American Railway Engineering Association latticed channel columns, Types *A* to *E* (1915 Series) inclusive, none failed by buckling of the channels between lacing-bars. The slenderness ratios for these columns varied from 50 to 120, but the largest ratio, $\frac{p'}{r_c}$, for a single channel was only 16.

Whereas, from these fragmentary data no accurate conclusion can be drawn concerning the proper limiting value of $\frac{p'}{r_c}$ in latticed channel columns, the data indicate that no failures primarily due to buckling occurred for values of $\frac{p'}{r_c}$ less than 25. Furthermore, in none of the cases under consideration, where the ratio, $\frac{p'}{r_c}$, was greater than 26, is there any evidence of marked reduction in strength which can be directly attributed to local buckling of the channels.

*47.—Unsupported Length of Channels in Columns Tied with Batten-Plates.*—Table 31 shows the principal results of the American Railway Engineering Association tests of 1918 on channel columns tied with widely spaced battens. Unfortunately, this series of tests did not include columns with battens spaced closely enough to develop the full strength of the column. By plotting the ratio, $\frac{A}{B}$, in Column (7), Table 31, against the slenderness ratio

for a single channel (Column (3)), it appears that if the battens had been spaced so that the ratio of the unsupported length to least radius of gyration for a single channel $\left(\frac{p'}{r_c}\right)$ was 28, the full strength of the column would have been secured. However, since the unsupported channels of batten-plate columns are subjected to heavy bending stresses* when the load becomes slightly oblique to the channel webs, the proper ratio for $\frac{p'}{r_c}$ for such channels should be the objective of further tests.

TABLE 31.—EFFECT OF UNSUPPORTED LENGTH OF CHANNELS ON CENTRALLY LOADED COLUMNS OF AMERICAN RAILWAY ENGINEERING ASSOCIATION SERIES.

| Column No. | Slenderness Ratios. | | | Average maximun unit load for three columns, in pounds per square inch, $A$. | Average maximum unit load of adequately laced columns of same length and sectional area, in pounds per square inch, $B$. | Ratio, $\frac{A}{B}$. |
|---|---|---|---|---|---|---|
| | Whole column, $\frac{l}{r}$. | Single channel, $\frac{p'}{r_c}$. | Ratio, $\frac{\frac{l}{r}}{\frac{p'}{r_c}}$. | | | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 1 *K* | 85 | 66 | 0.77 | 26 900 | 34 000 | 0.79 |
| 1 *I* | 85 | 84 | 0.99 | 25 400 | 34 000 | 0.75 |
| 1 *L* | 85 | 81 | 0.95 | 24 100 | 34 300 | 0.70 |
| 1 *M* | 120 | 121 | 1.01 | 15 100 | 30 800 | 0.49 |

*48.—Unsupported Length in Segmental Columns.*—In Bouscaren's tests on Keystone columns† failure occurred by bulging of the segments between the rivets when the unsupported length of the segment was 15 in. No bulging took place when the unsupported length was reduced to 12 in. For the latter length the ratio, $\frac{p'}{r_c}$, for the segment was 27.

Bouscaren's tests also indicate that the strengths of his Keystone columns (metal, ¼ to ⅜ in. thick) were improved by reducing the pitch of the rivets to 3 in. near the ends of the columns (see Columns Nos. 3, 8, 25 and 31).

*49.—Unsupported Width of Plates in Columns.*—Aside from the early work of Hodgkinson‡ and Fairbain,§ and the recent tests of the U. S. Bureau of Standards made in connection with the building of the Delaware River Bridge,‖ there is little experimental data relating to the effect of width of unsupported plate on its resistance to axial compression.

In Column (10) of Table 10¶ of the Progress Report of 1926 is recorded the ratio,

$$\frac{\text{Unsupported width of plate between rivet lines}}{\text{Thickness of plate}}$$

* See "Stresses in Channels of Batten-Plate Columns," Table 12.

† *Transactions*, Am. Soc. C. E., Vol. IX (1880), p. 447.

‡ "The Britannia and Conway Tubular Bridge," by E. Clark, Lond., 1850; also, the Report of Commission Appointed to Inquire into Application of Iron Railway Structures, *Parliamentary Paper* 29, Lond., 1849.

§ *Philosophical Transactions*, Vol. 140, pp. 677–725, 1850.

‖ *Technologic Paper No. 327*, U. S. Bureau of Standards.

¶ *Transactions*, Am. Soc. C. E., Vol. 89 (1926), pp. 1546–1547.

for the box-sections, consisting of two channels and two cover-plates reported in Tests of Metals and Other Materials (1884). From these data it will be noted that the short wrought-iron columns with 8-in. channels and 12½ by ⅝-in. plates, had ratios of unsupported width to thickness ranging from 35 to 40.4. The maximum unit loads, $P/A$, for these columns varied between 32 100 and 35 000 lb. per sq. in., the maximum fiber unit stress calculated from loads and deflections varied between 35 100 and 37 600 lb. per sq. in., and all columns save Column No. 379 failed by deflection. Column No. 379, having plates with a ratio of unsupported length to thickness equal to 23 and a ratio of unsupported width to thickness equal to 40.4, failed by buckling of plates at a unit load of 35 000 lb. per sq. in., which probably is fully as high as the yield point of the material.

The evidence from these column tests indicates that the ratios of unsupported width of plate to thickness of 40 or less caused no lessening of the axial resistance to compression of these wrought-iron plates.

In connection with the design of the towers for the Delaware River Suspension Bridge, tests were made by R. S. Johnston,* of the U. S. Bureau of Standards, on fourteen I-columns 10 ft. long, consisting of web-plates, 35 in. wide, stiffened on the long edges by four 6 by 4 by ⅝-in. flange angles attached with 4-in. legs outstanding. The web-plate thickness ranged from ⅜ in. to 1½ in. in thickness. In four of the thicker webs two plates were stitch-riveted together. The ends of the columns were stiffened by 4 by 4 by ½-in. angles and filler-plates, and were carefully milled to afford true bearings for the load. Longitudinal deformations were taken on four gauge lines located on the flange angles. The lateral buckling of the web-plates was measured at nine points across each member (four points on the angles and five on the web-plate) on lines spaced 3 in. apart along the member.

Table 32 gives the principal results of these tests. Measurements of the initial contours of these plates show that there were pronounced initial waves or buckles in the web-plates, ⅝ in. and less in thickness. The largest waves were noted in Webs *G1A-3*, *G2-4*, and *G3A-3*. In Web *G3A-3* waves having crests ⅛ in. on either side of the axial plane were observed. The contours of the thicker web-plates were more nearly plane. In the tests of these thin web members the presence of these initial waves (crookedness) contributed to premature buckling.

Another noteworthy influence which caused some variation in these tests was the edge restraint produced by the 4 by 6-in. angles. The load buckle curves indicate that this restraint was sufficient to hold the thin, but not the thick, plates in line. Therefore, in Table 32 the ratio of width to thickness of plate has been recorded both for a width of 26 in. between inner rivet lines (the usual specifications stipulation) and for a width of 35 in., which was the full width of these members.

In Fig. 38 the ratios,

$$\frac{\text{Proportional limit from the load buckle curve of the column}}{\text{Proportional limit of the web-plate material}}$$

* *Technologic Paper No. 327*, U. S. Bureau of Standards.

TABLE 32.—Comparison of Proportional Limits of Columns as Determined from Load Buckle Curves with Proportional Limit of Material, Delaware River Bridge Tests.

| Column No. | Web thickness, in inches. | Unit Stresses, in Pounds per Square Inch, at Proportional Limit of: Load buckle curve for column, $C$. | Web material, $M$. | Ratio, $\frac{C}{M}$. | Ratio of Unsupported Width to Thickness of Plate, $\frac{w}{t}$, Considering the Width, in Inches. 26 | 35 |
|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| $G$ 1 | 3/8 | 20 000 | 47 000 | 0.43 | 69 | 93 |
| $G$ 1 $A$ 3 | 3/8 | 18 000 | 32 500 | 0.55 | .... | .... |
| $G$ 2-4 | 1/2 | 21 000 | 36 800 | 0.57 | 52 | 70 |
| $G$ 2 $A$-4 | 1/2 | 25 500 | 42 000 | 0.61 | .... | .... |
| $G$ 3-1 | 5/8 | 35 000 | 37 000 | 0.95 | 42 | 56 |
| $G$ 3 $A$-3 | 5/8 | 27 000 | 32 000 | 0.84 | .... | .... |
| $G$ 4-4 | 3/4 | 40 000 | 39 500 | 1.01 | 35 | 47 |
| $G$ 4 $A$-4 | 3/4 | 38 000 | 38 000 | 1.00 | .... | .... |
| $G$ 5-1 | 1 | 36 000 | 34 000 | 1.06 | 26 | 35 |
| $G$ 5 $A$-1 | 1 | 34 000 | 31 500 | 1.08 | .... | .... |
| $G$ 6-1 | 2 by 1/2 = 1 | 36 000 | 36 800 | 1.08 | 26 | 35 |
| $G$ 6 $A$-1 | 2 by 1/2 = 1 | 38 000 | 42 000 | 0.91 | .... | .... |
| $G$ 7-1 | 2 by 3/4 = 1 1/2 | 37 000 | 39 500 | 0.94 | 17 | 23 |
| $G$ 7 $A$-1 | 2 by 3/4 = 1 1/2 | 41 000 | 38 000 | 1.08 | .... | .... |

found in Column (5) Table 32, have been plotted against the ratio,

$$\frac{\text{Width of the member } (w)}{\text{Thickness of the web-plate } (t)}$$

Fig. 38 shows that in most cases the proportional limit of members having webs 3/4 in. or more in thickness, was reached prior to buckling. The record indicates that variability in material and crookedness were influential in creating the low buckling values of Webs $G6A$-1 and $G7$-1. Web $G3$-1, a fairly plane 5/8-in. web-plate, carried 95% of the proportional limit of the material before buckling became pronounced.

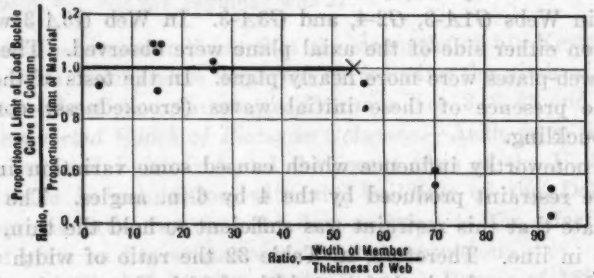



Fig. 38.

A formula for the calculation of the unit load, $p$, at which lateral buckling will occur in an axially loaded plate obtained from Bryan's discussion* on stability of ship plating under axial thrusts, is,

$$p = \frac{\pi^2 E t^2}{3 (1 - m^2) w^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

* *Proceedings*, Mathematical Soc., Vol. 22, p. 54, 1890.

If $m = 0.25$ and $E = 30\,000\,000$ lb. per sq. in., this may be changed to,

$$\frac{w}{t} = \frac{10\,300}{p\sharp} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

In these tests the average proportional limit of the web-plate material was 37 700 lb. per sq. in. If that value be substituted in the equation for $p$, $\frac{w}{t}$ is found to be 53. This limiting value is indicated by the cross in Fig. 38. For this ratio, elastic failure would occur simultaneously due to compression and buckling. Considering $w$ equal to the full width of the member, this limiting ratio of $\frac{w}{t}$ based on a perfect plate and ideal supporting conditions is slightly less than $\frac{w}{t}$ for the $\frac{3}{8}$-in. plates and about 13% less than $\frac{w}{t}$ for the $\frac{3}{4}$-in. plates. From the existing data, it appears:

1.—If a plate is subjected to thrusts in the direction of its longitudinal axis and restrained along its long edges, it may fail in either of two ways: (*a*) If the width of the plate is not too great it will fail by direct compression, and failure will commence when the fiber stress reaches the elastic limit of the material; (*b*) if the width is large the plate will fail by lateral buckling before the unit load reaches the elastic limit of the material in compression.

2.—The maximum ratio of width to thickness, $\frac{w}{t}$, which a structural steel plate may have without reduction of its compression strength has been conservatively stated by Fairbain to be 30. The more recent tests indicate that for web-plates the limit of $\frac{w}{t}$ may tentatively be set at 50, if $w$ is taken as the full width of the member; and at 40, if $w$ is taken as the distance between rivet lines. If the stiffness of the edge supports is high, $w$ should be taken equal to the distance between the rivet lines. When the web-plate is more than $\frac{5}{8}$ in. in thickness, and the restraint along the edges is low, $w$ should be made equal to the full width of the web-plate.

3.—Bryan's formula may be used to calculate the theoretical limit of $\frac{w}{t}$, for metal plates by substituting the elastic limit for $p$, but suitable reduction should be made to allow for crookedness and lack of uniformity of plates.

4.—Stitch-riveted double web-plates appear to be as strong as single-web plates of equal quality and thickness.

*50.—Unsupported Width of Outstanding Flanges in Compression.*—From analysis supplemented by tests, Roark* has shown that if a flange of uniform thickness, $t$, and perfectly restrained along one edge, is subjected to uniform compressive fiber stress, the unsupported edge will buckle laterally when the unit load, $p$, exceeds,

$$p = 0.6\,E\left(\frac{t}{w'}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

* *Engineering Record*, Vol. 68, p. 722; Vol. 74, p. 592.

in which, $w'$ is the unsupported width of the outstanding flange. If $p$ be made the proportional limit of the material, the ratio, $\frac{w'}{t}$, at which failure due to direct compression and lateral buckling would simultaneously occur, may be found. Using a proportional limit of 30 000 lb. per sq. in. for structural steel, $\frac{w'}{t} = 24$. Hence, allowing for imperfections in such material, if the unsupported width of the outstanding flange is less than about 20, the flange should develop full compressive strength before buckling.

### 3.—Strength and Stiffness of Lateral Connections

*51.—Lateral Connections.*—The adequacy and efficiency of lateral connections between the longitudinal components of a column is one of the most important elements of column design, and one of the most difficult to determine either from analysis or experimentation. Assuming integral action, the tests and analysis heretofore given indicate that the strength of a column can be closely calculated if the end conditions and the yield point of the material are known. Integral action of built-up columns depends, however, on the effectiveness of lateral connections, and this is a function not only of the strength of such connections, but also of their stiffness or rigidity. The theoretical analysis given in Section (D) shows that there are two quite different conditions under which the shearing stress to be carried by lateral connections may be large: (*a*) Relatively long columns bending in single curvature; and (*b*) relatively short columns which are subjected to heavy shears by the bending moments developed at the joints, either from the general distortion of the structure, or by unequal or oblique bearings. Columns with truly centrally applied loads are subjected to only small shears, even when the initial bending may be considerable, but a truly central load is a condition hardly to be realized in the case of a column forming a member of a truss. The worst case of shear stress is probably the relatively short column subjected to moments of opposite sign at two ends, giving the effect of an obliquely applied compression load.

*52.—Laced and Batten-Plate Columns.*—The oblique loading tests described in Part I were undertaken for the particular purpose of securing information regarding the stiffness and strength of lacing and batten-plates of relatively short columns under oblique loads. The results there obtained under shearing stresses of 2 to 5% of the direct stress are useful as bearing on this problem. The relative stiffness is clearly shown by the shearing distortions. The batten-plate columns were naturally the most flexible and the weakest, but the shearing stress of 5% is very high, and is not likely to occur except in the case of very short columns or under abnormal conditions. For more normal values, such as would occur in the ordinary case, the batten-plate column may be a satisfactory and economical design. More study and experimentation are needed on this subject.

The laced channel columns under oblique loading showed no unexpected behavior. The very thin lacing failed at a value corresponding to its strength

separately determined. Otherwise, the full strength of the channels was developed.

*53.—Strain Measurements in Lacing-Bars.*—Careful strain measurements were made on the lacing-bars of the columns described in Part I under both central and oblique loading, and similar measurements have also been made by other investigators. Such measurements give valuable information regarding stresses due to eccentricities and other causes not subject to calculation, and which have a bearing on practical design.

Although there is no shear stress in an ideal column axially loaded, various defects in the column and method of loading cause such stress in the centrally loaded column of practice. Information regarding the importance of such stresses in laced channel columns is afforded from the tests of Talbot and Moore,* from the American Railway Engineering Association series,† and from the recent tests at the University of Wisconsin.

All the columns herein considered had single lacing save the Talbot and Moore Column No. 2(*a*), which had double lacing riveted at mid-length. The columns tested by Talbot and Moore had pin ends with pin axes parallel to the lacing; those of the American Railway Engineering Association series had square ends, and those tested at the University of Wisconsin were loaded through roller bearings with the axes of the rollers normal to the lacing. Additional data on the properties of the Wisconsin columns may be found in Table 1 and Fig. 4. From the strain measurements, considering $E$ to be 29 000 000 lb. per sq. in., the unit stress, $S_b$, in the most stressed bar of each of thirty different columns, has been calculated and recorded in Column (8) of Table 33. The corresponding excess shears, $V'$, found in Column (9), Table 33, were obtained from the relation,

$$V' = S_b A_b \sin \theta_b \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

in which $A_b$ is the cross-sectional area of the lacing-bars in a given panel, or a column, and $\theta_b$ is the inclination of the bar to the column axis. The ratios of these calculated shears to the load on the column, $\frac{V'}{P}$, are shown in Column (10), Table 33.

Fig. 39, compiled from the data of Table 33, indicates that the lacing-bars of short columns are likely to be more severely stressed than the bars of long columns. This is undoubtedly due to inequalities in end bearings as heretofore mentioned, which would cause greater shear stresses in short, than in long, columns.

Further evidence of shear stresses in excess of the nominal amount is afforded by the results of Table 34. These data were obtained from the oblique loading tests made at Wisconsin. In a similar manner, the shears, $V'$, corresponding to the most stressed bar in each column, were calculated and recorded in Column (7), Table 34. Then the excess shears, $V'$, and the ratios, $\frac{V'}{P}$, were entered in Columns (10) and (11), Table 34.

* *Transactions*, Am. Soc. C. E., Vol. LXV (1909), p. 202.
† *Proceedings*, Am. Ry. Eng. Assoc., Vol. 16, p. 636; Vol. 19, p. 789.

TABLE 33.—MAXIMUM UNIT STRESSES IN LACING-BARS CALCULATED FROM MEASURED STRAINS AND CORRESPONDING TRANSVERSE SHEARS FOR AXIALLY LOADED COLUMNS.

| Column No. (1) | Series of tests. (2) | Area of column, in square inches. (3) | $\frac{l}{r}$ for Column No. (4) | Lacing-Bars. Number measured. (5) | Lacing-Bars. Nominal size, in inches. (6) | Unit load on column, $\frac{P}{A}$, in pounds per square inch. (7) | Maximum unit stress, $S_b$, in lacing-bars,* in pounds per square inch. (8) | Excess shear, $V'$, in pounds. (9) | Ratio, $\frac{V'}{P}$. (10) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | University of Illinois. | 18.76 | 37 | 52 | 1 by 1/4 | 10 000 | + 8 200 | 3 700 | 0.020 |
| 1 | " | 18.76 | 37 | 36 | 1 1/4 by 7/16 | 10 000 | + 3 000 | 2 900 | 0.016 |
| 2a | " | 17.64 | 38 | 32 | 2 1/2 by 3/8 | 10 000 | + 3 900 | 5 200 | 0.029 |
| 17 | A. R. E. A. | 9.56 | 85 | 6 | 2 1/4 by 3/8 | 15 000 | +10 500 | 15 080 | 0.105 |
| 58 | " | 9.56 | 85 | 4 | 2 1/4 by 3/8 | 31 000 | +23 900 | 34 300 | 0.116 |
| 58 | " | 9.56 | 85 | 4 | 2 1/4 by 3/8 | 15 000 | − 1 450 | 2 080 | 0.014 |
| 61 | " | 9.56 | 85 | 6 | 2 1/4 by 3/8 | 16 000 | + 1 090 | 1 560 | 0.010 |
| 66 | " | 9.56 | 120 | 6 | 2 1/4 by 3/8 | 31 000 | +34 100 | 48 900 | 0.165 |
| 66 | " | 9.56 | 120 | 6 | 2 1/4 by 3/8 | 28 000 | − 1 810 | 2 600 | 0.010 |
| 74 | " | 9.56 | 50 | 6 | 2 1/4 by 3/8 | 16 000 | − 1 450 | 2 080 | 0.014 |
| 191 | " | 9.56 | 85 | 6 | 2 1/4 by 3/8 | 30 000 | + 2 540 | 3 640 | 0.013 |
| 192 | " | 9.56 | 85 | 6 | 2 1/4 by 3/8 | 17 000 | + 1 450 | 2 080 | 0.015 |
| 197 | " | 9.56 | 85 | 6 | 2 1/4 by 3/8 | 16 000 | − 1 090 | 1 560 | 0.010 |
| 25 | " | 12.5 | 120 | 5 | 2 1/4 by 3/8 | 17 000 | − 1 810 | 2 600 | 0.012 |
| 25 | " | 12.5 | 120 | 5 | 2 1/4 by 3/8 | 30 000 | + 2 900 | 4 160 | 0.011 |
| 60 | " | 12.5 | 85 | 6 | 2 1/4 by 3/8 | 16 000 | + 1 450 | 2 080 | 0.010 |
| 62 | " | 12.5 | 85 | 6 | 2 1/4 by 3/8 | 16 000 | + 1 450 | 2 080 | 0.010 |
| 67 | " | 12.5 | 120 | 6 | 2 1/4 by 3/8 | 15 000 | − 1 090 | 1 560 | 0.008 |
| 68 | " | 12.5 | 120 | 6 | 2 1/4 by 3/8 | 28 000 | + 2 540 | 3 640 | 0.010 |
| 72 | " | 12.5 | 50 | 6 | 2 1/4 by 3/8 | 16 000 | − 1 090 | 1 560 | 0.008 |
| 73 | " | 12.5 | 50 | 6 | 2 1/4 by 3/8 | 16 000 | + 1 450 | 2 080 | 0.010 |
| 73 | " | 12.5 | 50 | 6 | 2 1/4 by 3/8 | 34 000 | + 3 620 | 5 190 | 0.012 |
| 172 | " | 9.56 | 120 | 6 | 2 by 1/4 | 15 000 | + 725 | 615 | 0.004 |
| 173 | " | 9.56 | 120 | 6 | 2 by 1/4 | 32 000 | + 4 350 | 3 700 | 0.012 |
| 174 | " | 9.56 | 120 | 6 | 2 by 1/4 | 21 000 | − 2 900 | 2 460 | 0.012 |
| 174 | " | 9.56 | 120 | 6 | 2 by 1/4 | 33 000 | + 5 800 | 4 980 | 0.016 |
| 199 | " | 9.56 | 85 | 5 | 2 by 1/4 | 5 000 | + 725 | 615 | 0.013 |
| 200 | " | 9.56 | 85 | 5 | 2 by 1/4 | 15 000 | + 725 | 615 | 0.004 |
| 204 | " | 9.56 | 85 | 5 | 2 by 1/4 | 5 000 | + 725 | 615 | 0.013 |
| 201 | " | 12.5 | 85 | 6 | 2 by 1/4 | 15 000 | − 3 620 | 3 080 | 0.016 |
| 201 | " | 12.5 | 85 | 6 | 2 by 1/4 | 19 000 | + 1 450 | 1 230 | 0.005 |
| 202 | " | 12.5 | 85 | 6 | 2 by 1/4 | 15 000 | + 2 900 | 2 460 | 0.013 |
| 202 | " | 12.5 | 85 | 6 | 2 by 1/4 | 24 000 | + 4 350 | 3 700 | 0.012 |
| 203 | " | 12.5 | 85 | 6 | 2 by 1/4 | 15 000 | + 725 | 615 | 0.003 |
| 4 | University of Wisconsin | 11.52 | 22 | 32 | 2 1/4 by 9/16 | 14 000 | + 5 070 | 3 780 | 0.023 |
| 5 | " | 11.30 | 22 | 32 | 2 1/2 by 1/4 | 14 000 | − 1 940 | 2 080 | 0.013 |
| 6 | " | 11.30 | 22 | 33 | 2 1/2 by 5/8 | 14 000 | − 3 380† | 5 650 | 0.036 |
| 6 | " | 11.30 | 22 | 33 | 2 1/2 by 5/8 | 14 000 | + 2 560 | 4 270 | 0.027 |

* Calculated from strain measurements made only on the outer faces of the bars except in the Illinois tests where they were made on opposite faces. Tension, +; compression, −.

† Bar had an initial bow at middle of 0.01 in. outward.

From a consideration of the $\frac{V'}{P}$ ratios (Table 33), for loads less than the maximum, it will be observed that only 5 columns out of 30 tested exhibited strains corresponding to values of $V'$, larger than $0.02P$. For loads less than the maximum, only 2 of the 335 bars under observation exhibited strains corresponding to values of $V'$ in excess of $0.03P$. The high value, $V' = 0.105P$, for Column No. 17, at a unit load of 15 000 lb. per sq. in., appears abnormal. This conclusion is substantiated by the fact that the strain in the corresponding bar diminished considerably when the column was subjected to higher loads. In one bar of Column No. 58 and one of Column No. 66, high strains were measured at loads near the maximum, but there is no evidence recorded that weakness in these bars caused failure.

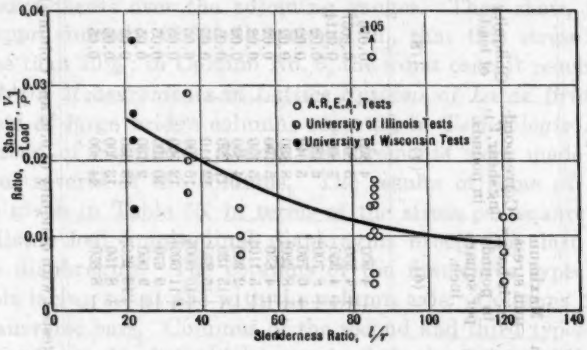



FIG. 39.—RATIO OF SHEAR TO LOAD CALCULATED FROM MOST STRAINED LACING-BAR IN CENTRALLY LOADED COLUMNS.

Only one failure, that reported by Talbot and Moore of their Column No. 1, when equipped with 1 by ¼-in. lacing-bars under a "slightly" oblique load of 150 000 lb. (8 060 lb. per sq. in.), is attributed solely to weakness in the lacing-bars. Since these flimsy bars when separately tested failed under compressive loads of 2 100 lb., Talbot and Moore calculated the equivalent shear in the column at bar failure to have been $V' = 0.025P$.

The data in Columns (6) and (11) of Table 34 show that in most cases where the calculated stresses and excess shears were high, the bars had initial bends which would contribute an increase to the strain measurements. Since no measurements of crookedness in the lacing were reported in the University of Illinois or American Railway Engineering Association tests, no estimate of the effect on measured strains can be made. It appears likely, however, from the results of Table 34, that the high values recorded in Table 33 for certain American Railway Engineering Association columns were enlarged by crookedness of the bars.

Referring again to the results of the Wisconsin tests on obliquely loaded laced columns, it will be recalled that the maximum stresses in excess of these calculated were found in tension bars (Table 9). Furthermore, it will be recalled that in only the abnormally thick bars of Column No. 6, carrying small strains which were difficut to measure accurately, did the compression bars exhibit stresses much in excess of the load stresses.

* Calculated from strain measurements made only on the outer faces of the bars except in the Illinois tests where they were made on opposite faces. Tension, +; compression, −.
† Bar had an initial bow at middle of 0.01 in. outward.

TABLE 34.—MAXIMUM STRESSES IN LACING-BARS CALCULATED FROM MEASURED STRAINS AND CORRESPONDING SHEARS IN EXCESS OF NOMINAL SHEARS DUE TO OBLIQUE LOADING.

| Column No. | Series of tests. | Lacing-Bars. | | Unit load on column, $\frac{P}{A}$, | Maximum unit stress, $S_b$, in bars,* | Corresponding shear, $V_1$, | Obliquity of load. | Nominal shear, $V$, | Excess shear, $V' = V_1 - V$, | Ratio, $\frac{V'}{P}$. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Number tested. | Nominal size, in inches. | in pounds per square inch. | in pounds per square inch. | in pounds. | | in pounds. | in pounds. | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) |
| 1 | University of Illinois | 21 | 1¼ by 7/16 | 10 000 | — 4 900 | 4 800 | 0.016 | 3 000 | 1 800 | 0.009 |
| 2a | " " " | 32 | 2½ by ⅜ | 10 000 | + 4 350 | 5 800 | 0.0057 | 1 000 | 4 800 | 0.027 |
| | " " " | 32 | 2½ by ⅜ | 10 000 | + 4 350 | 5 800 | 0.0114 | 2 000 | 3 800 | 0.021 |
| 4 | University of Wisconsin | 33 | 2¼ by 5/16 | 14 000 | + 8 700† | 6 480 | 0.02 | 3 230 | 3 250 | 0.020 |
| | " " " | 33 | 2¼ by 5/16 | 14 000 | + 6 760 | 5 040 | 0.02 | 3 230 | 1 810 | 0.011 |
| 5 | " " " | 32 | 2½ by ¼ | 14 000 | + 8 220‡ | 8 830 | 0.02 | 3 160 | 5 670 | 0.036 |
| | " " " | 32 | 2½ by ¼ | 14 000 | + 4 830 | 5 190 | 0.02 | 3 160 | 2 030 | 0.013 |
| 5a | " " " | 28 | 2¼ by ¼ | 14 600 | — 5 110‡ | 5 150 | 0.02 | 3 390 | 1 760 | 0.010 |
| 6 | " " " | 33 | 2½ by ⅜ | 14 000 | — 2 900 | 4 850 | 0.02 | 3 160 | 1 690 | 0.011 |
| 4 | " " " | 28 | 2¼ by 5/16 | 14 600 | +15 130 | 11 290 | 0.05 | 8 390 | 2 900 | 0.017 |
| | " " " | 28 | 2¼ by 5/16 | 17 600 | +16 270 | 12 110 | 0.05 | 10 110 | 2 000 | 0.010 |
| | " " " | 32 | 2½ by ¼ | 14 000 | +15 000§ | 16 100 | 0.05 | 7 910 | 8 190 | 0.052 |
| 5 | " " " | 32 | 2½ by ¼ | 14 000 | +11 600† | 12 500 | 0.05 | 7 910 | 4 590 | 0.029 |
| | " " " | 32 | 2½ by ¼ | 14 000 | — 9 670 | 10 400 | 0.05 | 7 910 | 2 490 | 0.016 |
| 5a | " " " | 28 | 2¼ by ¼ | 14 600 | — 9 060 | 9 130 | 0.05 | 8 465 | 665 | 0.004 |
| | " " " | 28 | 2¼ by ¼ | 19 600 | +13 250‖ | 13 350 | 0.05 | 11 370 | 1 980 | 0.009 |
| 6 | " " " | 28 | 2½ by ⅜ | 14 600 | + 8 150 | 13 600 | 0.05 | 8 225 | 5 375 | 0.033 |
| | " " " | 28 | 2½ by ⅜ | 19 600 | — 8 850 | 14 800 | 0.05 | 11 050 | 3 750 | 0.017 |

* Strains measured along two faces of bars except Columns Nos. 4 and 6 tested with obliquity of 0.02 and Column No. 5; tension, +; compression, —.

† Bar had an initial bow at middle, 0.14 in. inward.

‡ " " " " " " " 0.01 in. "

§ " " " " " " " 0.07 in. "

‖ " " " " " " " 0.01 in. outward.

These results indicate that under large definitely known shears the stresses in the lacing-bars tend to approach the calculated values under the higher loads, but in design the excess stresses shown in Table 34, Column (11), indicate that the obliquity should be liberally estimated. Under nominally central loads the stresses in the lacing-bars will be considerable and larger for the shorter columns. The data of Table 33, Column (10), indicate a shearing stress generally of $V' = 0.015P$ to $0.02P$, with larger values for the shorter lengths.

In order to secure a rough estimate of the magnitude of the stress differences in channels of centrally loaded laced columns, and thus determine the stress transfer which must be made by the lacing, the average fiber stresses near the stay-plates of Columns Nos. 4, 5, 5*A*, and 6 were calculated from strain measurements over the adjoining gauges. They show, for unit loads, $P/A$, of approximately 15 000 lb. per sq. in., that this stress difference was usually less than 10%; in Column No. 6, the worst case, it reached 20 per cent.

*54.—Strain Measurements in Lattice Systems of Large Bridge Columns.*— In the tests of large bridge columns reported in *Technologic Paper No. 101*, U. S. Bureau of Standards, strain measurements were made on the lattice members of several of the columns. The results of some of these measurements are given in Table 35, in terms of the stress per square inch. All the columns listed had longitudinal diaphragms except the last two. All had transverse diaphragms. The columns of the first three types were provided with double lacing set at 45° with the column axis. Columns of the first type had no transverse bars. Columns of the second and third types had transverse angles at each panel point which restrained the lateral spreading of the outer ribs. Column $US_2MS_1$ (carbon steel) had four ribs and was provided with a cover-plate on one side. On the opposite side the outer flanges were tied with a single system of lacing-bars. These had independent ends, and served to connect each outer rib with the adjacent inner rib. Bars were set at an angle of 60° with the column axis. The inner ribs were connected by the tie-plates placed at each intersection of the lattice-bars. Column $LC_0LC_1$ (carbon steel) had four ribs which were tied on both flanges in a manner similar to Column $US_2MS_1$.

Considering the first type, without transverse bars, there is little resistance, except near the batten-plates, to the outward bending of the flanges which results from the pantograph action of the latticing when the column is under compressive stress. The result is that the lattice-bars show very little stress (1 500 to 2 000 lb. per sq. in. as a maximum), except in the panel next to the batten-plate where the maximum stress observed is 18 400 lb. per sq. in., or about 30% of the column stress. Even at the transverse diaphragm the stresses in the lattice-bars were very small, showing the same outward bulging as at other points.

The columns of the next two types, having transverse bars, show considerable compressive stresses in all lattice-bars and tensile stresses in all transverse bars. The tensile stresses in the batten-plates (in a transverse direction) are relatively large. These results show the general tendency of transverse bars to maintain the main ribs in line better than latticing without transverse bars

TABLE 35.—UNIT STRESSES IN THE LACING SYSTEMS OF LARGE BRIDGE COLUMNS.

(*Technologic Paper No. 101*, U. S. Bureau of Standards. Unit stresses in Columns (10) to (14) were calculated from $S = 29\,500\,000 \times$ unit strain; $t$ = tension; $c$ = compression.)

| Column No. (1) | Shape of cross-section. (2) | Area of cross-section, in square inches. (3) | Ratio, Ultimate strength / Yield point (4) | Bracing of Column. Lattice. (5) | Bracing of Column. Transverse diaphragm. (6) | Unit Load, in 1 000 Pounds per Square Inch on. Long diaphragm. (7) | Battens. (8) | Column, $\frac{P}{A}$. (9) | Lattice. Maximum. (10) | Lattice. Mean. (11) | Transverse Bars. Maximum. (12) | Transverse Bars. Average. (13) | Battens, average. (14) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $U_2U_3$ | | 88.8 | 1.11 | 2½-in. channels, ¼-in. web. | Three ¼-in. plates, three 5/16-in. plates. | 5/16-in. plates. | ¼-in. plates. | 58.0 | 18.4*c* | .... | .... | .... | .... |
| $U_3U_4$ | | .... | 1.11 | | | | | 32.0 | 6.7*c* | .... | .... | .... | .... |
| | | | | | | | | 58.0 | 9.6*c* | .... | .... | .... | .... |
| | | | | | | | | 30.0 | 3.7*c* | .... | .... | .... | .... |
| *HC* | | 74.6 | 0.94 | Transverse 4 × 3 × 5/16-in. angles, diagonal 4 × ¼-in. bars. | Three ¼-in. plates. | 7/16-in. plates. | ¼-in. plates. | .... | ..... | .... | .... | .... | .... |
| *MY* | | 74.9 | 1.03 | | | | | 45.0 | 11.9*c* | 8.8*c* | 8.8*t* | 7.6*t* | 18.4*t* |
| 31*HC* | | 76.9 | 0.98 | | | | | 30.0 | 9.0*c* | 5.0*c* | 5.4*t* | 4.5*t* | 12.4*t* |
| 32*S* | | 76.9 | 0.92 | | | | | .... | ..... | .... | .... | .... | .... |
| 34*CS* | | 76.9 | 0.88 | | | | | .... | ..... | .... | .... | .... | .... |
| | | | | | | | | 30.0 | 6.5*c* | 4.2*c* | 6.3*t* | 4.8*t* | 11.1*t* |
| *HC*1 | | 75.4 | 1.08 | Transverse, 4 × 3 × 5/16-in. angles diagonal 3 × ¼-in. bars. | Three ¼-in. plates. | 7/16-in. plates. | ¼-in. plates. | .... | ..... | .... | .... | .... | .... |
| *MY*1 | | 75.3 | 1.04 | | | | | 40.0 | ..... | .... | 7.4*t* | 4.7*t* | 17.0*t* |
| 31*HC*1 | | 77.2 | 1.09 | | | | | 30.0 | ..... | .... | 6.6*t* | 3.8*t* | 11.4*t* |
| 32*S*1 | | 77.2 | 0.92 | | | | | .... | ..... | .... | .... | .... | .... |
| 34*CS*1 | | 77.2 | 0.96 | | | | | 40.0 | 13.2*c* | 9.9*c* | 10.3*t* | 8.4*t* | 14.8*t* |
| | | | | | | | | 30.0 | 7.2*c* | 6.2*c* | 10.0*t* | 7.4*t* | 10.2*t* |

TABLE 35.—(*Continued.*)

| Column No. (1) | Shape of cross-section. (2) | Area of cross-section, in square inches. (3) | Ratio, Ultimate strength / Yield point (4) | Bracing of Column. Lattice. (5) | Bracing of Column. Transverse diaphragm. (6) | Unit Load, in 1 000 Pounds per Square Inch on. Long diaphragm. (7) | Battens. (8) | Column, $\frac{P}{A}$ (9) | Lattice. Maximum. (10) | Lattice. Mean. (11) | Transverse Bars. Maximum. (12) | Transverse Bars. Average. (13) | Battens, average. (14) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $US_2MS_1(C)$ | | 56.0 | * | Transverse tie-plates ¼-in., diagonal 3 × ¼-in. bars. | Four ¼-in. plates. | ..... | 5/16-in. plates. | 25.0 | 10.7$c$ | .... | .... | .... | .... |
| $LC_0LC_1(C)$ | | 118.2 | 0.78 | Tie plates ¼-in., diagonal 3 × 1¾ × ¼-in. angles. | Four ¼-in. plates. | ..... | 5/16-in. plates. | 30.0 | †2.6$c$ | .... | .... | .... | .... |

* Failed by lateral bending of forked ends.
† Only one bar measured.

and thus to prevent some of the bending stresses which occur in the latter type near the batten-plates. Neither the longitudinal diaphragm nor the transverse diaphragm appear to have had much effect on this action.

The strain-gauge measurements on the lattice system of Column $Us_2MS_1$ and Column $LC_0LC_1$ were not sufficient to afford a detailed study of the stresses. From the measurements made on Column $US_2MS_1$, it is apparent that the lattice-bars suffered considerable flexure in their own plane, also that the average unit stress in these bars was small. The manner of failure of Column $LC_0LC_1$ (carbon) and Column $LC_0LC_1$ (Mayari) suggests that the lattice system was insufficient to develop integral action between the several ribs, and that there was considerable secondary bending stress caused at the ends of the lattice-bars by the rigidity of the riveted connections. The ratio of ultimate strength to yield point, as shown in Column (4) of Table 35, is a maximum for Type 1, and nearly uniform for the various columns of Types 2 and 3. The last type was relatively weak, due apparently to the weakness of the lattice system, no longitudinal web being present to assist in carrying the shear.

Additional information concerning the lateral bulging of column ribs under the pantograph action of a double lattice system and the constrictions produced by tie-plates at the ends is afforded by measurements on Columns $TC4_2$, $TC5_2$, and $TC6_2$, which were tested in 1912 for the New Quebec Bridge.* Columns $TC4_2$ and $TC5_2$ consisted of two built-up channel sections with transverse diaphragms at the pin-plates and double lattice without transverse bars. Column $TC5_2$ had some intermediate diaphragms also. Each rib in these columns consisted of two 4 by 4-in. by $\frac{5}{8}$-in. angles turned outward and a 22-in. by $\frac{3}{4}$-in. web-plate. The lattice for Column $TC4_2$ consisted of $2\frac{1}{2}$-in. by $\frac{7}{16}$-in. bars; for Column $TC5_2$ it was made of 4-in. by $\frac{7}{16}$-in. bars.

Column $TC6_2$ had two I-shaped ribs each made of four 4 by 4-in. by $\frac{5}{8}$-in. angles united by a 24-in. by $\frac{3}{8}$-in. web-plate. The ribs were connected along their axes by a longitudinal diaphragm, 16 in. by $\frac{3}{8}$ in. in section, which was attached to the ribs by four 4 by 3-in. by $\frac{5}{8}$-in. angles. A double-lattice system made of 3-in. by $\frac{1}{4}$-in. bars was used to tie each pair of flanges. Transverse diaphragms were also supplied at intervals along the column.

By means of the strain measurements taken on transverse gauge lines separated about 1 ft. along the axis of the columns, it was found that spreading at the edge of the tie-plate for Columns $TC4_2$ and $TC5_2$ amounted to about 0.003 in., and at $1\frac{1}{2}$ ft. from the plates, it was about 0.01 in. In Column $TC6_2$, equipped with the longitudinal diaphragm, the strains were about one-fourth of these values. The measurements showed that the ribs were bent concave outward at the tie-plates and convex outward for the length of the first panel of lattice. Calculations of stress showed that the bending stress at the tie-plate augmented the direct compression in the ribs of Columns $TC4$ and $TC5$ by about 20 per cent.

*55.—The Effect of Diaphragms on Column Strength.—*

Longitudinal Diaphragms.—A good example of the value of a longitudinal diaphragm is afforded by a comparison of Columns $TC4$, $TC5$, and $TC6$ of the

* See The Quebec Bridge Report, Vol. 1, p. 207.

Quebec Bridge tests. The general make-up of these columns has already been described in discussing stresses in lattice. The lengths of these columns were the same; all were equipped with double lattice and transverse bars; but only Column *TC*6 had a longitudinal diaphragm. The principal results of these tests are summarized in Table 36. Each result represents two tests.

TABLE 36.—COMPARISON OF COLUMNS, QUEBEC BRIDGE.

| Column No. | Ratio, unsupported width of web-plate to thickness (estimated). | Relative ratio, weight to cross-sectional area. | Ratio, ultimate strength to yield point of metal. | Relative strengths in proportion to weight. |
|---|---|---|---|---|
| *TC*4 | 23 | 1.00 | 0.566 | 1.00 |
| *TC*5 | 23 | 1.05 | 0.559 | 0.94 |
| *TC*6 | 21 | 1.09 | 0.669 | 1.08 |

Mention has already been made of the effect which the longitudinal diaphragm exerted to lessen the lateral bulging of the flanges at the end batten-plates in Column *TC*6.

Attention is also called to the comparisons in Table 37. Although there are many variables other than the effect of the longitudinal diaphragm which affect these comparisons, yet it seems probable that the influence of the diaphragm was the most important factor in raising the efficiency of the column in which it was used.

In test of Column *BHC*1 (*Technologic Paper No. 101*, U. S. Bureau of Standards), measurements were made of the axial and transverse strains in the longitudinal diaphragm at the middle of a panel where the transverse tension in the diaphragm would be expected to be greatest. At $P/A = 30\,000$ lb. per sq. in., the unit axial compression was 0.00111, and the unit lateral extension was 0.000375. If Poisson's ratio was 0.30 and $E = 29\,000\,000$ lb. per sq. in. for this column, then the axial compression was 31 600 lb. per sq. in. and the transverse tension, 2 340 lb. per sq. in. These results show that the diaphragm was participating in carrying the load and also sustaining a small amount of transverse tension. The ribs were spaced 16½ in. on centers, the diaphragm was $\frac{7}{16}$ in. thick, and a double lattice of 3 by ¼-in. bars, with 4 by 3-in. by $\frac{7}{16}$-in. angle transverse ties, was used. Assuming the tensile stress uniform over a length of diaphragm equal to the rib-spacing, the total transverse pull in that portion was $16.5 \times \frac{7}{16} \times 2\,340 = 16\,900$ lb., or about 0.73% of the column load.

Stresses at Transverse Diaphragms.—The ratio of the spacing of transverse diaphragms to the unsupported widths of the adjoining web-plates, and the position of these diaphragms with respect to the lattice intersections, are factors which probably greatly affect the stresses near the diaphragms. Data containing a systematic arrangement of these variables is wanting, but strain measurements made during the Quebec Bridge tests are of importance in this connection.

Columns *TC*4 and *TC*5 were equipped with transverse diaphragms which intersected the ribs at panel points of the lattice. Readings of strain in the

outer fibers of the rib flanges showed that the compressive stresses on these fibers for gauges spanning the diaphragm were from 7 to 14% higher than the stresses in adjoining gauges on the same fibers. These data show that the diaphragm offered restraint against the outward thrust of the lattice-bars. Consequently, the rib flanges were thus bowed concave outward at the diaphragm in much the same way as at the end battens. Measurements on Column *TC*6 which had I-shaped ribs and a longitudinal diaphragm, failed to show any marked discrepancy between strains across the transverse diaphragms and elsewhere.

TABLE 37.—Showing Comparative Strengths of Large Built-Up Columns With and Without Longitudinal Diaphragms.

| Column No. | Report. | $\frac{l}{r}$. | Ratio, $\frac{\text{Unsupported width of web-plate}}{\text{Thickness (estimated)}}$ | Ratio, $\frac{\text{Ultimate strength}}{\text{Yield point of material, } A}$ | Mean ratio, $A$. | Type of failure. |
|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| **Columns with Diaphragms.** | | | | | | |
| *U2U3* | *Technologic Paper No. 101*, U. S. Bureau of Standards. | 25 | 9 | 1.11 | .... | Flanges buckled |
| *U8U9* | | 37 | 9 | 1.11 | .... | " " |
| *HC* | | 44 | 7 | 0.94 | .... | Buckled at center |
| *MY* | | 44 | 7 | 1.03 | .... | Bending |
| 31*HC* | | 44 | 7 | 0.98 | .... | " |
| 32*S* | | 44 | 7 | 0.92 | .... | Center web buckled |
| 34*CS* | | 44 | 7 | 0.88 | .... | Center web and lacing buckled |
| *HC*1 | | 36 | 10 | 1.03 | .... | Bending |
| *MY*1 | | 38 | 10 | 1.04 | .... | " |
| 31*HC*1 | | 38 | 10 | 1.09 | .... | Diaphragm buckled |
| 32*S*1 | | 38 | 10 | 0.92 | .... | Center web buckled |
| 34*CS*1 | | 38 | 10 | 0.96 | .... | Buckling and bending |
| *TX*18*A*1 | Quebec Bridge. | 38 | 14 | 0.97 | .... | Flanges at splice |
| *TX*18*A*2 | | 38 | 14 | 1.00+ | .... | No failure |
| *TX*18*B*1 | | 38 | 14 | 1.00+ | .... | " " |
| *TX*18*B*2 | | 38 | 14 | 1.00+ | .... | " " |
| *TX*17-1 | | 78 | 34 | 0.78 | .... | Ribs buckled at splice |
| *TX*17-2 | | 78 | 34 | 0.79 | .... | " " " " |
| *TX*18-1 | | 58 | 34 | 0.83 | .... | Bending |
| *TX*18-2 | | 58 | 34 | 0.86 | .... | " |
| | | | | | 0.96 | |
| **Columns without Longitudinal Diaphragms.** | | | | | | |
| *LCoLC*1(*C*) | *Technologic Paper No. 101*, U. S. Bureau of Standards. | 28 | 13 | 0.78 | .... | Plates and angles buckled |
| *LCoLC*1(*M*) | | 27 | 24 | 0.93 | .... | " " " " |
| *TX*16*C*1 | Quebec Bridge. | 52 | 17 | 0.90 | .... | Ribs buckled at splice |
| *TX*16*C*2 | | 52 | 17 | 0.81 | .... | " " " " |
| *TX*16*N*1 | | 52 | 11 | 0.81 | .... | Ribs buckled |
| *TX*16*N*2 | | 52 | 11 | 0.88 | .... | " " |
| No. 5 | Howard. | 47 | 19 | 0.86 | .... | Web-plate buckled |
| | | | | | 0.85 | |

In Column $U_2U_3$ (*Technologic Paper No. 101*, U. S. Bureau of Standards), the diaphragms were placed at the intersection of the lattice (mid-points of panels) at intervals of two panel lengths. In Column $U_8U_9$, the end diaphragms were similarly placed, but the center diaphragm was attached at panel points of the lattice. Measurements of strain along the edges of the center diaphragm in the plane of the lattice gave the results shown in Table 38. It will be noted that the stresses are somewhat greater in the $\frac{3}{8}$-in. diaphragm of Column $U_8U_9$ than in the $\frac{1}{4}$-in. diaphragm of Column $U_2U_3$. This discrepancy is probably due to the difference in the positions of the diaphragms with respect to the lattices.

TABLE 38.—STRAINS IN DIAPHRAGMS.

| Column No. | Unit load, $P/A$, in pounds per square inch. | Average unit strain on edge of diaphragm. | Calculated unit stress in diaphragm, in pounds per square inch. |
|---|---|---|---|
| $U_2U_3$ | 10 000 | 0.00027 | 8 100 |
| | 32 000 | 0.00103 | 30 500 |
| | 44 000 | 0.00155 | 45 500 |
| $U_8U_9$ | 10 000 | 0.00039 | 11 400 |
| | 30 000 | 0.00125 | 37 000 |
| | 45 000 | 0.00186 | 55 000 |

## SUMMARY OF RESULTS

The results of tests and the theoretical studies contained in this report may be summarized briefly, as follows:

(1) When correctly interpreted, the average tensile yield point of the material is a satisfactory measure of the ultimate strength of columns, except for columns having high values of the slenderness ratio, in which buckling takes place at stresses below the yield point.

(2) The shearing rigidity of columns varies greatly as between columns with solid webs, columns with lacing, and columns with batten-plate connections.

(3) The strength of single lacing-bars in resisting column shear can be determined approximately by tests on single bars, in the manner set forth in the report.

(4) Columns the component parts of which are connected by batten-plates are relatively weak in shear, and under loads involving heavy shearing stress, will be subjected to relatively high stress in the extreme fibers of the individual segments. Under loads producing small shearing stresses, the tests are not conclusive as to the advisability of batten-plate columns.

(5) Under loads the eccentricities of which are known, the results of tests check very closely with theoretical results.

(6) Eccentric load tests on H-columns showed a very considerable excess of ultimate strength over that determined on the basis of yield point in the case where the eccentricity was in a direction perpendicular to the web, so that the most stressed fibers were on the edges parallel to the web. This excess strength was, in general, greater the smaller the slenderness ratio, $\frac{l}{r}$.

(7) On a theoretical basis, the flexibility of a column in the plane of the lacing due to shearing distortion is not of great influence on the ultimate

strength of columns under eccentric loads. Such reduction of strength as may be caused by this increased flexibility can be overcome by an increase in the corresponding radius of gyration.

(8) The results shown in the Progress Report of 1926 to the effect that the tensile and compressive yield points are very nearly equal, are confirmed by the tests herein described.

(9) The yield point of the material varies considerably according to the location of the specimens in the section. For H-columns, the specimens cut from the intermediate portions of the flanges appear to represent the section best. For channels, the web specimens were most representative. For thin plates, not much difference was found between specimens cut from the edges and those cut from the center.

(10) The value of the modulus of elasticity of the column as a whole is nearly constant, ranging only about 5% more or less than 29 500 000 lb. per sq. in.

(11) The theoretical analysis of a column forming a part of a riveted truss indicates that the strength of the column as regards bending in the plane of the truss is much greater than that of a pivoted end column, and approaches rather closely the strength of a pivoted end column of one-half the length under eccentric load, with eccentricity corresponding to the secondary bending moments assumed in the analysis.

(12) In the case of columns forming compression members of riveted structures, the effect of eccentricity due to crookedness and the effect of weight as regards bending in the plane of the truss are relatively small, the effect of crookedness being less than one-half that due to the same amount of crookedness in a pivoted end column of one-half the length.

(13) The crookedness of the columns tested was, in most cases, less than 1/1500 of the length. All columns were fabricated under standard American Railway Engineering Association specifications.

(14) The obliquity of the ends of columns designed to transmit largely by end-bearing is a matter of importance. As measured by the slope of one end with respect to the other, the average obliquity of the H-columns tested was found to be about 0.003.

(15) A study of the existing data relative to the failure of individual column components shows that in laced channel columns, no failures occurred where the ratio, $\frac{p_1}{r_c}$, was less than 25. The tests studied were, however, too limited in number to be at all conclusive.

(16) The tests made by the U. S. Bureau of Standards for the Delaware River Bridge Commission indicate that a safe ratio of plate width to thickness for structural steel is about 40, provided the width is measured between rivet lines and the plates are well restrained at the edges.

(17) Under perfect conditions of support at one edge, outstanding structural steel plates or flanges may be expected to carry the full proportional limit stress of about 30 000 lb. per sq. in. without buckling, provided the ratio of outstanding width to thickness is not greater than 20.

(18) The stresses in lacing are likely to be considerably greater than those due to calculations from shear; in centrally loaded columns (shear = 0) generally they are greater by an amount from 0.015 to 0.02 $P$, and in obliquely loaded columns, by 0.015 to 0.03 $P$. For loads approaching the maximum, the stresses in the lacing-bars approach more nearly the calculated values.

(19) The action of double lacing without transverse bars forces the ribs of a column apart, and introduces considerable bending stresses near the stay-plates. With transverse bars and double lacing, this action can be reduced, but the lacing will then receive considerable compressive stress, and this arrangement involves details which are undesirable in some respects.

(20) Tests on large bridge members indicate difficulty in securing integral action of the several ribs of a large member by means of lattice-bars only. Longitudinal diaphragms or cover-plates are more effective.

## Suggestions Relative to Future Work

From the studies made thus far, it seems evident that further investigation of columns may well be considered in two parts: (1) The strength and effectiveness of lacing, batten-plates, and diaphragms; effect of form of section, outstanding flanges, etc.; unsupported length of component parts; and other details affecting integrity of action; and (2) the conditions of the column as a part of a structure, its end conditions, crookedness, and the effect of other members thereon.

The first group of studies can best be made with end conditions definitely known, as by the use of roller-bearing blocks; the second group can only be made by studies and observations of actual columns and trusses. If details are designed to secure with certainty integrity of action, and then if the conditions of the column in the truss are known, it would seem that the application of theoretical principles is the rational process of design and is quite as well warranted as in the design of beams. The attempt to study the entire problem by the customary flat-end test is a slow process, and does not yield results that are readily analyzed and applied.

Further needed information can be secured regarding the first group of problems by a moderate expenditure for test specimens and laboratory work; the second part of the problem is more difficult, and will probably require a considerable amount of field work.

Respectfully submitted,

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S. H. Widdicombe,
Special Committee on Steel Column Research.

May 7, 1928.

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PAPERS AND DISCUSSIONS

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# BAFFLE-PIER EXPERIMENTS ON MODELS OF PIT RIVER DAMS

## Discussion*

BY I. C. STEELE AND R. A. MONROE, MEMBERS, AM. SOC. C. E.†

I. C. STEELE‡ AND R. A. MONROE,§ MEMBERS, AM. SOC. C. E. (by letter).‖—Floods of unusual magnitude occurred in Pit River during the latter part of March, 1928, and afforded the opportunity of observing and photographing the action of the baffle-piers at both Pit No. 3 and Pit No. 4 Dams. The maximum flow over Pit No. 3 Dam was approximately 15 000 sec-ft., with a depth of over-pour of 6.3 ft. (Fig. 64). At Pit No. 4 Dam the discharge was approximately 17 000 sec-ft. and the maximum depth of over-pour was 11.5 ft. (Fig. 65). The stilling action of the baffle-piers at both dams was highly satisfactory.

It should be observed (Fig. 66) that at Pit No. 3 Dam the two end splitter piers are only half-piers and the sloping wing-walls were not provided with baffles. At the left wing-wall the water which was deflected from the end baffle-pier combined with the water passing directly between the half-pier and the wall and intermittently climbed the wall (Fig. 64). This action was expected but is not considered to be of serious moment because the volume of water which rides the wall is quite small. It may be necessary at some future time to increase the side-wall protection at this particular locality by placing rip-rap on the bank above the existing concrete wing-wall.

At Pit No. 4 Dam it was observed that, for the flows noted, no better results occur on the depressed apron side, which indicates that the deeper stilling-pool is of little consequence. Unfortunately, the baffle-pier on the shallow side next to the right wing-wall was not constructed because of its location under the diversion flume. In addition, the right wing-wall is sloping as contrasted with the vertical side of the heavy division wall at the left side of the depressed apron. As a consequence there is created, along the right wing-wall, wave action which is not noticeable across the remaining width of the apron.

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* Discussion of the paper by I. C. Steele and R. A. Monroe, Members, Am. Soc. C. E., continued from October, 1928, *Proceedings*.

† Authors' closure.

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‖ Received by the Secretary, October 13, 1928.

An inspection was made of the baffle-piers and the apron below Pit No. 3 Dam shortly after the flood had subsided to determine what damage had occurred. Fig. 66 indicates the location and extent of certain pitting and erosion which had taken place since the completion of the structure. About midway between each of the splitter piers small holes from 3 to 12 in. deep had developed apparently by the grinding action of rocks caught and held by the eddy which probably existed at this point. Approximately 75 cu. yd. of rock and débris were left on the apron at the completion of the construction work.

The faces and edges of the concrete piers show no erosion whatever except that two of the corners of the down-stream pier near the left wing-wall were chipped to a depth of 3 or 4 in., presumably by large rocks from slides which developed on the hillside above the wall. Several large rocks which evidently came from this location were found lying on the apron. These slides were due to a saturated condition of the hillside occasioned by heavy rains and, in time, this slope will become stabilized.

The efficiency of the baffle-pier designs in preventing erosion is indicated by the condition of the river bed immediately below the concrete apron. An area of limited extent on the right side has been scoured out approximately 1 ft. in depth. In the center of the channel, rocks and gravel have piled up several feet above the apron level. This material undoubtedly came from the débris which was left on the apron when the dam was constructed and from the slides on the hillside above the wing-wall, as previously noted. The area on the left side of the channel has not been disturbed materially.

The experiments mentioned by Mr. Ewald* confirm the results of similar experiments in the Pit model tests. It is believed that baffles of the shapes and in the locations shown in Mr. Ewald's experiments might form an obstruction which would be subject to considerable shock. In the Pit River experiments it was soon observed that the best results were obtained by forcing the water to dissipate its energy within itself, the splitter and baffle-piers being placed rather far apart and acting more as guides to create cross-currents which would be largely directed against each other. Observations, although confined to surface indications, indicated that the water throughout its entire depth was under great turmoil, and that the major portion of the kinetic energy was being destroyed by this action. Generally speaking, it is admitted that obstructions to the free and direct flow of the water away from the dam should be avoided, but it is obvious that special conditions encountered at a particular dam site must govern. As previously stated, the foundation and river-bed conditions of both Pit No. 3 and No. 4 Dams were quite unusual.

In the Pit model experiments the control weir mentioned by Mr. Ewald and others was placed at the lower end of the apron and at the point of free fall to the stream bed below. It was thought that this would simulate the natural barrier offered to flood waters by the stream bed below the aprons of the main structures. No such control weirs were actually constructed and, consequently, no secondary drop exists.

* *Proceedings*, Am. Soc. C. E., April, 1928, Papers and Discussions, p. 1225.

FIG. 64.—BAFFLE-PIERS AND APRON OF PIT RIVER DAM NO. 3, UNDER DISCHARGE OF 12 000 SECOND-FEET.

FIG. 65.—LOOKING DOWN ON BAFFLE-PIERS OF PIT RIVER DAM NO. 4, UNDER DISCHARGE OF 10 000 SECOND-FEET.

Mr. Scheidenhelm* correctly states that "the most reliable criterion for comparison of the various means of preventing scour is the action on the stream bed itself", and more conclusive evidence regarding the advantages or disadvantages of this design will be available after several years of actual performance. Results of three years' operations of Pit No. 3 Dam and one year of Pit No. 4 Dam indicate that the structures as designed are functioning in a very satisfactory manner.

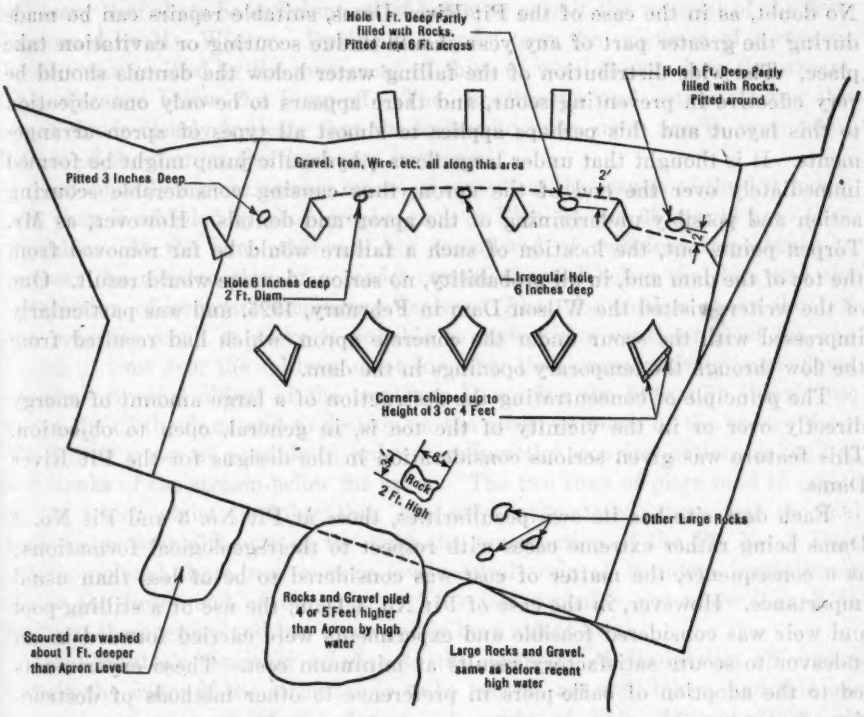
Hole 1 Ft. Deep Partly filled with Rocks. Pitted area 6 Ft. across
Hole 1 Ft. Deep Partly filled with Rocks. Pitted around
Gravel, Iron, Wire, etc. all along this area
Pitted 3 Inches Deep
2'
Hole 6 Inches deep 2 Ft. Diam.
Irregular Hole 6 Inches deep
Corners chipped up to Height of 3 or 4 Feet
Rock
2 Ft. High
Other Large Rocks
Rocks and Gravel piled 4 or 5 Feet higher than Apron by high water
Scoured area washed about 1 Ft. deeper than Apron Level
Large Rocks and Gravel, same as before recent high water

FIG. 66.—CONDITION OF THE APRON OF PIT NO. 3 DAM, MAY, 1928.

It is felt that the appearance of the surface and movement of currents of the water, a study of the action in and about the piers, and, what is perhaps more to the point, the smoothness of the flow immediately down stream, should afford a reliable indication of the effectiveness of the stilling device. A close study of the photographs of both tests and actual floods, such as Figs. 64 and 65, cannot but impress one with the excellent results from the standpoint of destruction of kinetic energy.

The experiments described by Mr. Sherman† are very instructive. The principal differences between the design of the baffles of the Gatun Spillway Dam and those of the Pit River Dams, lie in the shapes and positions of the piers and in the bucket of the dam. It is thought that the Pit River design will cause a destruction of a greater percentage of the energy within the water itself. Published data on the Gatun Spillway were studied with a

* *Proceedings*, Am. Soc. C. E., April, 1928, Papers and Discussions, p. 1232.
† *Loc. cit.*, January, 1928, Papers and Discussions, p. 363.

great deal of interest in connection with the preparation of designs for the Pit River Dams.

The data submitted by Mr. Torpen* describing the tests for the Bull Run Dam are exceedingly interesting and enlightening and the plan adopted appears to afford a very satisfactory solution to a difficult problem. It will be very instructive to learn of the scouring effect of the fast moving water over the apron, particularly in the vicinity of the lower end of the bucket. No doubt, as in the case of the Pit River Dams, suitable repairs can be made during the greater part of any year should undue scouring or cavitation take place. The wide distribution of the falling water below the dentals should be very effective in preventing scour, and there appears to be only one objection to this layout and this perhaps applies to almost all types of apron arrangements. It is thought that under large flows a hydraulic jump might be formed immediately over the end of the apron, thus causing considerable scouring action and possibly undermining of the apron and dentals. However, as Mr. Torpen points out, the location of such a failure would be far removed from the toe of the dam and, in all probability, no serious damage would result. One of the writers visited the Wilson Dam in February, 1925, and was particularly impressed with the scour under the concrete apron, which had resulted from the flow through the temporary openings in the dam.

The principle of concentrating the destruction of a large amount of energy directly over or in the vicinity of the toe is, in general, open to objection. This feature was given serious consideration in the designs for the Pit River Dams.

Each dam site has its own peculiarities, those at Pit No. 3 and Pit No. 4 Dams being rather extreme cases with respect to their geological formations; as a consequence, the matter of cost was considered to be of less than usual importance. However, in the case of Pit No. 4 Dam, the use of a stilling-pool and weir was considered feasible and experiments were carried forward in an endeavor to secure satisfactory results at minimum cost. These experiments led to the adoption of baffle-piers in preference to other methods of destruction of energy.

The splitter piers for the Pit River Dams are provided with 12 by 12-in. by ½-in. steel-plate noses, and no concern has been felt regarding the possible danger from floating logs. This might be more of a problem in other sections of the country.

The data submitted by Mr. Parshall† relative to the Halligan Dam on the North Fork of the Cache La Poudre River are very interesting and indicate that in some cases the scour on an unprotected foundation below a relatively high dam may be of little consequence. The determining factor is without doubt the hardness or the resistance of the foundation to erosion. This cannot always be discovered by inspection, and, in some instances, over-pour dams founded on apparently hard rock have suffered extensive erosion. This particularly applies in cases where the over-pour is not confined to the center of the dam, but is allowed to spill across the entire crest length, in which case erosion of the canyon walls and the abutments is probable.

* *Proceedings*, Am. Soc. C. E., March, 1928, Papers and Discussions, p. 857.
† *Loc. cit.*, May, 1928, Papers and Discussions, p. 1553.

The description by Mr. Winter* of the Martin Dam Spillway which embodies a return-flow conduit at the apron, is of interest. Doubtless such an installation will reduce the effective head at the end of the apron. The directing of the water sheet downward at jump-off seems important to prevent the over-pour sheet from hurdling the piers and it is believed that for all flows over the Pit River Dams the water sheet will plunge into the basin waters rather than be deflected and perhaps carried along the surface. It is also necessary that there be sufficient relief below to permit the reversal of the flow mentioned by Mr. Winter. During the model tests the existence of a reverse current was verified by the movement of chips of wood thrown into the stream some distance below the jump-off. The experiments and action during the recent floods clearly indicate that the hydraulic jump more nearly approaches the toe of the dam as the discharge increases.

Undoubtedly a more precise means of recording the results of the Pit model tests than is afforded by photographs, such as Figs. 64 and 65, would have been desirable. At the time, this was not considered of sufficient importance to warrant the construction of a model stream channel below the dam and the installation of accurate measuring devices. Also, the storage above the test dam was too small to make it possible to maintain a constant head for any length of time over the spillway crest for other than comparatively small flows.

The principal object of the tests was to confine the hydraulic jump close to the toe of the dam, and to destroy on the apron as much of the energy from the falling water as possible in order to reduce the scouring action on the bed and banks of the stream below the apron. The two rows of piers tend to create basins which assist in doing this. As previously stated, there was no final back-water weir although at the time the construction of the Pit No. 3 Dam was completed there was a very low natural weir of boulders immediately below the lower end of the apron. This was probably not more than 3 ft. high on the average and has since partly been swept away.

Mr. Winter's remarks† relative to the necessity of dissipating all energy within the water if the structure is to be lasting, are well taken, and this was the primary purpose of the design. A study of Figs. 64 and 65 clearly indicates a tremendous boiling and cross flow and illustrates the effect of the water impinging on itself and a comparatively even flow at the lower end of the aprons. All the energy cannot, of course, be so dissipated and some wear or scouring of the concrete is anticipated. In this regard it is interesting to note that during the model tests only two six-penny nails were required to hold the curved baffle-piers in position on the apron. However, in the final plans, the concrete piers and apron were designed and reinforced to withstand the full force of the water sheet across the entire cross-section area of the piers. This did not lead to any unusual or extreme construction. A part of the apron slab was designed to act with the pier to prevent possible over-turning. No fear is felt that the apron will be broken up by the action of the water. Should any of the concrete protective work show signs of dangerous wear, it is accessible for repair during the major part of the year, as the power house takes the entire river flow, except during flood periods in the spring months.

* *Proceedings*, Am. Soc. C. E., May, 1928, Papers and Discussions, p. 1558.
† *Loc. cit.*, p. 1560.

Mr. Bauman* has submitted a very interesting mathematical analysis of a subject which is particularly difficult to treat analytically. His basic formulas for determining the length of river bed which should be protected below a dam are well established and should be of assistance to engineers who in the future may be confronted with problems of this character. In particular, his method will be of value in pointing the way for further research, but in the final analysis it is believed that investigation by means of models and observation of completed structures will be necessary to obtain data which can be relied on with certainty.

Reference to the notched or dental weir method of protecting the down-stream channel from scour has been made by several discussors and there seems no doubt that this device might be used to advantage in many cases.

Mr. Stevens† has submitted an enlightening discussion particularly in that he presents data obtained from a number of tests on models using the dental or notched weir to prevent erosion. Apparently, the situation at Leaburg had been carefully analyzed before deciding to use the dental weir to the exclusion of other methods of preventing scour. This series of experiments is especially noteworthy in that data were secured giving the complete profile of the water surface under the various tests. The effect of the scour on the stream bed was also determined. It is to be noted that the solid weir did not give as satisfactory results as the notched type, which fact could be inferred from the Pit River model tests. The data obtained as to the height and location of the dentals to give the best results should be of assistance to future investigators. The observation that a considerable percentage of submergence was necessary before the up-stream water level was affected, was to have been expected from results obtained in other investigations, but might easily be overlooked by an engineer not familiar with this phenomenon.

In conclusion, it might be observed that each discussor presents some particular method of handling the problem of scour below dams, but that none of the discussors apparently has utilized a baffle-pier design similar to the type used below the Pit River Dams. The basic idea of the Society's Special Committee on Irrigation Hydraulics in having this paper published was to bring out additional information regarding other successful methods of preventing scour below dams, and it is felt that in the many discussions offered much valuable data have been accumulated and that a great deal of attention is being given to this problem.

The authors wish to express their appreciation for the many contributions to their paper. Acknowledgments are also due the following engineers who were in immediate charge of the tests: Walter Dreyer, Assistant Engineer in Charge of Design, Grover C. Green, Field Engineer, Clinton de Witt, Assistant Engineer, and Arthur W. Kidder, Assistant Field Engineer, all Associate Members of the Society. The engineering for Pit No. 3 and Pit No. 4 projects was under the direction of A. H. Markwart, M. Am. Soc. C. E., Vice-President in Charge of Engineering, Pacific Gas and Electric Company. John D. Galloway, M. Am. Soc. C. E., was Consulting Engineer on both dams.

* *Proceedings*, Am. Soc. C. E., August, 1928, Papers and Discussions, p. 1807.

† *Loc. cit.*, October, 1928, Papers and Discussions, p. 2303.

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# PAPERS AND DISCUSSIONS

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## SILTING OF THE LAKE AT AUSTIN, TEXAS

### Discussion*

By T. U. Taylor, M. Am. Soc. C. E.†

T. U. Taylor,‡ M. Am. Soc. C. E. (by letter).§—The discussions of the silting of actual reservoirs have been disappointing, because practically all descriptions of the measurements made have come from the arid and semi-arid West, and these are largely confined to the Southwest.

However, the results have been valuable beyond expectation. The writer, after spending more than a third of a century in studying silting of reservoirs and in making actual measurements on existing reservoirs, has had some of his cherished ideas exploded. The first of these was the assumption on his part that there was a direct connection between run-off and the amount of silt deposited in the reservoir. The data contributed by Mr. Robinson have proved that in some reservoirs there is slight connection.‖ A very dry, parched water-shed with a sudden run-off will produce a much greater amount of silt than a much heavier rain on a wet water-shed.

It has been claimed that lakes spreading out over their banks and having widths many times that of the river channel, will not silt as rapidly as channel lakes. This is not substantiated by experience at the old or the new lakes at Austin, Tex. The width of the new lake on the average was three times that of the river channel, two-thirds of it being over the relatively flat uplands near the dam. For the stretch extending from the dam to a point 1 003 ft. up stream the channel part of the lake has silted to 84% of its capacity in 1913, while that part on the nearly flat valley which was formerly a farm, has silted to more than 114% of the original capacity. Next to the bank on the east side there has been added (in the period from 1913 to 1926) a strip of land of an average width of 327 ft. and a length along the river bank of 1 027 ft., covering

* Discussion of the paper by T. U. Taylor, M. Am. Soc. C. E., continued from November, 1928, *Proceedings*.

† Author's closure.

‡ Dean of Eng., Univ. of Texas, Austin, Tex.

§ Received by the Secretary, November 19, 1928.

‖ *Proceedings*, Am. Soc. C. E., August, 1928, Papers and Discussions, p. 1939.

an area of 14.5 acres, all outside the channel area. One mile above the dam on the west bank a more intensive silting outside the channel area has taken place. At this point, a boathouse resting on high columns, under which there was a depth of 11 ft. of water in 1913, is now "high and dry" on 16 ft. of silt at least 200 ft. from the water's edge. In all, there are three broad marginal areas on the west side of the lake, aggregating a total length of 5.2 miles and containing silt to the amount of more than 100% of the original capacity. On the east margin there are five curved strips aggregating a total length of 7.5 miles where the lake spreads out over old farm lands. In this total length, that part outside the channel is more than 111% in every case, while that in the channel proper is less than 88% of the original capacity.

In 1913, at a point 1 000 ft. up stream from the dam, the water was 3.2 ft. deep on the east side. In 1928, the silt was 7.8 ft. above the crest of the dam, making a total silting at this section of 11.0 ft. on the east bank. In the center of the present channel there is a depth of water of 5.5 ft. where, in 1913, there was a depth of 51.42 ft., or 45.92 ft. of silt in the middle of the old and present channel.

A study of all the reservoirs for which data can be obtained, convinces the writer that silting is no respecter of channel or valley. The broad reservoirs of McMillan, Zuni, and Lake Worth, all bear out the fact that valley reservoirs, if the width of the channel is insignificant with respect to the width of the lake, will collect silt. On the other hand, the reservoirs at Austin, Lake Pennick, and the Lower Rio Grande, bear out the fact that the channel will also collect silt.

The silt cakes and cracks on the Rio Grande near San Benito are almost exact duplicates of those at Austin. The behavior of these cakes will convince any one that the "hog wallow" prairies of Texas were produced by exceedingly dry seasons during which the ground broke open and left cracks from 2 to 3 in. wide. Later, rains would soften and break the edges which would sink into the cracks and leave a hill in the center and a hollow or depression between the cakes. While the distance between "hog wallows" is about 4 ft. from crest to crest, the silt cakes and cracks are not more than one-third of this.

The most striking example of the fact that valley reservoirs collect silt is the case of Lake Worth, near Fort Worth, Tex. The original volume of Lake Worth in 1915—when the reservoir first began functioning—was 47 177 acre-ft. In July, 1928, thirteen years afterward, the writer made a complete survey from the dam to the head of the lake, which gives the total capacity as 33 340 acre-ft. The lower sections of the lake (south of "9-Mile Bridge") were kept separate for comparison; the silting is shown in Table 12.

The main channel of the river, from its entrance to the dam, because of its meanders, is 10 miles long. Its total surface area is 112 acres, or only 3% of the total area of the lake. The total amount of silting in the 15 years has been 13 837 acre-ft., and the silting in the channel proper is proportionate to that in the valley, that is, in the lake area outside the channel. Some parts of the valley have filled 100%, while in no place has the channel been filled to the brim.

The writer has come to the deliberate conclusion that the ordinary freshets or small floods have little effect on the silting, and that it is extraordinary floods, water-spouts, tornadoes, hurricanes, and cloudbursts falling on a relatively dry water-shed that contribute the larger part of the silt to impounding reservoirs. The flood in the spring of 1922 contributed more silt to the new lake at Austin in 2 weeks than had accumulated in the previous 5 years.

TABLE 12.—CAPACITY OF LAKE WORTH.

| Location. | WATER CAPACITY, IN ACRE-FEET. | | AREA, IN ACRES. | MEAN. | DEPTH, IN FEET. |
|---|---|---|---|---|---|
| | 1915. | 1928. | 1915. | 1915. | 1928. |
| Below 9-Mile Bridge... | 38 364 | 29 880 | 2 590 | 14.81 | 11.54 |
| Above 9-Mile Bridge... | 8 813 | 3 460 | 1 180 | 7.47 | 2.93 |
| Entire lake..... | 47 177 | 33 340 | 3 770 | 12.51 | 8.84 |

However, if the silt is assumed to have been deposited at a uniform rate during the 13 years of the life of Lake Worth, the capacity in 1928 is 70.67% of the full 1915 capacity; and, at the same rate, it will be reduced by 1941 to 49.95%, or about 23 000 acre-ft.

During November, 1928, the writer visited the Lower Rio Grande Valley with a view of seeing at first hand the conditions outlined in the discussion of Mr. Welty.* In 1925, the Rio Grande, true to form, cut through one of the old sags referred to by Mr. Welty, and left the pumping plants of the San Benito Company and of Harlingen, Tex.—1¾ miles up stream—more than ½ mile from the river. The river cut across a U-shaped section, Fig. 21(*a*), and left the two pumping plants on the old loop of the river. This forced each company to use the old section of the river from the plant to the new river course as a forebay or feed canal for the pumps.

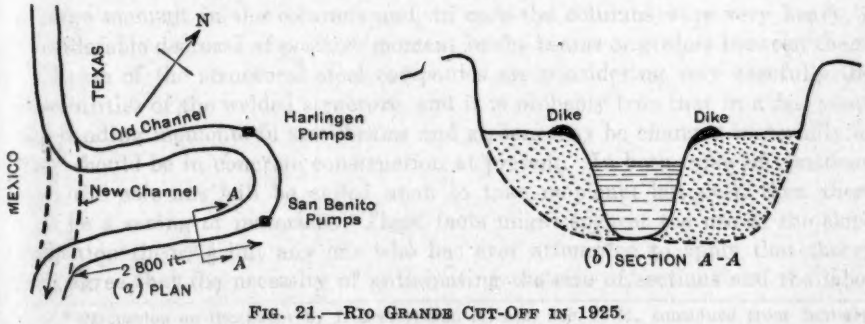
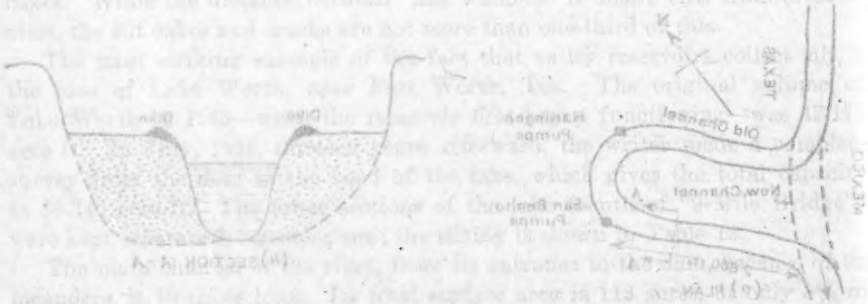



FIG. 21.—RIO GRANDE CUT-OFF IN 1925.

The silt from the Rio Grande is an ever-present menace to the San Benito Company in closing its forebay or source of supply. An 8-in. pump is kept at

* *Proceedings*, Am. Soc. C. E., October, 1928, Papers and Discussions, p. 2359.

work in the old channel ten months of the year. The middle third of the old river is used as a channel (Fig. 21) and the "slush" is deposited on each side by the hydraulic-fill method. The pump is kept on a barge and works from it. The silt is stirred into a slush consisting of 85% water, which is pumped over a berm and is kept from flowing back into the river by a small dike on the river side of the berm. This method is used throughout the entire 2 800 ft. to the new river.



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# CONTINUOUS BEAMS OVER THREE SPANS

## Discussion*

BY MESSRS. R. A. CAUGHEY, JOHN B. SNETHLAGE, AND T. D. MYLREA.

R. A. CAUGHEY,† M. AM. SOC. C. E. (by letter).‡—In the presentation of this paper, Mr. Oesterblom has well emphasized the need of more care in the computation of moments in beams and girders continuous over several supports. He has also done well in presenting Table 3§ and in giving a treatment for continuous beams of variable moments of inertia. In spite of all this the writer fears that this or other similar papers stop somewhat short of the goal which structural engineers would like to reach.

When continuous slabs or joists are supported by girders, the torsional rigidity of which is comparatively small, the theorem of three moments is applicable; but when beams or girders of any kind frame into columns, it seems that some other means of computing the moments should be adopted. This would hold in the case of the construction mentioned by Mr. Oesterblom if the girders were placed in the transverse direction of the building and joists of one type or other were placed parallel to the length of the building. Under such arrangement one would find, by using the slope deflection theory, a large moment in the columns and, in case the columns were very heavy, a considerable decrease of positive moment in the beams or girders between them.

Some of the structural steel companies are considering very carefully the possibilities of the welded structure, and it is probably true that in a few years the bending moments in steel beams and girders may be changed materially as they should be in concrete construction at present. In both types of construction the columns will be called upon to take moment; but even then there will be a saving of materials. These facts might suggest the use of the slope deflection theory,‖ but any one who has ever attempted to apply that theory will agree that the necessity of anticipating the size of sections and the labor

* Discussion on the paper by I. Oesterblom, M. Am. Soc. C. E., continued from January, 1929, *Proceedings*.

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‡ Received by the Secretary, October 18, 1928.

§ *Proceedings*, Am. Soc. C. E., March, 1928, Papers and Discussions, p. 719.

‖ *Bulletin No. 108*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

involved in the solution of large numbers of simultaneous equations will rule it out of consideration in most cases. The writer believes, however, that with certain approximations this method can be adopted, and that it will give results sufficiently close for practical use—much closer than some of the arbitrary coefficients in common use and far closer than will be obtained by the theorem of three moments. The writer has prepared a series of curves in order to apply the method rapidly enough to make it practicable.

Fig. 7 shows moment diagrams* and shear values obtained by loading two different panels of an unsymmetrical building frame with a uniform load.† In both cases, the effect of the load in any given panel is negligible beyond those immediately adjacent to the loaded panel. The highest value of moment outside the limits mentioned is that in Column $L\,Q$ (Fig. 7($b$)), but very little computation is necessary to show that for any reasonable load on the girder, $G\,H$, the moment in $L\,Q$ will produce very small unit stresses in the column and that they will be maximum at the end of the column where it is stayed against buckling. This will be true generally for other members similarly located.

This would suggest dividing the structure into several parts similar to that shown in Fig. 8($a$) and considering each part with its load separately. In the inspection of quite a large number of moment diagrams it was observed that the inflection points on columns are generally located about eight-tenths of the height above or below the floor-beams or girders. The inflection points on the horizontal members may be located from Fig. 8($b$) which was made up from several conditions of loading on symmetrical and unsymmetrical frames. As will be noticed the location of the inflection point seems to be a function of the ratio of the lengths of loaded and adjacent spans.

The application of the slope deflection theory to such a structure becomes a very simple problem as there are only two equations to be made up and solved simultaneously. In order to simplify the derivation of equations to be used, $\Sigma K_1$ will be taken as the sum of the $K$-values of all the members meeting the loaded beam on the left; $K_2$ will be the $K$ for the loaded beam and $\Sigma K_3$ the sum of the $K$-values of all members meeting the right end of the loaded beam. The structure can then be considered as a three-span continuous beam, $X\,A\,B\,Y$ (Fig. 8($a$)), hinged at the ends, $X$ and $Y$.

Applying the slope deflection equations:

$$M_{AX} = E \Sigma K_1 (3\,\theta_A) \quad \ldots\ldots (35)$$

$$M_{AB} = 2\,E\,K_2 (2\,\theta_A + \theta_B) - C_{AB} \quad \ldots\ldots (36)$$

$$M_{BA} = 2\,E\,K_2 (\theta_A + 2\,\theta_B) + C_{BA} \quad \ldots\ldots (37)$$

$$M_{BY} = E \Sigma K_3 (3\,\theta_B) \quad \ldots\ldots (38)$$

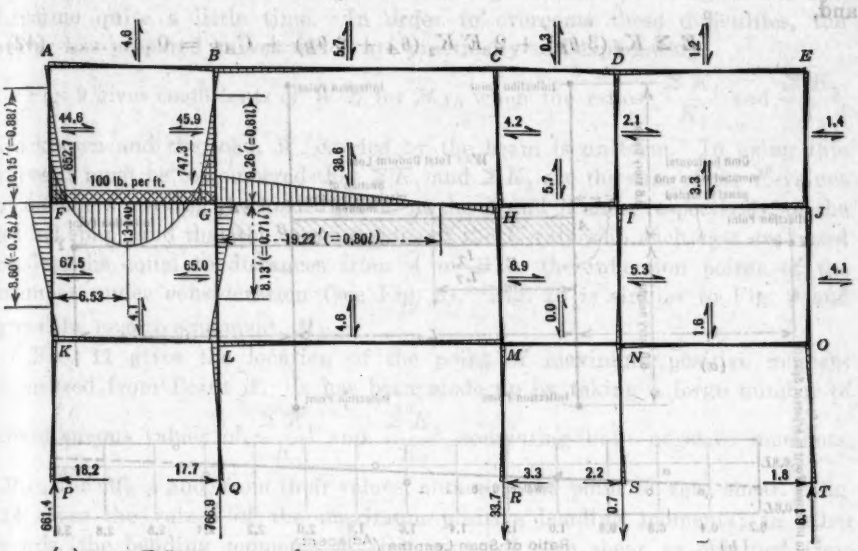
In order to have equilibrium at $A$ and $B$:

$$M_{AX} + M_{AB} = 0 \quad \ldots\ldots (39)$$

and,

$$M_{BY} + M_{BA} = 0 \quad \ldots\ldots (40)$$

* These curves by E. E. Ebling, Assoc. M. Am. Soc. C. E., are obtained by the slope deflection method and are taken from a thesis prepared under supervision of the writer and submitted for the degree of Master of Science at the Iowa State Coll.



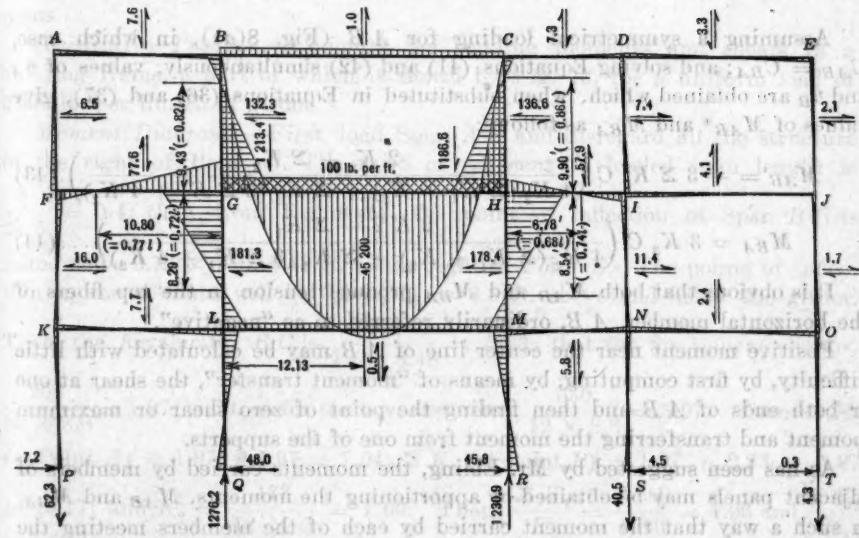




FIG. 7.—MOMENT DIAGRAMS AND SHEAR IN AN UNSYMMETRICAL BUILDING FRAME.

Substituting the values given by Equations (35), (36), (37), and (38), in Equations (39) and (40):

$$E \Sigma K_1 (3\theta_A) + 2 E K_2 (2\theta_A + \theta_B) - C_{AB} = 0 \ldots\ldots\ldots (41)$$

and,

$$E \Sigma K_3 (3\theta_B) + 2 E K_2 (\theta_A + 2\theta_B) + C_{AB} = 0 \ldots\ldots\ldots (42)$$

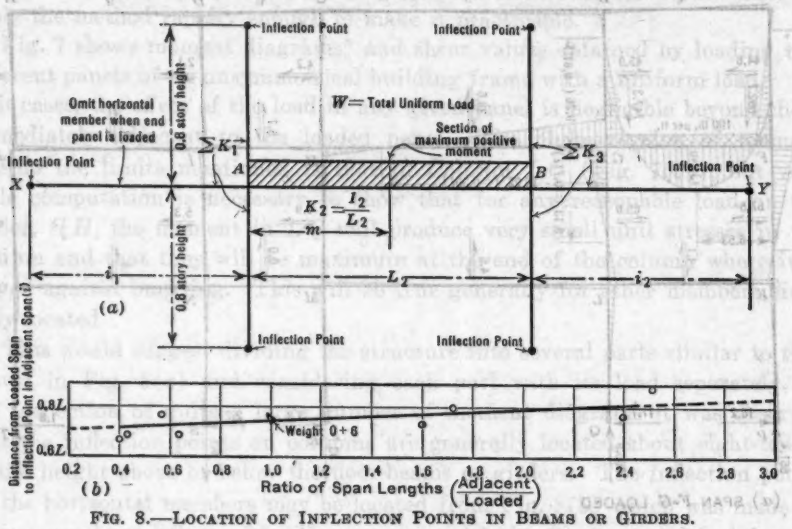



FIG. 8.—LOCATION OF INFLECTION POINTS IN BEAMS OR GIRDERS.

Assuming a symmetrical loading for $A\,B$ (Fig. 8($a$)), in which case, $C_{AB} = C_{BA}$; and solving Equations (41) and (42) simultaneously, values of $\theta_A$ and $\theta_B$ are obtained which, when substituted in Equations (36) and (37), give values of $M_{AB}$* and $M_{BA}$ as follows:

$$M_{AB} = -3 \Sigma K_1 C \left( \frac{2 K_2 + \Sigma K_3}{4 K_2 (\Sigma K_1 + K_2) + \Sigma K_3 (3 \Sigma K_1 + 4 K_2)} \right) \ldots (43)$$

$$M_{BA} = 3 K_3 C \left( \frac{2 K 2 + \Sigma K_1}{4 K_2 (\Sigma K_1 + K_2) + \Sigma K_3 (3 \Sigma K_1 + 4 K_2)} \right) \ldots (44)$$

It is obvious that both $M_{AB}$ and $M_{BA}$ produce tension in the top fibers of the horizontal member, $A\,B$, ordinarily referred to as "negative".

Positive moment near the center line of $A\,B$ may be calculated with little difficulty, by first computing, by means of "moment transfer", the shear at one or both ends of $A\,B$ and then finding the point of zero shear or maximum moment and transferring the moment from one of the supports.

As has been suggested by Mr. Ebling, the moments carried by members of adjacent panels may be obtained by apportioning the moments, $M_{AB}$ and $M_{BA}$, in such a way that the moment carried by each of the members meeting the end of the loaded girder is proportional to its $K$-value. These latter moments act to oppose the moments, $M_{AB}$ and $M_{BA}$, thus holding the joint in equilibrium.

*It will be noted that $M_{AB}$ and $M_{BA}$ have opposite signs. For the method of determining signs and applying the slope deflection method generally, *see Bulletin No. 108*, Eng. Experiment Station, Univ. of Illinois.

*Explanation of Curves for Computing Moments.*—In actual practice it may be necessary, in order to secure the proper sections for beams and columns of a building, to design preliminary sections and revise them at least once and probably oftener. This would involve a large amount of computation and consume quite a little time. In order to overcome these difficulties, the writer has prepared curves which are practically self-explanatory.

Fig. 9 gives coefficients of $W L$ for $M_{AB}$ when the ratios, $\frac{\Sigma K_1}{K_2}$ and $\frac{\Sigma K_3}{K_2}$, are known and the load, $W$, carried by the beam is uniform. In using this curve it must be remembered that $\Sigma K_1$ and $\Sigma K_3$ are the sum of the $K$-values of members meeting the loaded girder at the $A$ and $B$ ends, respectively, of the loaded span; and that the computations of the $K$-values in each case are based on lengths equal to distances from $A$ or $B$ to the inflection points of the member under consideration (see Fig. 8). Fig. 10 is similar to Fig. 9 and gives the negative moment, $M_{BA}$.

Fig. 11 gives the location of the point of maximum positive moment measured from Point $A$. It has been made up by taking a large number of simultaneous values of $\frac{\Sigma K_1}{K_2}$ and $\frac{\Sigma K_3}{K_3}$, computing both negative moments, $M_{AB}$ and $M_{BA}$, and, from their values, obtaining the point of zero shear. Fig. 12 gives the values of the maximum positive bending moments; in other words, the bending moments at the points of zero shear as obtained from Fig. 11. Figs. 9, 10, 11, or 12 apply in the case of either end or intermediate spans.

*Illustrative Problem.*—Compute the bending moments and shears in a building frame, a part of which is shown by Fig. 13, for a uniform load of 2 000 lb. per lin. ft. of girder.

*Moment Diagrams.*—First, load Span $A B$ and disregard all the structure to the right of Point $C$. The ratio of adjacent to loaded span length is $\frac{9}{22.5} = 0.4$; then, from Fig. 8 ($b$), the point of inflection of Span $B C$ is found to be $0.72 \times 9 = 6.48$ ft. to the right of Point $B$. The points of inflection in the columns will be $0.8 \times 11 = 8.8$ ft. above and below the girder. Then, the $K$-value for $B C$ is $\frac{173}{12 \times 6.48} = 2.23$; that for the upper columns, $\frac{208}{12 \times 8.8} = 1.97$; and that for the lower columns, $\frac{630}{12 \times 8.8} = 5.97$. Then, $\Sigma K_1$ (at Point $A$) $= 1.97 + 5.97 = 7.94$; $\Sigma K_3$ (at Point $B$) $= 1.97 + 2.23 + 5.97 = 10.17$; and $K_2 = \frac{173}{12 \times 9} = 1.60$. Then, $\frac{\Sigma K_1}{K_2} = \frac{7.94}{1.60} = 4.96$ and $\frac{\Sigma K_3}{K_2} = \frac{10.17}{1.60} = 6.36$.

From Fig. 9,

$$M_{AB} = 0.0723 \times 2\,000 \times 22.5 \times 22.5 = 73\,200 \text{ ft-lb.}$$

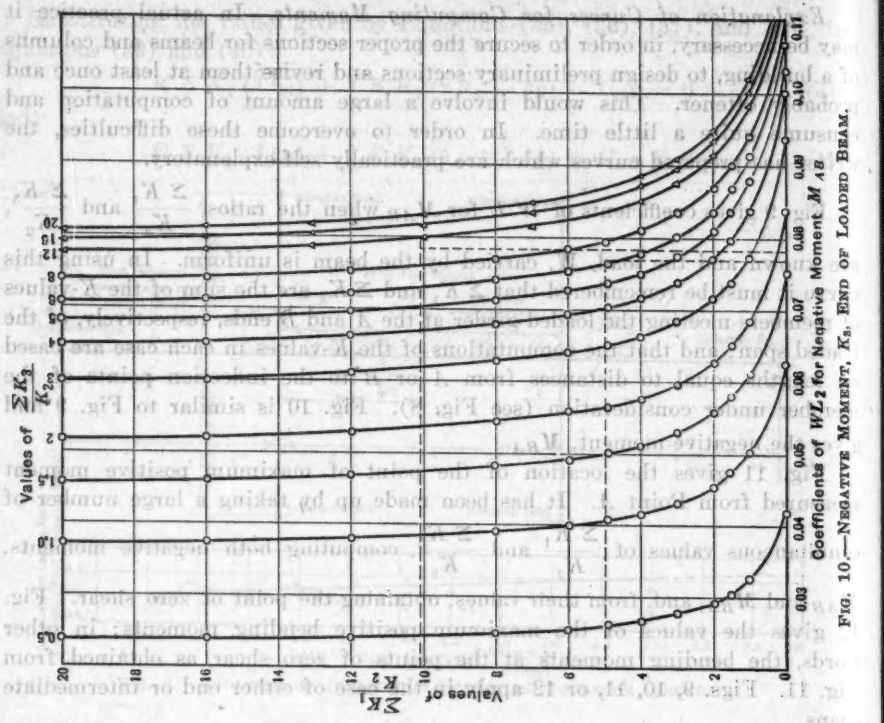



FIG. 10.—NEGATIVE MOMENT, $K_3$, END OF LOADED BEAM.

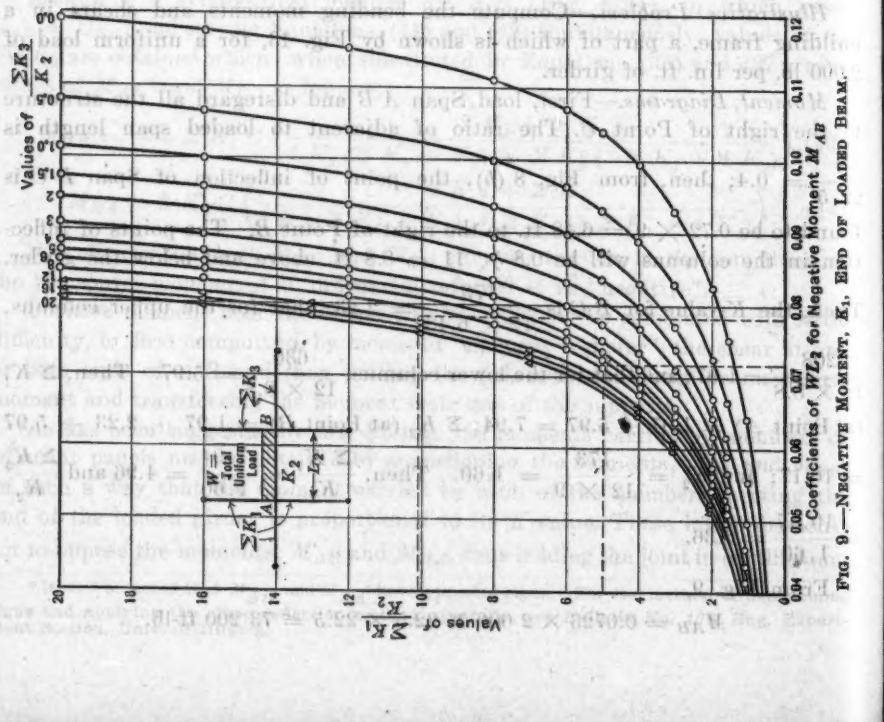



FIG. 9.—NEGATIVE MOMENT, $K_1$, END OF LOADED BEAM.

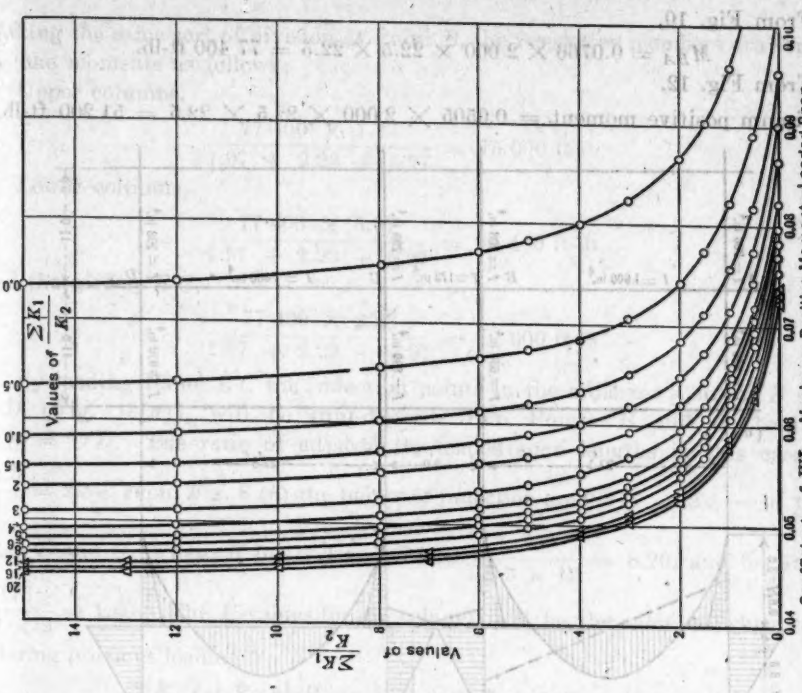



FIG. 12.—NEGATIVE MOMENT, $K_1$, END OF LOADED BEAM.

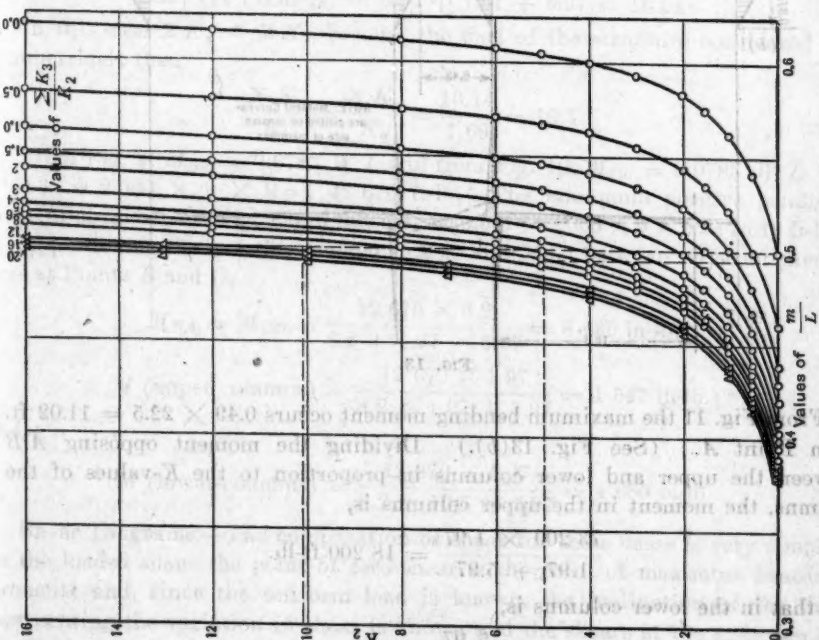



FIG. 11.—DISTANCE TO POINT OF MAXIMUM POSITIVE BENDING MOMENT—($m$).

FIG. 9.—NEGATIVE MOMENT, $K_1$, END OF LOADED BEAM.

FIG. 10.—NEGATIVE MOMENT, $K_3$, END OF LOADED BEAM.

From Fig. 10,

$$M_{BA} = 0.0766 \times 2\,000 \times 22.5 \times 22.5 = 77\,400 \text{ ft-lb.}$$

From Fig. 12,

Maximum positive moment $= 0.0505 \times 2\,000 \times 22.5 \times 22.5 = 51\,200$ ft-lb.

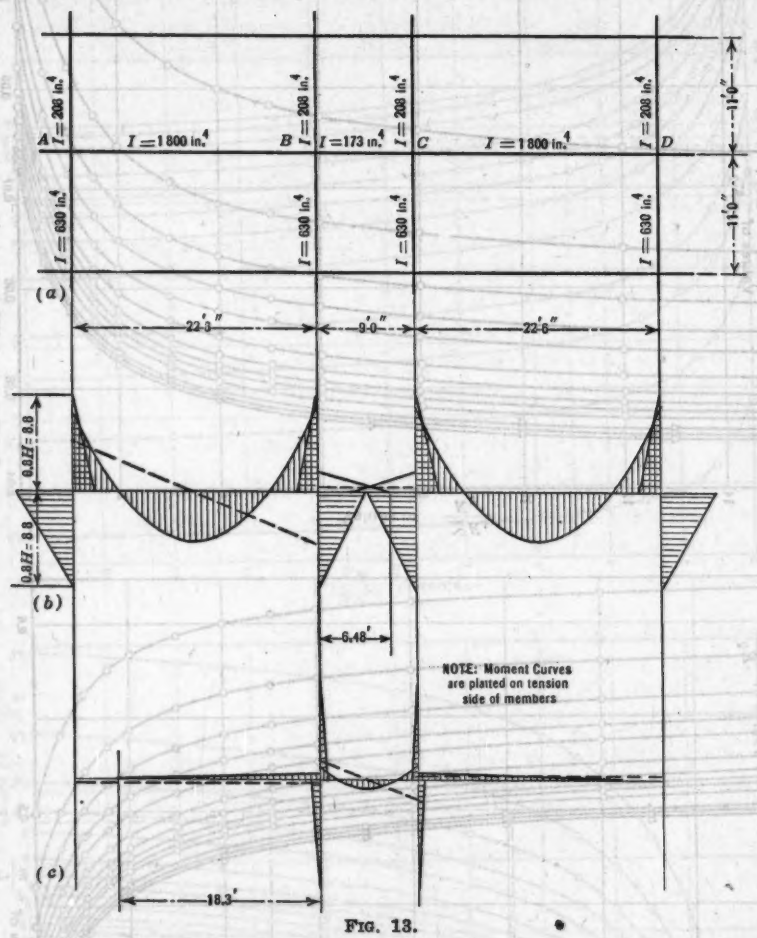



FIG. 13.

From Fig. 11 the maximum bending moment occurs $0.49 \times 22.5 = 11.02$ ft. from Point $A$. (See Fig. 13($b$).) Dividing the moment opposing $AB$ between the upper and lower columns in proportion to the $K$-values of the columns, the moment in the upper columns is,

$$\frac{73\,200 \times 1.97}{1.97 + 5.97} = 18\,200 \text{ ft-lb.}$$

and that in the lower columns is,

$$73\,200 \times \frac{5.97}{7.94} = 55\,000 \text{ ft-lb.}$$

Making the same sort of division at Point $B$, the respective members are found to take moments as follows:

Upper columns,

$$\frac{77\,400 \times 1.97}{1.97 + 2.23 + 5.97} = 15\,000 \text{ ft-lb.}$$

Lower columns,

$$\frac{77\,400 \times 5.97}{1.97 + 2.23 + 5.97} = 45\,400 \text{ ft-lb.}$$

and the girder, $B\,C$,

$$\frac{77\,400 \times 2.23}{1.97 + 2.23 + 5.97} = 17\,000 \text{ ft-lb.}$$

By loading Panel $B\,C$ the inflection points in the adjacent spans, $A\,B$ and $C\,D$ (Fig. 13($a$)), will be equi-distant from Points $B$ and $C$, because $A\,B = C\,D$. The ratio of adjacent to loaded span lengths in this case is $\frac{22.5}{9} = 2.5$. From Fig. 8 ($b$) the points of inflection are $0.815 \times 22.5 = 18.3$ ft. from $B$ and $C$. Then, $K$ for $B\,A$ and $C\,D = \frac{1\,800}{18.3 \times 12} = 8.20$, and for $B\,C$, $\frac{173}{12 \times 9} = 1.60$. The $K$-values for the columns will be the same as when considering previous loading.

$$\Sigma K_1 \text{ (at Point } B) = 8.20 + 1.97 + 5.97 = 16.14$$

In this case $\Sigma K_3 = \Sigma K_1$, because the part of the structure considered is symmetrical; then,

$$\frac{\Sigma K_1}{K_2} = \frac{\Sigma K_3}{K_2} = \frac{16.14}{1.60} = 10.1$$

From Fig. 9, $M_{BC} = 0.0783\ W\,L$ and from Fig. 10, $M_{CB} = 0.0783\ W\,L = 0.0783 \times 2\,000 \times 9 \times 9 = 12\,670$ ft-lb. The maximum positive bending moment at mid-span (Fig. 12) in this case is, $0.047 \times 2\,000 \times 9 \times 9 = 7\,610$ ft-lb.

Apportioning the resisting moments, $M_{BC}$ and $M_{CB}$, between adjacent members at Points $B$ and $C$,

$$M_{BA} = M_{CD} = \frac{12\,670 \times 8.2}{8.2 + 1.97 + 5.97} = 6\,430 \text{ in-lb.}$$

$$M \text{ (upper columns)} = \frac{12\,670 \times 1.97}{8.2 + 1.97 + 5.97} = 1\,547 \text{ in-lb.;}$$

and,

$$M \text{ (lower columns)} = \frac{12\,670 \times 5.97}{8.2 + 1.97 + 5.97} = 4\,680 \text{ in-lb.}$$

Shear Diagrams.—The computation of shear for these cases is very simple. In the loaded spans the point of zero shear is the point of maximum bending moment; and, since the uniform load is known, the inclination of the line representing the variation in shear is known and the shears at the ends can be obtained by multiplying the length of right and left segments of the loaded beam by the uniform load per unit of length. In this case the point of

maximum bending with Span $A\,B$ loaded is $0.495 \times 22.5 = 11.15$ ft. from the left end. The left and right segments of the beam are, therefore, 11.15 and 11.35 ft., and the shears at Points $A$ and $B$ are $11.15 \times 2\,000 = 22\,300$ and $11.35 \times 2\,000 = 22\,700$, respectively.

In case of the adjacent spans with $A\,B$ loaded, the shear is constant and equal to the maximum end moment divided by the distance from the end to the point of inflection; in this case, $\frac{M_{BC}}{i} = \frac{17\,000}{6.48} = 26.15$.

With Span $B\,C$ loaded, the shears at the ends of the loaded span are the same, since the structure and load are symmetrical, and are equal to $4.5 \times 2\,000 = 9\,000$ lb., the shears in the adjacent spans being $\frac{6\,430}{18.3} = 355$.

The shear diagrams are drawn in dotted lines on Fig. 13 (*b*) and (*c*) for Spans $A\,B$ and $B\,C$ loaded; that for Spans $C\,D$ loaded will be opposite hand to that for Span $A\,B$ loaded.

Combination of Diagrams.—The maximum moment and shear for any given member of the frame at any given point may be obtained by combining the results given in the shear and moment diagrams of Fig. 13 in such a way as to produce the desired maximum.

*Loads Other Than Uniform.*—In case the "total load, $W$," is to be placed on a given span as one or more concentrated loads placed symmetrically, the moments, $M_{AB}$ and $M_{BA}$, of Equations (43) and (44) may be computed by substituting for the constant, $C$, another constant instead of $\frac{W\,L}{12}$, which is used for uniform load only and which represents the negative moment at the ends of a fixed beam. The constants which may be substituted for $\frac{W\,L}{12}$ are the values of the negative moments at the ends of a symmetrically loaded fixed beam carrying the load, $W$, distributed as desired. This substitution suggests computing first the ratio of the value of $C$ with the new loading condition to $\frac{W\,L}{12}$ and multiplying the values of the negative moments at the ends of the loaded girder found from Figs. 9 and 10 by the ratio thus obtained. With these moments computed, all necessary moments and shears may be calculated in a manner similar to that used in the example given for uniform loads.

As to the accuracy of the results obtained by this method and by the curves submitted, the writer has checked the results obtained by Mr. Ebling and has found that in four loading conditions in a very unsymmetrical building, the moments in the girder of the loaded panel (at ends and near mid-span) obtained by the curves have checked Mr. Ebling's slope deflection results with a maximum error of 7.6 per cent. The moments in question taken from the curves have occurred, of course, on the working line of the building frame, which is, in this case, on the center line of the columns. The moment at the face of the column would be actually somewhat less than that at the center line, and since higher unit stresses are usually allowed in a monolithic build-

ing frame in sections of members adjacent to supports, it would seem that the 7.6% variation mentioned, would cause no trouble.

The check has not been as good on members in which the moment has been obtained by proportion between the values of $K$, because the maximum error in one case—and in only one case—reached 14 per cent. All checks, however, have been close enough in the writer's opinion to warrant the use of these curves or similar ones in place of arbitrary coefficients or even the three-moment equation, when there is a possibility of the columns and beams (or girders) forming a rigid frame. This opinion is based on the fact that the allowable increase in unit stresses at the ends of members, together with a decrease in moment at the face of the members into which they frame, as compared to the moment at the center line of the latter would absorb, in practically all cases, a difference as great as 14%; in some cases much more.

JOHN B. SNETHLAGE,* ASSOC. M. AM. SOC. C. E. (by letter).†—The author has certainly given a clear idea of how he thinks continuous beams over three spans ought to be designed, and the results are most efficiently laid down in tables which cover a sufficient number of span varieties to warrant usefulness. Little is to be added to his general treatment of the problem. It is well known that normally the conditions at the supports are not as he assumes them to be and that restraining moments at the ends counteract the theoretical ones.

The author makes a severe attack on conventional moment factors, of building codes, joint committee regulations, etc., and attempts to illustrate their shortcomings with an example‡ which seems to make their use prohibitive.

When his method gives the correct negative moment as —4 490 ft-lb. in the center span, and the conventional moment-factor method, according to him, calls for + 940 ft-lb. (see Table 4),‡ the writer can hardly believe that there can be compensating features to prevent failure. In the writer's opinion the author does not apply these moment factors as they were intended.

As shown in Table 4, the correct maximum moments for the conditions given, are: + 5 740 ft-lb. for outer spans; —4 490 ft-lb. for center span; and —5 770 ft-lb. for center supports.

By applying the conventional moment factors, a positive moment is found in the outer spans equal to $\frac{1}{10} \times 127 \times 21.75^2 = 6\,000$ ft-lb., and at the center supports, not some kind of an average as the author states is used, but a negative moment of the same absolute value as the positive moment, or —6 000 ft-lb. (See Fig. 15.)

This negative moment prevails over the entire center span only to be reduced by the positive moment of the center span, or $\frac{1}{12} \times 147 \times 8.3^2 = 850$ ft-lb.

* Civ. and Hydr. Engr., Hawthorne, N. J.

† Received by the Secretary, November 23, 1928.

‡ *Proceedings*, Am. Soc. C. E., March, 1928, Papers and Discussions, p. 723.

Therefore, this span must be reinforced for a negative moment, in the middle, of 6 000 — 850 = 5 150 ft-lb.

Comparison of these figures with Table 4 shows that the moment factor method requires 4% more steel at the center supports and 4.5% more steel at the outer spans, against which hardly any objection can be raised. On the center span, however, this waste is 15% which seems excessive; but in reality it is of little importance, as that negative moment increases very rapidly toward the center supports.

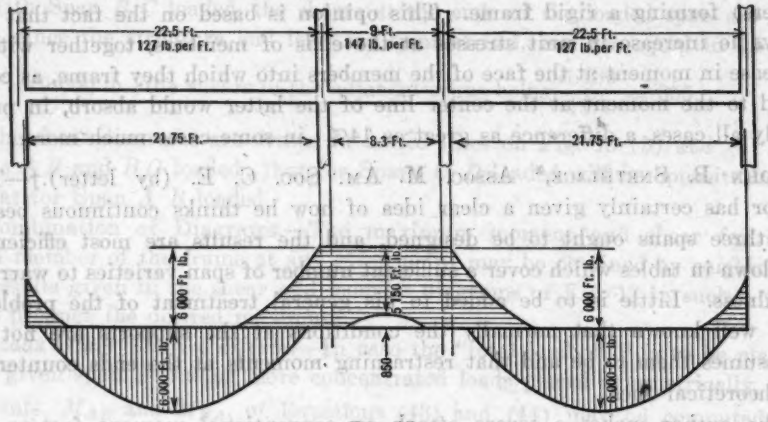



FIG. 15.

Moreover, in detailing the steel it will not be found possible to follow the fluctuation accurately by decreasing and again increasing the amount of steel over so short a distance as 9 ft., and most likely the steel from the negative moment at one center support will be continued over the entire center span to the other support.

This demonstrates clearly what the writer believes to be the proper application and true interpretation of the use of the conventional moment factors of building codes, joint committee regulations, etc., and although by no means ideal, they certainly do not deserve such unreasonable criticism.

T. D. MYLREA,* M. AM. SOC. C. E. (by letter).†—The author is certainly justified in his criticism of the practice of using arbitrary moment coefficients in many cases to which they do not apply, and is to be commended for his efforts toward simplifying the methods of finding the true moment coefficients in a three-span continuous beam. The labor involved and the time consumed in computing the true coefficients by the usual methods are undoubtedly responsible for the present reliance on arbitrary coefficients. As the author points out, this is particularly true in the usual analytic solution when a beam of more than three spans is encountered. Here, the graphical solution of Messrs. Nishkian and Steinman‡ may prove more expeditious.

* Prof. of Building Constr., Carnegie Inst. of Tech., Pittsburgh, Pa.

† Received by the Secretary, November 28, 1928.

‡ "Moments in Restrained and Continuous Beams by the Method of Conjugate Points", by L. H. Nishkian and D. B. Steinman, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 1.

The analytic solution, however, may be approached from a standpoint distinctly different from that taken by the author, and one which permits of the solution of multi-span beams with comparative ease. The kernel of this method is suggested in Mr. Hadley's discussion.* It consists of distributing throughout the beam the unbalanced, or excess, fixed-end moments. The method Mr. Hadley proposes is not entirely new, however, for the solution by distributing unbalanced moments has been taught by Hardy Cross, M. Am. Soc. C. E., for several years.

Furthermore, Professor Cross' method is applicable to beams of any number of spans, as well as to continuous bents, and the labor involved is small. Moreover, by this method there is no need of the scheduling of equations and certain numerical factors, as both the author and Mr. Hadley do, in which way the computer is apt to lose sight of the physical beam. The writer has often used this method with much satisfaction in his own work and feels that Professor Cross should publish the complete details for the benefit of the profession.†

* *Proceedings*, Am. Soc. C. E., November, 1928, Papers and Discussions, p. 2559.

† A paper by Professor Cross covering his method has been accepted for publication by the Society. EDITOR.

The analytic solution, however, may be approached from a standpoint distinctly different from that taken by the author, and one which permits of the solution of multi-span beams with comparative ease. The kernel of this method is suggested in Mr. Hadley's discussion.* It consists of distributing through the beam the unbalanced, or excess, fixed-end moments. The method Mr. Hadley proposes is not entirely new, however, for the solution by distributing balanced moments has been taught by Hardy Cross, M. Am. Soc. C. E., for several years.

Furthermore, Professor Cross' method is applicable to beams of any number of spans, as well as to continuous beams, and the labor involved is small. Moreover, by this method there is no need of the solution of equations and certain material factors, as both the author and Mr. Hadley do, in which way the computer is apt to lose sight of the physical beam. The writer has often used the method with much satisfaction in his own work and feels that Professor Cross should publish the complete details for the benefit of the profession.†

* Proceedings, Am. Soc. C. E., November, 1923, Papers and Discussions, p. 2393.

† A paper by Professor Cross covering his method has been accepted for publication by the Society.—EDITOR.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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# PAPERS AND DISCUSSIONS

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## HYDROSTATIC UPLIFT IN PERVIOUS SOILS

### Discussion*

BY H. DE B. PARSONS, M. AM. SOC. C. E.†

H. DE B. PARSONS,‡ M. AM. SOC. C. E. (by letter).§—A full and frank discussion increases knowledge, and this is especially true when the subject is one of experimentation and research. The writer, therefore, thanks those who have criticized his paper. Some of the discussions require no comment, and consideration of the others will be as brief as the facts will permit.

Mr. Meem‖ infers that the pistons or contact rods were not pressed or forced into the sand beds so as to secure full contact, but were simply rested on the sand. The experimental rods were forced down firmly, and their unit loading was as heavy as that under many buildings. The cans were pressed down by hand to compress the sand beneath their bases and establish definite areas of contact. In some of the tests made at Rensselaer Polytechnic Institute, the apparatus was allowed to stand, heavily loaded, for 18 to 48 hours, but the results were substantially the same as when the tests were made (with saturated sand) shortly after arranging the apparatus. Mr. Meem's experiments are interesting, but seem, to the writer, to embody dynamic rather than strictly hydrostatic conditions. Professor Terzaghi's discussion¶ should be read with Mr. Meem's contribution.

Mr. Gallogly stated** that the apparatus seemed to illustrate the conditions surrounding a river caisson only, and that the tests do not seem to cover all the cases of foundations constructed in pervious soils below ground-water level. The writer would like to call attention to the fact that his tests were made to determine whether the hydrostatic uplift acted on the full area of

* Discussion on the paper by H. de B. Parsons, M. Am. Soc. C. E., continued from December, 1928, *Proceedings*.

† Author's closure.

‡ Cons. Engr., New York, N. Y.

§ Received by the Secretary, December 29, 1928.

‖ *Proceedings*, Am. Soc. C. E., August, 1928, Papers and Discussions, p. 1972.

¶ *Loc. cit.*, p. 1985.

** *Loc. cit.*, p. 1983.

the foundation base, or only a part, called the "effective area". This being so, it would seem that the results apply to any kind of structure founded below water level.

The discussion by Professor Terzaghi is both instructive and constructive. The writer was unaware of any tests published in the English language before his paper, and he undertook this research because repeated demands were being made for information by American engineers.

The writer found, at the beginning, the difficulty of excluding the air content from the sand; and it was one of the causes which delayed the final tests for about nine months, as the air produced erratic results. A test was made by inserting a rubber tube in the sand bed, one end being against the base of a can and the other joining a glass tube having the open end above the water level. The water level in the containing vessel was then raised, and the time noted for the water in the glass tube to come to the same level. It took many hours for the water in the tube to rise, and it frequently did not quite reach the level of the water in the container. The writer and his aides recognized this as air, and the technique was changed to saturate or try to saturate the sand.

The tests made at Stevens Institute of Technology for gravel, confined sand, and sand not confined, appeared to show that the depth of material below the base of the contact rod had an influence on the effective area, as shown in Fig. 5.* This result appears to have been substantiated by the high-head tests at Rensselaer Polytechnic Institute.

All tests were repeated under similar conditions, and the results averaged. (See Tables 1 to 7.†) The results with the gravel confirmed those with the sand, as regards the influence of the depth of the material. The gravel was put in the container with water, and the rod pounded down, like a battering ram, on the gravel, to compact it before completely filling the container with water. On account of the relatively large voids between the pebbles and the manipulation used, it would seem that air was practically excluded. In view of the cases cited by Professor Terzaghi,‡ the writer can reply only by saying that the conclusion, regarding the depth of material required under the base to get substantially 100% uplift, was in accordance with the results as found.

An explanation of the phenomenon may be somewhat as follows: Under the load applied by the contact rod, or by that of a base of an ordinary structure, there can be no appreciable deformation of the sand or gravel particles, although the particles are firmly pressed together in close contact. The weight is supported on the contact areas between the base and the top layer of the particles. There was no hydrostatic head on these areas, as the contact excluded the water. The top layer (that is, the one next to the base) was supported by other layers of particles, between which there were contact areas on which the hydrostatic head also did not act. If the particles had been arranged in perfect alignment, uniform in size and of oval form, as in Fig. 9,§

* *Proceedings*, Am. Soc. C. E., April, 1928, Papers and Discussions, p. 951.
† *Loc. cit.*, pp. 945, 947-948, 950, 953-955.
‡ *Loc. cit.*, August, 1928, Papers and Discussions, p. 1985.
§ *Loc. cit.*, p. 1988.

the effective area would be smaller than the area of the base by an infinitely small amount, corresponding to the elastic deformation of the particles, as stated by Professor Terzaghi; if, with a similar arrangement, the contact areas of the particles were flat and of measurable extent, the effective area would be less than the total area of the base, since the head would act only on that part between the contact areas, no matter how deep the sand was under the base.

The fact is that the particles varied widely in size and shape, and were in every conceivable position relative to one another; and the contact areas on the top and the bottom of a particle would not be always in the same vertical line. The head, acting on the top half of a particle exposed to the water, would give a downward resultant, to which force would be added the weight of the particle. Likewise, the head acting on the bottom half would give an upward resultant, the head being greater due to the vertical thickness of the particle. This increase in head would be extremely small for one particle, but would be measurable as the layers increased. It would seem reasonable that some depth of material would be required before the exposed areas on the bottoms of the particles, with their hydrostatic heads, would be sufficient to overcome the exposed areas on the tops with their heads, so as to transmit the uplift to the base through the points of contact. While appreciating Professor Terzaghi's criticism, it is conceivable that an analytical treatment might not be as accurate as the results of high-head tests.

Mr. Feld* is correct in stating that the sand friction tests, given in Table 1† and shown graphically in Fig. 3,‡ are not to be applied to other work. The condition of the sand is described in the paper.§ The water levels for the can tests (low-head tests) were within a few inches of the top edges of the cans.

Mr. Holmes states‖ that in the high-head tests there was "some replacement by water when the pistons were actuated". The pistons were stationary during the test. When a piston commenced to move, the test was over.

The water pressure in the sand or gravel body built up as the head was increased, but without movement, as the water can be considered as incompressible under the pressures used. The only chance for "flow" was entrapped air, and this has been discussed in the writer's reply to Professor Terzaghi.

* *Proceedings*, Am. Soc. C. E., October, 1928, Papers and Discussions, p. 2381.
† *Loc. cit.*, April, 1928, Papers and Discussions, p. 945.
‡ *Loc. cit.*, p. 946.
§ *Loc. cit.*, p. 944.
‖ *Loc. cit.*, December, 1928, Papers and Discussions, p. 2707.

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# PAPERS AND DISCUSSIONS

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## EVALUATION OF WATER RIGHTS

### Discussion*

BY C. E. GRUNSKY, PAST-PRESIDENT, AM. SOC. C. E.

C. E. GRUNSKY,† PAST-PRESIDENT, AM. SOC. C. E. (by letter).‡—The author has attempted to find a simple, generally applicable method of ascertaining the value of a water right. The attempt is laudable, but the effort will be in vain. One main difficulty with the particular method suggested lies in the fact that no rule can be found for apportioning the aggregate value of all intangible elements to the individual elements. Moreover, under the American system of controlled rates, the intangible elements of public utilities acquire value only when rates are fixed sufficiently high. If the earnings will not yield income in excess of operating expenses plus the ordinary cost of money, there will be no intangible values.

A water right, in many respects, is similar to a franchise. It is a grant by the public. It may have indefinite or only a limited life. Its acquisition may have cost something or nothing. Under certain circumstances the water right may acquire a recognized market value, as will be the case if all the water that is available is needed for irrigation.

Frequently in acquiring a water right, it becomes necessary to quiet adverse rights. It may be necessary, for example, to compensate riparian owners for the actual or the prospective damage to their property which would result from an up-stream diversion. All legitimate expenditures made in acquiring a water right (with due allowance for any change which may have taken place in the purchasing power of the dollar) should appear in the rate base.

The term, "rate base", as herein used, must not be confounded with market value. The aggregate market value of a public utility property results from its earnings and its earnings, in turn, depend on rates. The rates should have some fairly definite relation to the rate base. The value resulting from the rates cannot be used.

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* Discussion of the paper by John E. Field, M. Am. Soc. C. E., continued from January, 1929, *Proceedings*.

† Cons. Engr. (C. E. Grunsky Co.), San Francisco, Calif.

‡ Received by the Secretary, December 18, 1928.

Both the Courts and the rate-fixing authorities have definitely assumed the attitude that, when water rights are involved, earnings should be allowed with due consideration of the fact that the water right may have value; but this principle is not thus expressed in the decisions. As the author states, no method has yet been found for evaluating water rights which would be acceptable to the Courts, public service commissions, or valuation experts.

Sometimes recourse is had to a comparison with the cost of developing water in the same general region in which a water right is to be valued. The results are rarely satisfactory, because in determining such average value, some costs which are above the average and others that are below the average, will be taken into account. It would be evidently absurd to give those whose development has cost in excess of the average a negative value. In places where water is scarce and water rights have been made a matter of bargain and sale, a definite basis can sometimes be found for estimating value. In such cases, the earnings that result from the use of the water become the measure of its value, and this value, in consequence, may assume a wide range. A determination of the value of water rights in such sections of the country will have an effect on the value of rights elsewhere, the question naturally being asked, "Why, if water commands a high price in one section of the country, should it not also command a similar price under comparable conditions elsewhere?" It seems obvious that whoever makes a successful development of water and puts it to beneficial use has done something worth while for society and is entitled to a fair reward. It would not be unreasonable, therefore, to make some allowance in the earnings, preferably based on the ordinary regional costs of developing water which, when capitalized, would be the value of the water right.

If this principle were accepted by the rate-fixing authorities it would result in removing indefiniteness relating to the value of water at its source. The value of a water right, if thus allowed and intentionally recognized and established by the rate-fixing authorities, would not be subject to such uncertainty, nor to the wide range so notable at present. It sometimes happens that, when all the water available is applied to beneficial use, the cost of development of some of the supply is far below that of the remainder. In such cases the value of the water right—if the cost of development is low—may be very large in comparison with the value (at its source) of the other water used in the same district. In such cases, strategic value is recognized, and this can be determined more or less satisfactorily by the comparison-of-cost method.

Another factor which deserves consideration when a water right is to be evaluated is the time during which the right may be exercised, and still another factor is the length of time that it may take to put the water right to full use. Attention is called to these matters merely to show the complexity of the problem of determining the value of a water right. If development is timely and economically sound, its value—in excess of such part thereof as may be based on cost—is so merged with the value of all other intangible elements that separation is necessarily more or less arbitrary. Personal judgment of experts in each particular case must determine the allocation of the aggregate of all intangible values, which, moreover, is rarely of material significance.

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# PAPERS AND DISCUSSIONS

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## THE DESIGN OF TALL BUILDING FRAMES TO RESIST WIND

### Discussion*

By H. W. Bolin, Assoc. M. Am. Soc. C. E.

H. W. Bolin,† Assoc. M. Am. Soc. C. E. (by letter).‡—This paper, like some others on the subject of wind-bracing for tall buildings, compares the results obtained by the use of the method therein outlined with those obtained by Fleming's Method No. I to show that these results are much more accurate when tested by those obtained by the use of the slope deflection method.

The writer has always felt that Smith's Approximate Method, which is called Method No. IV by Messrs. Wilson and Maney§ is an approximate or "workable" method that is superior to Fleming's Method No. I. It is less tedious and is adaptable to computation in tabular form. As regards the accuracy of this method, it is placed by Messrs. Wilson and Maney on the same plane as Fleming's Method No. I.‖ Professor Wilson has gone so far as to recommend the use of Method No. IV for the design of wind-bracing in usual or ordinary types of building frames, due attention being given to "exterior" and "interior" columns.¶ Professor Tachu Naito also suggests** that Method No. IV be used for the design of building frames to resist wind, except that he proposes modifications of the assumptions of this method, as follows:

(*a*) That the point of contraflexure for the first story columns be at six-tenths the story height rather than midway; and,

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* Discussion on the paper by Albert Ward Ross, Jun. Am. Soc. C. E., and Clyde T. Morris, M. Am. Soc. C. E., continued from January, 1929, *Proceedings*.

† Cons. Structural Engr., Oakland, Calif.

‡ Received by the Secretary, November 24, 1928.

§ *Bulletin No. 80*, Univ. of Illinois Eng. Experiment Station, p. 7.

‖ *Loc. cit.*, p. 47.

¶ *Engineering and Contracting*, March 24, 1920, p. 316.

** *Bulletin*, Seismological Soc. of America, June, 1927, p. 68.

## TABLE 19.—Column Moments.

| Story. | Column A. | | | | | Column B. | | | | | Column C. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Slope deflection. | Method No. IV. | Error. | | | Slope deflection. | Method No. IV. | Error. | | | Slope deflection. | Method No. IV. | Error. | | |
| | | | Method No. IV. | Ross Method. | Fleming Method No. I. | | | Method No. IV. | Ross Method. | Fleming Method No. I. | | | Method No. IV. | Ross Method. | Fleming Method No. I. |
| 7 | 145.1 | 151.2 | + 4 | − 2 | −28 | 288.4 | 302.4 | + 5 | + 1 | −19 | 334.2 | 302.4 | −10 | − 4 | + 27 |
| 6 | 147.0 | 154.5 | + 5 | − 1 | −28 | 288.6 | 309.0 | + 7 | + 3 | −18 | 335.7 | 309.0 | − 8 | − 2 | + 29 |
| 5 | 151.7 | 169.1 | + 2 | − 4 | −30 | 291.5 | 338.2 | + 6 | + 2 | −18 | 340.8 | 338.2 | − 9 | − 3 | + 27 |
| 4 | 152.9 | 166.6 | +11 | + 4 | −33 | 318.8 | 333.2 | + 6 | + 2 | −19 | 366.2 | 333.2 | − 8 | − 1 | + 28 |
| 3 | 163.2 | 190.8 | + 4 | − 3 | −29 | 324.4 | 381.6 | + 4 | 0 | −20 | 376.6 | 381.6 | −10 | − 4 | + 25 |
| 2 | 153.3 | 316.7 | + 8 | + 2 | −26 | 314.8 | 633.4 | + 6 | + 2 | −19 | 364.0 | 506.8 | − 9 | − 2 | + 27 |
| M | 155.2 | 277.1 | + 7 | + 1 | −27 | 319.3 | 554.2 | + 4 | 0 | −20 | 371.1 | 277.1 | −10 | − 4 | + 25 |
| 1 | 156.4 | 409.8 | +22 | +14 | −16 | 300.1 | 819.6 | +27 | +22 | − 1 | 350.4 | 655.6 | + 9 | +16 | + 53 |
| 0 | 244.6 | 327.8 | −22 | −25 | −46 | 392.1 | 655.6 | − 3 | − 6 | −24 | 475.3 | 786.8 | −20 | −16 | + 18 |
| | 301.3 | 393.4 | −16 | −19 | −35 | 565.8 | 786.8 | −10 | −13 | −30 | 688.4 | | −26 | −23 | + 4 |
| | 199.3 | | +59 | +39 | + 1 | 438.5 | | +44 | +47 | −10 | 355.9 | | −11 | − 3 | +100 |
| | 277.8 | | 0 | −13 | +76 | 584.8 | | − 5 | − 4 | +35 | 325.5 | | −15 | − 7 | + 40 |
| | 250.7 | | +10 | − 3 | +96 | 567.4 | | − 2 | − 1 | +39 | 224.2 | | +24 | +34 | + 13 |
| | 276.6 | | +48 | +30 | −10 | 565.4 | | +44 | +48 | − 7 | 558.4 | | −27 | −20 | + 36 |
| | 333.8 | | − 2 | +22 | −25 | 668.0 | | − 2 | 0 | −21 | 898.6 | | −27 | − 1 | + 14 |
| | 486.1 | | −19 | −10 | −55 | 567.6 | | +39 | +12 | − 7 | 882.8 | | − 6 | − 5 | + 37 |
| | 594.5 | | −34 | −11 | −63 | 607.8 | | +30 | +11 | −13 | 905.8 | | −13 | − 5 | + 26 |

## TABLE 20.—Girder Moments.

| Floor. | Girder *a*. | | | | | Girder *b*. | | | | | Girder *c*. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Slope deflection. | Method No. IV. | Error. | | | Slope deflection. | Method No. IV. | Error. | | | Slope deflection. | Method No. IV. | Error. | | |
| | | | Method No. IV. | Ross Method. | Fleming Method No. I. | | | Method No. IV. | Ross Method. | Fleming Method No. I. | | | Method No. IV. | Ross Method. | Fleming Method No. I. |
| 7 | 292.1 | 305.7 | + 4 | − 1 | −28 | 314.8 | 305.7 | − 3 | + 4 | −17 | 341.5 | 305.7 | −11 | − 1 | +75 |
| | 262.3 | | +16 | 0 | −20 | 328.0 | | − 7 | − 4 | −20 | | | | | |
| 6 | 304.6 | 323.6 | + 6 | 0 | −27 | 333.4 | 323.6 | − 3 | + 4 | −17 | 360.5 | 323.6 | −10 | − 1 | +73 |
| | 277.0 | | +17 | 0 | −20 | 346.7 | | − 7 | − 3 | −21 | | | | | |
| 5 | 316.6 | 335.7 | + 6 | − 1 | −28 | 350.2 | 335.7 | − 4 | + 3 | −18 | 377.0 | 335.7 | −11 | − 2 | +72 |
| | 289.2 | | +16 | − 1 | −21 | 363.4 | | − 8 | − 4 | −21 | | | | | |
| 4 | 311.6 | 354.7 | +14 | + 8 | −21 | 338.1 | 354.7 | + 5 | +13 | − 9 | 367.2 | 354.7 | − 3 | + 7 | +89 |
| | 281.5 | | +26 | + 9 | −13 | 354.0 | | 0 | + 4 | −14 | | | | | |
| 3 | 545.9 | 444.2 | −19 | −22 | −44 | 525.3 | 444.2 | −15 | − 2 | −27 | 564.3 | 444.2 | −21 | −14 | +54 |
| | 432.4 | | + 8 | −20 | −29 | 599.5 | | −26 | −26 | −36 | | | | | |
| 2 | 477.2 | 593.8 | +24 | + 9 | +40 | 589.9 | 593.8 | + 1 | +24 | −12 | ..... | ..... | ..... | ..... | ..... |
| | 433.4 | | +37 | +10 | +54 | 681.3 | | −13 | − 5 | −24 | | | | | |
| *M* | 527.2 | 686.9 | +30 | +14 | +40 | 659.9 | 686.9 | + 4 | +28 | −13 | ..... | ..... | ..... | ..... | ..... |
| | 472.6 | | +45 | +17 | +57 | 782.2 | | −12 | − 4 | −27 | | | | | |
| 1 | 819.8 | 721.2 | −12 | − 9 | −43 | 620.8 | 721.2 | +16 | + 3 | − 6 | 1 070.4 | 721.2 | −33 | − 5 | +24 |
| | 615.1 | | +17 | − 5 | −24 | 659.6 | | + 9 | − 7 | −11 | | | | | |

## TABLE 21.—Girder Shears.

| Floor. | Girder *a*. | | | | | Girder *b*. | | | | | Girder *c*. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Slope deflection. | Method No. IV. | Error. | | | Slope deflection. | Method No. IV. | Error. | | | Slope deflection. | Method No. IV. | Error. | | |
| | | | Method No. IV. | Ross Method. | Fleming Method No. I. | | | Method No. IV. | Ross Method. | Fleming Method No. I | | | Method No. IV. | Ross Method. | Fleming Method No. I. |
| 7 | 38.7 | 42.6 | +10 | 0 | −24 | 57.8 | 54.8 | − 5 | 0 | −18 | 32.0 | 28.5 | −11 | − 2 | +75 |
| 6 | 40.6 | 45.1 | +11 | 0 | −24 | 61.1 | 58.1 | − 5 | 0 | −19 | 33.8 | 30.3 | −10 | − 1 | +73 |
| 5 | 42.3 | 46.7 | +10 | − 1 | −24 | 64.2 | 60.3 | − 6 | − 1 | −20 | 35.3 | 31.5 | −11 | − 2 | +72 |
| 4 | 41.4 | 49.5 | +20 | + 1 | −17 | 62.2 | 63.7 | + 2 | + 9 | −12 | 34.4 | 33.3 | − 3 | + 7 | +89 |
| 3 | 68.3 | 62.0 | − 9 | −21 | −37 | 101.1 | 79.9 | −21 | −15 | −32 | 52.9 | 41.7 | −21 | −14 | +54 |
| 2 | 63.5 | 82.7 | +30 | +10 | +46 | 114.3 | 106.5 | − 7 | + 8 | −19 | ..... | ..... | ..... | ..... | ..... |
| *M* | 69.8 | 93.0 | +33 | +15 | +48 | 129.6 | 123.5 | − 5 | +11 | −20 | ..... | ..... | ..... | ..... | ..... |
| 1 | 100.1 | 100.5 | 0 | − 7 | −35 | 115.0 | 129.4 | +12 | − 2 | − 9 | 100.4 | 67.5 | −33 | − 4 | +24 |

(*b*) That the shear in the exterior columns in the first story only be 80% of that in the interior columns rather than 50% as in all stories above the first.

Confusion exists as to the differences of Fleming's Methods Nos. I, II, and III and Method No. IV, due perhaps to the use of such expressions as "cantilever method", "portal method", "modified portal method", "equal shear method", etc. By some writers Method No. IV has been placed in the same category as Fleming's Methods Nos. II and III. In their conclusions, Messrs. Wilson and Maney state that Fleming's Methods Nos. II and III are so inaccurate that they should never be used. Method No. IV is specifically excepted.* Fleming's Methods Nos. II and III are both based on the assumption that the shear is the same on all columns of a story, whereas Method No. IV assumes that the shears on the "interior" columns are equal and the shear on each "exterior" column is equal to one-half the shear on an interior column.

TABLE 22.—COLUMN MOMENTS.

| Story. | Column *A*. | | | | | Column *B*. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Slope deflection. | Method No. IV. | Error. | | | Slope deflection. | Method No. IV. | Error. | | |
| | | | Method No. IV. | Ross Method. | Fleming Method No. I. | | | Method No. IV. | Ross Method. | Fleming Method No. I. |
| 12 | 3.45 | 3.24 | − 6 | − 6 | − 8 | 6.79 | 6.48 | − 4 | − 4 | − 3 |
| | 2.84 | | +14 | +14 | +12 | 6.26 | | + 3 | + 4 | + 5 |
| 11 | 3.96 | 3.60 | − 9 | −10 | −12 | 7.42 | 7.20 | − 3 | − 3 | − 1 |
| | 3.46 | | + 4 | + 3 | + 1 | 7.60 | | − 5 | − 4 | − 3 |
| 10 | 3.87 | 3.96 | + 3 | 0 | − 1 | 8.50 | 7.92 | − 7 | − 6 | − 5 |
| | 3.53 | | +12 | + 9 | + 9 | 7.85 | | + 1 | + 2 | + 3 |
| 9 | 4.38 | 4.32 | − 1 | − 4 | − 2 | 8.68 | 8.64 | − 1 | + 1 | 0 |
| | 4.32 | | 0 | − 2 | − 1 | 8.61 | | 0 | + 1 | + 1 |
| 8 | 3.70 | 4.68 | +27 | +24 | +26 | 8.10 | 9.36 | +16 | +17 | +16 |
| | 5.64 | | −17 | −17 | −17 | 10.62 | | −12 | −12 | −11 |
| 7 | 4.80 | 5.04 | + 5 | + 5 | + 3 | 10.17 | 10.08 | − 1 | − 1 | 0 |
| | 5.00 | | + 1 | + 1 | − 1 | 10.33 | | − 3 | − 3 | − 2 |
| 6 | 6.25 | 6.34 | + 1 | + 2 | − 1 | 11.94 | 12.67 | + 6 | + 6 | + 7 |
| | 6.93 | | − 9 | − 6 | −11 | 12.97 | | − 2 | − 4 | − 2 |
| 5 | 7.04 | 6.83 | − 3 | 0 | − 4 | 13.52 | 13.65 | + 1 | − 1 | + 1 |
| | 6.90 | | − 1 | − 1 | − 2 | 13.50 | | + 1 | + 1 | + 1 |
| 4 | 7.32 | 7.32 | 0 | 0 | − 1 | 14.67 | 14.63 | 0 | 0 | 0 |
| | 7.32 | | 0 | + 3 | − 1 | 14.46 | | + 1 | 0 | + 1 |
| 3 | 8.36 | 7.81 | − 7 | − 4 | − 9 | 15.62 | 15.61 | 0 | − 1 | 0 |
| | 7.56 | | − 3 | + 3 | + 1 | 15.17 | | + 3 | + 3 | + 3 |
| 2 | 9.42 | 9.52 | + 1 | + 1 | − 2 | 18.87 | 19.04 | + 1 | + 1 | + 1 |
| | 8.96 | | + 6 | +15 | + 3 | 19.92 | | − 5 | − 8 | − 4 |
| 1 | 14.90 | 14.14 | − 5 | − 6 | − 7 | 20.92 | 28.27 | +35 | + 4 | +37 |
| | 22.67 | | −38 | − 3 | −39 | 25.68 | | +10 | + 6 | +11 |
| | | | Method No. IV Modified. | | | | | | | |
| 1 | 14.90 | 15.09 | + 1 | .... | .... | 20.92 | 18.83 | −10 | .... | .... |
| | 22.67 | 22.60 | − 1 | .... | .... | 25.68 | 28.25 | +10 | .... | .... |

Method No. IV is adaptable to irregular frames such as those in which all columns are not continuous for the full height and to frames of unequal spans.

* *Bulletin No. 80*, Univ. of Illinois Eng. Experiment Station, p. 47.

TABLE 23.—GIRDER MOMENTS.

| Floor. | Moment $M_{AB}$. | | | | | Moment $M_{BA}$. | | | | | Moment $M_{BB}$. | | | | | Method No. IV Modified. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Slope deflection. | Method No. IV. | Error. | | | Slope deflection. | Method No. IV. | Error. | | | Slope deflection. | Method No. IV. | Error. | | | |
| | | | Ross Method. | Fleming Method No. I. | Method No. IV. | | | Ross Method. | Fleming Method No. I. | Method No. IV. | | | Ross Method. | Fleming Method No. I. | Method No. IV. | |
| 12 | 6.12 | 6.12 | 0 | −2 | 0 | 5.72 | 6.12 | 0 | +5 | 7 | 6.52 | 6.12 | 0 | +3 | 6 | |
| 11 | 6.82 | 6.84 | 0 | −2 | 0 | 6.38 | 6.84 | 0 | +5 | 7 | 7.29 | 6.84 | 0 | +2 | 6 | |
| 10 | 7.47 | 7.56 | −1 | −3 | −1 | 7.06 | 7.56 | 0 | +3 | 7 | 8.21 | 7.56 | −1 | −1 | 7 | |
| 9 | 7.32 | 8.28 | −2 | −2 | −5 | 7.59 | 8.28 | −2 | +6 | 9 | 8.90 | 8.28 | −2 | −2 | 7 | |
| 8 | 7.98 | 9.00 | +10 | +14 | −13 | 7.67 | 9.00 | +10 | +17 | 17 | 8.98 | 9.00 | +11 | +17 | 6 | |
| 7 | 10.49 | 9.72 | −7 | −9 | −7 | 9.75 | 9.72 | −7 | −2 | 0 | 10.04 | 9.72 | −8 | −2 | −12 | |
| 6 | 11.25 | 11.33 | −2 | −1 | −1 | 10.47 | 11.33 | −2 | −6 | 9 | 11.79 | 11.33 | −2 | −6 | 4 | |
| 5 | 13.97 | 13.16 | −3 | −8 | −6 | 12.67 | 13.16 | −3 | −2 | 4 | 13.83 | 13.16 | −3 | −2 | 5 | |
| 4 | 14.21 | 14.14 | 0 | −1 | −1 | 12.80 | 14.14 | 0 | −10 | 11 | 15.37 | 14.14 | −1 | −4 | 8 | |
| 3 | 15.59 | 15.12 | −2 | −5 | −3 | 14.31 | 15.12 | −1 | −6 | 13 | 15.80 | 15.12 | −1 | −9 | 4 | |
| 2 | 16.92 | 17.33 | −2 | −2 | −2 | 15.33 | 17.33 | −2 | −9 | 18 | 18.63 | 17.33 | −1 | −14 | 6 | |
| 1 | 23.98 | 23.66 | −6 | −3 | −1 | 20.42 | 23.66 | −9 | −14 | 16 | 20.62 | 23.66 | −12 | −19 | 14 | |
| 1 | 23.98 | 24.61 | .... | .... | +3 | 20.42 | 24.61 | .... | .... | +80 | 20.62 | 24.26 | .... | .... | −96 | |

Judgment as to "exterior" and "interior" columns is required in cases of frames in which all columns are not continuous for the full height.

The writer has calculated the bending moments and shears for the examples* given in the paper by Method No. IV and has prepared tables showing how these results compare with results of the method outlined in the paper and Fleming's Method No. I. Table 19 gives the column moments, Table 20, the girder moments, and Table 21, the girder shears, for the lower seven stories of the American Insurance Union Building. It will be seen that Method No. IV compares more favorably with the slope deflection method and the Ross method than Fleming's Method No. I. Tables 22 and 23 give column moments and girder moments, respectively, for the lower twelve stories of the illustrative problem used by Messrs. Wilson and Maney. The results obtained by the use of the modifications proposed by Professor Tachu Naito are also shown. Favorable results are also obtained when moments calculated by Method No. IV are compared to those of Method No. I.†

The writer believes that for ordinary building frames the use of the Ross method, or similar methods, is not justified by the accuracy obtained, that Method No. IV gives reasonably accurate results; and that its use insures good bracing without undue expense. He believes that Method No. IV should be used in preference to Fleming's Method No. I. For unusual types of structures, such as the American Insurance Union Building, the Ross method, or even the slope deflection method may be used.

* *Proceedings*, Am. Soc. C. E., May, 1928, Papers and Discussions, p. 1419.

† *Bulletin No. 80*, Univ. of Illinois Eng. Experiment Station, Table 25, p. 81. A study of Tables 28, p. 84, and 31, p. 87, of this *Bulletin* again shows that Method No. IV is generally more accurate than the Fleming Method No. I.

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# PAPERS AND DISCUSSIONS

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## THE ISTHMIAN CANAL SITUATION

### Discussion*

BY MESSRS. WILLIAM B. LANDRETH, AND EDUARDO ICAZA.

WILLIAM B. LANDRETH,† M. AM. SOC. C. E. (by letter).‡—The author has done a great service by giving a clear and concise statement of Isthmian Canal conditions. Taking the last four years in Table 1§ as fairly typical of the yearly Panama tonnage, the annual percentage of gain or loss is found to be as follows: 1924, 1.30% less than 1923; 1925, 5.95% less than 1924; 1926, 12.60% more than 1925; and 1927, 10.70% more than 1926. On the basis of total tonnage, the four-year gain in 1927 is 15.65% of the 1923 tonnage.

The annual report of the State Superintendent of Public Works for 1927, gives the tonnage of the New York State Barge Canal for 1919 to 1927, inclusive. Making comparisons on the same basis as for Panama, the annual percentage of gain or loss is found to be as follows: 1924, 1.30% more than 1923; 1925, 15.30% more than 1924; 1926, 1.07% more than 1925; and 1927, 8.97% more than 1926. Likewise, on the basis of the total tonnage, the four-year gain in 1927 is found to be 28.69% of the 1923 tonnage.

During the canal season of 1928 the gain over that of 1927 has been 19.58 per cent. While the net tonnage on the Panama Canal is much greater than on the Barge Canal, the total gain on the Panama Canal for 1924 to 1927, inclusive, was 15.65% and on the Barge Canal for the same years it was 28.69 per cent. For two of the years the Panama Canal lost traffic, while the Barge Canal gained for each of the four years.

The Barge Canal passes through a populous section of the State and, therefore, a large number of pleasure boats pass through it. During the 1928

* This discussion (of the paper by Hans Kramer, Assoc. M. Am. Soc. C. E., published in August, 1928, *Proceedings*, but not presented at any meeting of the Society), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Cons. Engr., Schenectady, N. Y.

‡ Received by the Secretary, December 8, 1928.

§ *Proceedings*, Am. Soc. C. E., August, 1928, Papers and Discussions, p. 1697.

canal season, to October 13, the pleasure boats and their lockages were as shown in Table 4.

TABLE 4.—STATISTICS ON THE NEW YORK STATE BARGE CANAL FOR 1928.

| Location. | Number of boats. | Number of lockages. |
|---|---|---|
| Near eastern end of Erie Canal | 2 454 | 650 |
| Near western end of Erie Canal | 144 | 130 |
| Near south end of Champlain Canal | 1 528 | 674 |
| North end of Champlain Canal | 689 | 505 |
| North end of Oswego Canal | 251 | 167 |
| Near north end of Cayuga-Seneca Canal | 193 | 135 |
| Total | 5 259 | 2 261 |

Probably 15 000 to 20 000 people find enjoyment on the pleasure boats; the lockages used by them equals that used by several million tons of cargo. One should remember that the Panama Canal can be operated for 12 months each year while the weather prevents the use of the Barge Canal for more than about 60% of the year.

An interesting comparison is made between the Panama and Nicaragua Canals in Table 3,* which includes a very concise statement of the salient points of the two routes. This table gives the time of transit at Panama as 6 to 8 hours and estimates it for the longer Nicaragua Canal at from 30 to 36 hours. This would mean a rate per mile of from 6.26 to 8.33 hours at Panama and from 4.86 to 5.83 hours at Nicaragua. The vertical rise at Nicaragua is about 30% more than at Panama, and differences in the navigation conditions would tend to make the rate at Nicaragua less than at Panama.

On the Barge Canal the motor ship, *Twin Ports* (259 ft. long, 42 ft. wide, and loaded to a draft of 9 ft.), passed from Waterford to Oswego, N. Y.—a distance of 187 miles—in 39 hours and 22 min., which is an average speed of 4.75 miles per hour. The boat passed through twenty-nine locks and 22 miles of lakes on the trip. Other similar boats have made the trip, going or coming, in 42 hours.

The estimate of from 30 to 36 hours for the transit through the 175 miles at Nicaragua seems to be conservative. The writer believes that the conclusions reached by the author are fair and that the paper will prove of great value, especially to those interested in canals.

EDUARDO ICAZA,† JUN. AM. SOC. C. E. (by letter).‡—The author has demonstrated beyond a shadow of a doubt that the logical method of supplying future needs for additional Isthmian Canal facilities (in so far as shipping is concerned) is the installation of more locks at Panama. Table 3§ gives an excellent comparison. Under "Feasibility of Constructions",‖ may be added

* *Proceedings*, Am. Soc. C. E., August, 1928, Papers and Discussions, p. 1703.
† Chf., Div., San. Eng. and Sanitation, Panama, Panama.
‡ Received by the Secretary, August 20, 1928.
§ *Proceedings*, Am. Soc. C. E., August, 1928, Papers and Discussions, p. 1703.
‖ *Loc. cit.*, p. 1705.

the statement that sanitation in the Panama Canal Zone is more favorable than in any other area in the tropics.

One thing has been omitted and that is the immediate and urgent need of a permanent non-movable steel highway bridge across the canal. It must not be forgotten that the canal has divided the Republic of Panama in two; that Panama has spent millions of dollars on new roads on both sides of the canal and that at present the only available means of crossing is (at one's own risk) by means of an inadequate ferry service.

Therefore, the Isthmian Canal situation is not wholly providing for future shipping. This, as the author has aptly stated,* is a problem that "belongs to posterity". An important part of the present Isthmian Canal situation is to provide a non-movable bridge across the great waterway.

* *Proceedings*, Am. Soc. C. E., August, 1928, Papers and Discussions, p. 1707.

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# PAPERS AND DISCUSSIONS

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## FINAL REPORT OF THE COMMITTEE OF THE STRUCTURAL DIVISION ON FLORIDA HURRICANE

### Discussion*

BY DAVID A. MOLITOR, M. AM. SOC. C. E.

DAVID A. MOLITOR,† M. AM. SOC. C. E. (by letter).‡—A careful reading of this report leads one to conclude that structures designed in accordance with modern practice for a 20-lb. wind pressure and the specifications of the American Institute of Steel Construction, would have sustained little or no irreparable damage by storms like the Florida hurricane. This is a source of great satisfaction to structural engineers, due fortunately to the fact that at least one fairly well-designed building (Daily News Building) was subjected to the severe test. Such damage as was done to the Meyer-Kiser and the Realty Board Buildings would undoubtedly have been avoided, had they received the same careful consideration, especially with regard to detailing the beam and column connections, as was given to the Daily News Building.

Therefore, the failures recorded cannot be attributed to lack of engineering knowledge respecting methods of wind analysis and designing of connection details, but must be charged to carelessness or ignorance on the part of designers who did not fully consider the information available to them. The writer has treated this subject more fully in a discussion§ of another paper.

The following comments are offered relative to the five points suggested by the Committee.‖

*1.—A Determination of the Wind Pressure Exerted by the Storm.*—While this might be impracticable, the actual experience gained from designs based on a 20-lb. wind pressure seems to justify adhering to this practice; but why

* Discussion of the Final Report of the Committee of the Structural Division on Florida Hurricane, continued from January, 1929, *Proceedings*.
† Structural Engr., Albert Kahn, Inc., Detroit, Mich.
‡ Received by the Secretary, November 28, 1928.
§ *Proceedings*, Am. Soc. C. E., January, 1929, Papers and Discussions, p. 189.
‖ *Loc. cit.*, August, 1928, Papers and Discussions, p. 1763.

not use a 25-lb. pressure for this particular storm-infested area in future designs? Each locality has a Weather Bureau record of wind velocity covering a period of 10 to 20 years, from which the maximum can be selected for use, as in designing tall chimneys. From this maximum velocity, the pressure, in pounds per square foot, can be computed from the formula,

$$p = 0.0033\,V^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which, $V$ is wind velocity, in miles per hour.

Records show maximum wind velocities ranging from 50 to 120 miles per hour, for various stations in the United States, but it would not seem prudent to use anything less than 60 miles per hour as a basis for computation even if weather reports gave a less value as the maximum. Except in some of the storm-swept areas on the Atlantic Coast and the Gulf of Mexico, 100 miles per hour is considered very conservative, and 87 miles is generally maximum for all inland points.

*2.—A Standard Method of Computing Wind Stresses in Tier Building Construction.*—Such a method has been used by many structural engineers, and it is hard to understand why less rational methods should be used at all, or why more complicated methods, offering no greater reliability, should be proposed now.

*3.—Standard Details Best Able to Resist Wind Stresses, Considering Both Stiffness and Strength.*—It is in this connection that the most flagrant violations of good practice have been committed, of which examples described in the report are representative. There is no valid reason for this, because insufficiency in design means incompetency on the part of designers. Many important steel frames of to-day represent first attempts by amateurs.

*4.—A Study to Determine Whether the Requirements for Wind Should not Be Made to Depend on the Ratio of Height to Width of a Building.*—When strength alone is considered, the specified unit stresses govern every design. As rigidity is a factor of prime importance, however, and may vary between wide limits, it follows that the load assumptions and allowable unit stresses should bear some relation to the ratio of height to width of the frame. In speaking of rigidity of the steel frame alone, as affected by its width (neglecting the effects of the architectural fill materials), it should be noted that width governs the cantilever stiffness only when complete cross-frames or diagonal systems are used. For the so-called rigid frame, the basal width does not materially affect the cantilever stiffness which latter depends largely on the number of columns and the column details in the wind-resisting bent.

*5.—The Strength of Masonry Walls and the Bearing They Have on Wind Bracing.*—This the writer interprets to mean the effect of the architectural fill materials, including concrete, stone, brick, or tile, etc., composing the spandrel filling, floors, partitions, and fireproofing of the entire building. The inherent rigidity of the steel frame alone, as designed with specifications of the American Institute of Steel Construction, would not afford tenantable tall buildings when subjected to ordinary 60-mile wind. Deflection calculations prove this. However, experience teaches that this practice does produce commercially satisfactory buildings, and, hence, the architectural fill materials must be credited with supplying the extra rigidity required in such structures.

Perhaps in time some measurements of lateral building displacements may become available for the American Insurance Union Building, at Columbus, Ohio, in which instruments for taking such observations were installed during construction. This would be most gratifying, as no reliable measurements of this kind appear to be recorded.

The idea of instituting a fellowship to study the entire problem of wind-bracing, by appointing a good student to work under the direct supervision of Professor Morris,* is rather unique, especially in view of the closing remarks "that the work of designing wind-bracing for tall narrow buildings should never be entrusted to novices." It might have been more fitting to suggest the appointment of a committee of experts, trained by education and experience, to develop a practical and rational method of designing wind-bracing, so that a good student who is most likely a novice, could produce, with some practice, a creditable design.

* *Proceedings*, Am. Soc. C. E., August, 1928, Papers and Discussions, p. 1765.

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# PAPERS AND DISCUSSIONS

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## COUNTY SEWER DISTRICT WORK IN OHIO AND ASSESSMENT OF COST ACCORDING TO BENEFITS

### Discussion*

BY MESSRS. F. D. STEWART AND W. M. OLSON.

F. D. STEWART,† ASSOC. M. AM. SOC. C. E.—The Ohio Statutes require the approval—by the State Department of Health—of all plans of public water supply and sewerage improvements within the State and, consequently, the Department is brought into close contact with the work of the various counties in installing these improvements. The speaker has been associated with that Department for about eight years and has been in a position to see the operation of the County Sewer District Law expand from its original small proportions to its present status.

It has been noted‡ by Mr. Bradbury that the bulk of the work being done by counties is in territory contiguous to the larger cities. The value of having sewerage and water improvements designed and installed in a logical and comprehensive manner and in conformity with the same utilities of the adjacent city cannot be over-estimated. To any one familiar with the difficulty of securing proper sanitation in suburban areas the advantages of such a central working authority, as this law provides, is apparent. Without proper sewerage and water supply the areas will abound with unsafe wells (either contaminated or subject to contamination), cesspools, septic tanks, and misused storm drains, with resultant nuisances. In counties well organized for this work, sanitary improvements now precede to a greater or less extent the actual occupation of the outlying areas of cities. It is true that the county officials, in establishing the boundaries of sewer districts and designing the improvements, must guard against being unduly influenced by the optimism of realtors, but if

* Discussion of the paper by E. G. Bradbury, M. Am. Soc. C. E., continued from January, 1929, *Proceedings*.

† Asst. Engr., State Dept. of Health, Columbus, Ohio.

‡ *Proceedings*, Am. Soc. C. E., September, 1928, Papers and Discussions, p. 2063.

good sound judgment is exercised no real injustice in the way of assessments will occur. Furthermore, the law as it stands to-day provides ample recourse to any property owner who does not consider the action of the county right or just, either in the inclusion of his property in the district or in the tentative assessment.

The law which provides for the inclusion of parts of municipalities in county sewer districts has been found to be of great advantage. This permits the solution of sewerage problems in accordance with topography without limitations of corporate boundaries. In most instances this has been of greatest value to the cities in providing sewer outlets for territory within the city boundaries inaccessible to existing sewerage improvements.

There are in Ohio a number of summer resorts located along the shore of Lake Erie and around small inland bodies of water to which there is a large influx of people during the summer season. These resorts are generally remote from the centers of population and their sanitation as it relates to water supply and sewerage presents a most perplexing problem. The outdoor privy vault, formerly associated with summer vacationing, is now barely tolerated, and the demand for modern conveniences is insistent. It is here that the County Sewer District Law can function to a distinct advantage as the areas are usually unincorporated. In fact, it has been found in Ohio that in no other way can proper sewerage and water supply improvements be installed. The real obstacle in securing installation of improvements is the item of cost. The property is usually of lower value than city property. Also, the use of the adjacent bodies of water for bathing may render the disposal of sewage difficult and costly. The problem, therefore, calls for the exercise of the best engineering skill and economy of design and construction.

The existence of the County Sewer District Law often provides a means of securing sewerage and sewage disposal for small municipalities which are hampered by statutory debt and tax limitations. The municipal authorities may petition the county commissioners to include the municipality as a whole or part of a county sewer district, and thereupon the county may install the improvements and assess the cost directly on the benefited property.

The law vests with the county commissioners the authority to initiate work under the County Sewer District Law. Nevertheless, it has become a custom in the State to require a petition from a percentage, and usually in the case of local improvements, a majority, of property owners to be benefited by immediate installations. Occasionally, a board of county commissioners will refuse to act in this regard for areas obviously in need of improvements, even on receiving petitions from a large percentage of property owners. Relief from such a situation is afforded by the Ohio General Code. This provides that whenever formal complaint by constituted authorities is made with reference to unsanitary conditions in any area outside of corporate municipalities, the State Director of Health shall investigate the complaint in order to substantiate the claim. If conditions are found as alleged, the State Department of Health may order the establishment of a county sewer district by the county commissioners—to include the area concerning which complaint has been made—and the installation of sewerage therein.

The law further provides that on failure to comply with such an order of the State Department of Health, the Act may be enforced by a writ of mandamus issued by any Court authorized to issue such writs. Similar action may be taken with respect to water supply improvements in accordance with Sections 6602-28 to 6602-30, inclusive. The law provides further for appeal from such an order, either to referee engineers to determine the necessity of the order or to the State Supreme Court by petition in error.

W. M. Olson,* Assoc. M. Am. Soc. C. E. (by letter).†—The author makes clear the admirable procedure in Ohio, by which suburban or rural property is assessed for water or sewer mains. Such procedure is impossible in Illinois. Existing laws enable incorporated cities, towns, and villages to construct water and sewer mains and to assess the cost on property benefited. Sanitary districts to provide for sewage disposal through a common outlet may be organized to include incorporated municipalities and unincorporated areas within three miles of included corporations, but no provision is made for the construction, by public means, of water mains or sewer laterals in any unincorporated territory.

The construction of sewers by private capital is possible, but rarely ever practiced because of the difficulty of getting a reasonably large proportion of individual property owners to agree to share the cost. In some cases, real estate operators developing a new area construct some sort of sewer system. Water pipe is sometimes laid by private capital, either by real estate operators, who may construct reasonably adequate systems offering some measure of fire protection; or by "water associations" which lay small pipe to provide domestic water service only to areas adjacent to a public water system.

Unfortunately, the State Department of Public Health is not authorized to exercise adequate supervision over the construction of sewers and water mains. Most municipalities co-operate voluntarily with the Department of Public Health. Experience has demonstrated, however, that private agencies cannot be relied upon to co-operate, and this makes the private construction of sewer and water utilities especially undesirable. To make matters worse, there is no provision for regulations covering zoning, housing, and building construction in unincorporated territory.

As a result of these laws, or this lack of laws, the following conditions may obtain in unincorporated territory: (1) Unscrupulous real estate men and "home builders" may learn that they are free to do practically as they please, and may proceed to do so with a vengeance; (2) entire areas may be built up solidly without sanitary sewerage or proper water supply; and (3) an area may be included in, and taxed by, a sanitary district but may not receive any direct benefit.

This is just what has happened in many places in Cook County. In 1921, Acacia Park, which was covered largely by truck gardens, was annexed to the Sanitary District of Chicago. About five years later, an ill-advised attempt to incorporate as a tiny village (120 acres) failed. In the absence of

* San. Engr., Cook County Dept. of Health, Chicago, Ill.
† Received by the Secretary, November 30, 1928.

proper regulation, it has been a builders' paradise. Houses have sprung up as if by magic. On November 8, 1928, a count of houses within a most densely built-up 80 acres, revealed 370 buildings. Of these, 11 were used for commercial purposes (7 one-story frame, 1 two-story frame, and 3 one-story brick), and 359 were residences (130 one-story frame, 72 one and one-half-story frame, 5 two-story frame, 112 one-story brick, and 40 one and one-half-story brick). Most of the houses are set close together on 30-ft. lots in groups separated by series of vacant lots. There are many good-looking small houses interspersed by a few architectural atrocities. The intensity of use of this area may be indicated by noting that there are 4.6 houses per acre. Zoning ordinances of eight of the better-class suburbs in the county (according to the Chicago Regional Planning Association), specify a maximum population density of 6 families per acre. A house on every lot in Acacia Park will approximate 8 families per acre.

Natural drainage, which is none too good, has been supplemented by farm tile, laid before subdivision, and by makeshift street drains laid since. Existing facilities are entirely inadequate to remove storm run-off promptly. There is a safe domestic water supply distributed privately from the Chicago System, but no fire protection is provided. The fire hazard is enormous.

Practically all the houses have modern plumbing, discharging, supposedly, to individual septic tanks, which, in turn, are connected to the drainage system. Many of the so-called septic tanks are no larger than ordinary kitchen catch-basins, and some houses undoubtedly discharge raw sewage into the road drains.

Extensive building construction has occurred in spite of the fact that reputable banks refuse to loan money for the construction of residences on lots not served by a proper sanitary sewer system, and by a public water supply affording adequate fire protection. Small local banks closely affiliated with building organizations financed the work. The sale of houses was accelerated by the true statement that "You will have no city taxes to pay".

In the spring of 1928, an adjacent area similar to Acacia Park was annexed to the City of Chicago. The Chicago Board of Local Improvements is already making preliminary studies of proposed sewerage and has found that a long and expensive outfall sewer will be required. Acacia Park is particularly unfortunate in not having been included in this annexation, since a comprehensive sewer plan for the entire built-up area probably could have been devised to use a single outlet. As it is, the City of Chicago can make no provision for Acacia Park in its proposed plan, since the law does not permit such construction.

As the result of the lack of sewerage, the usual unpleasant and unhealthful conditions are present. The relatively few basements are flooded after every rain of any consequence. Some enterprising owners have installed electric pumps to discharge the "water" into the alley. In wet weather, heavy trucks break through unpaved roads and crush the shallow drainage lines. Sewage effluent rises to the surface and ponds on adjacent low-lying land. If the breakage of a drainage line causes private septic tanks to overflow, interested persons

may repair the break. If not, a complaint may be made to the health officer. Untrapped storm-water inlets emit the odors of decomposing wastes lodged in tile lines laid to no particular grade or alignment and without manholes for cleaning. Even in cool weather, sewage odors may be observed at various places in the community. In hot weather, nuisances abound, and frequent complaints are made to the health officer. In the absence of a sewer system, there is but little for him to do. He cannot indict the whole community. The relative absence of typhoid fever may be explained only by the safe water and milk supplies and by a large measure of good fortune.

With the growth of Acacia Park, conditions will become worse. Private septic tanks, usually installed with the supposition that no cleaning will be necessary, will have to be cleaned to relieve clogging. Nuisances now grievous will become unbearable, and annexation to the city must follow. At that time, the waste of money on temporary sewage disposal will be made clear and will stand out as a convincing indictment of the financial policy which permitted the expenditure of large sums on extensive building operations previous to the construction of necessary sanitary utilities.

The foregoing is intended to show that, in the Chicago Region, the problem of providing necessary sanitary facilities for closely built-up unincorporated areas is acute, and as yet unsolved. New enabling legislation is needed. Procedure similar to that in Ohio should be authorized, or existing sanitary districts should be empowered to construct lateral sewers where necessary and to assess the cost against property benefited. Legislation prohibiting the sale of lots for residence purposes until water and sewer mains have been installed according to plans approved by the Department of Public Health might be justified by modern conditions, but would probably be considered an unwarranted invasion of the rights of private property.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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# PAPERS AND DISCUSSIONS

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## THE SEWAGE DISPOSAL WORKS OF DECATUR, ILLINOIS

### Discussion*

By Messrs. George W. Fuller, and John F. Skinner.

George W. Fuller,† M. Am. Soc. C. E.—Among the novel features of the Decatur plant, pre-aeration‡ perhaps comes first to mind as this is one of the first instances in America where that arrangement has been studied. It became a factor in respect to cutting down the load on the sprinkling filters. That plan of course has been studied in Europe in a few instances, particularly at Birmingham, England, where it has enabled the doubling of the load on the filters when it was found difficult to increase the available area or to get more money for construction. One hour is the period of pre-aeration at the Birmingham Plant and a portion of the excess sludge is aerated and conditioned in a tank. This provides about 16 to 18 hours of flow.

This pre-aeration may be applicable in places where ordinary sedimentation would be sufficient for perhaps twelve months in most years, but where occasionally there may be need, for a few months of very dry weather, of something beyond ordinary sedimentation. This idea relates to pre-aeration as a device for removing colloidal matter, not as a nitrate producer and not as a reducer of dissolved organic matters. There are plants, perhaps, like the one at Columbus, Ohio, at which advantage is taken of the diluting effect of stream flow available for a number of months of the year, and where during periods of low stream flow, there should be more treatment than is afforded by ordinary sedimentation.

Another arrangement serving the same general purpose is what Karl Imhoff, M. Am. Soc. C. E., calls contact aerators. These are used in half a dozen

* This discussion (of the paper by Samuel A. Greeley, M. Am. Soc. C. E., and William D. Hatfield, Assoc. M. Am. Soc. C. E., presented at the meeting of the Sanitary Engineering Division, Columbus, Ohio, October 13, 1927, and published in October, 1928, *Proceedings*), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Cons. Engr. (Fuller & McClintock), New York, N. Y.

‡ *Proceedings*, Am. Soc. C. E., October, 1928, Papers and Discussions, p. 2275.

plants in the Ruhr District, in Germany, to get a better removal of the organic matter than is possible by sedimentation. After flowing through those tanks, the sewage goes along the stream bed a short distance to the Rhine. During this travel the mixed Ruhr water and treated sewage become well aerated. However, the colloids caused official complaint from across the National Boundary in Holland where the fishermen complained bitterly of fungus growths interfering with their nets.

Whether pre-aeration as described by the authors as a means of lessening the load on the sprinkling filters is a better arrangement than contact aerators, the speaker does not know. It certainly suggests possibilities for the solution of some types of problems.

The pioneering aspects of the Decatur plant are interesting as regards the collection and burning of gas to eliminate odors.* That is an important step in advance. The observations in the paper as to lime† also impress the speaker because of experiences recently with efforts to handle some of the complications which arise at intervals with Imhoff tanks, particularly at a plant that went in service in the fall of 1926 at Newcastle, Pa.

After some months of operation during the winter, when no effort was made to increase the temperature of the sewage or sludge, there was a tremendous amount of foaming followed by acid production in the spring. This acidity was not materially affected by the addition of small doses of lime. About ten years ago, at Plainfield, N. J., acid production became so pronounced that additions of lime were almost futile. When the pH-value becomes considerably less than 6, say, 5.2, the addition of lime in small quantities is not effective. The only recourse is to put in sufficient lime to bring the pH-value up to about 7, or more.

The question of handling gas at Newcastle has proved bothersome in that the arrangements for collecting it were improvised after the contract had been let, and they permitted the clogging of the gas screens. A further complication resulted from the large quantity of scum. While the gas collector is submerged, nevertheless its arrangement permits the space beneath the gas exits to fill with floating matter, so that the original facilities for cleaning it are now inadequate. Hence, at that plant there were three problems: (1) Inadequate screening provided by 2-in. clear openings; (2) inadequate protection of the gas exits which could be improved by the use of gas screens that do not clog so readily; and (3) inadequate arrangement for flushing the scum that accumulates in the A-space beneath the covers of ordinary gas vents, which covers have a longitudinal slope to lead the gas to the several screened exits.

JOHN F. SKINNER,‡ M. AM. SOC. C. E.—This paper is very interesting and the speaker wishes to add a few words on one point that is based on recent experience in Rochester, N. Y. A section of that city is supplied with water from Lake Ontario by the Rochester and Lake Ontario Water Company. This water is coagulated, passed through pressure filters, and chlorinated.

* *Proceedings*, Am. Soc. C. E., October, 1928, Papers and Discussions, p. 2253, Fig. 4.
† *Loc. cit.*, p. 2268.
‡ Deputy City Engr., Rochester, N. Y.

The Water Company's intake is 1.5 miles west of the mouth of the Genesee River and an occasional east wind carries to the intake a current of river water polluted with sewage and industrial wastes. The plant of the Rochester Gas and Electric Corporation has been in the habit of discharging into the river, 8 miles from its mouth, an average of 20 000 gal. per day of ammonia-still waste containing about 350 lb. of phenol ($C_6 H_5 OH$). Whenever the polluted river water reached the intake, the chlorine dose was increased, and complaints of the taste were numerous.

It was thought that if this gas-house waste could be discharged into the city's intercepting sewer, passed through the main disposal plant (consisting of Imhoff tanks), and then discharged with the effluent into Lake Ontario, 7 000 ft. from shore, in water 50 ft. deep and 2 miles farther from the water-works intake than the mouth of the river, greatly increased dispersion would take place, and if the waste reached the water-works intake it would be greatly diluted.

In order to determine whether this waste could be safely handled in the tanks at the main plant without upsetting their action and their ability to digest sludge properly, a large-scale experiment was conducted at the City's small "Charlotte" Plant* where the discharge averaged 500 000 gal. daily. The Imhoff tank has an inside diameter of 38 ft. In this tank the sewage flow is semi-circumferential, and a steel dome was placed over the central opening and its lower edge sealed by setting the dome in an annular groove of concrete on the deck and filling this groove with water.†

The experiment was begun on June 3, and ended on September 2, 1927. Physical and chemical constants were first determined from four samples of normal sludge and later these were compared with fourteen samples of sludge after the addition of ammonia-still waste continuously. The ratio was 1 gal. of waste to 1 000 gal. of sewage, or 500 gal. of waste daily, containing about 8.75 lb. of phenol, to 4 166 000 lb. of sewage, or 2.1 parts of phenol per 1 000 000 parts of sewage by weight.

The conclusions from the experiment were that when phenol wastes are added to sanitary sewage at concentration of 2 parts per million of phenol, no harmful effects result. The operation of the Imhoff tank can be carried on as usual and the digestion of the sewage solids takes place rapidly and energetically. This was indicated by the following facts:

1.—The sludge was well digested and drained quickly on the drying bed.

2.—No change in physical constants resulted, except a high and clear dewatering.

3.—Chemical constants indicated a better stabilization and more complete digestion.

4.—Bacterial population showed an increase for both aerobic and anaerobic.

5.—Chemical analysis of dry sludge and ash showed small variations from normal sludge.

* *Transactions*, Am. Soc. C. E., Vol. 91 (December, 1927), p. 524.
† *Loc. cit.*, Vol. 88 (1925), p. 503, Fig. 4.

6.—The pH-value was normal.

7.—A high rate of gas, which was mainly methane (about 70%) and $CO_2$ (about 30%) resulted. The calorific value was 700 B.t.u.

The entire discharge of the Rochester Gas and Electric Corporation's ammonia-still waste into the sewer leading to the (main) Irondequoit Disposal Plant* is at a lower concentration than that of the experiment. The plan has been put into operation, no upsetting of the plant has occurred, and all complaints of taste in the water have ceased.

* *Transactions*, Am. Soc. C. E., Vol. 91 (December, 1927), p. 524.

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## LETTING CONSTRUCTION WORK BY COMPETITIVE BIDDING

### Discussion*

BY MESSRS. JOHN C. WAIT, EDWARD W. STEARNS, AND RICHARD S. KIRBY.

JOHN C. WAIT,† M. AM. SOC. C. E.—The author presents two conditions for the successful completion of contract work: (1) The choice of a capable and experienced contractor who is thoroughly reliable, financially and otherwise; and (2) the choice of a good surety company that is thoroughly able and willing to back him. Every one will agree that this would be an ideal condition for the owners, municipalities, States, and the contractors but, practically, it does not exist.

Especially, in the City of New York, some of the greatest projects and the biggest contracts have been undertaken by unknown contractors without personal finances. Some contractors have done New York's greatest work and have gone, penniless, to the grave. Some of the greatest buildings, bridges, tunnels, and subways have been successfully built by men who could scarcely be expected to undertake great work.

The policy that is recommended in this paper (see Principle (39))‡ is now developing in the municipalities and in the States; that is, the policy of investigating the financial strength and the experience of contractors. Public officials are being authorized by law to determine the reliability, responsibility, and capability of contractors and they are doing so to the fullest extent. The great Manhattan Bridge was built by a contractor who owed his bank thousands of dollars; who did not even have funds with which to begin the work; who was opposed by all the steel companies in North America; and yet who built the bridge in the shortest time in which any suspension bridge had ever been built, and there was not one single complaint or lawsuit that followed its completion.

* Discussion on the paper by Edward W. Bush, M. Am. Soc. C. E., continued from January, 1929, *Proceedings*.

† Counsellor at Law (Wait, Wilson & Newton), New York, N. Y.

‡ *Proceedings*, Am. Soc. C. E., November, 1928, Papers and Discussions, p. 2455.

Now, why select contractors who are successful, who have great financial and political backing, and who have great experience, when there are others who are not so fortunate nor so successful but who can carry out the contract. Some of the biggest public works being built to-day are by contractors who were not known to have financial backing or experience, and the contracts are being successfully completed. Low prices do not always come from big and successful contractors.

The speaker is in sympathy with a great many suggestions in this paper, but he believes that they apply largely to private work. Some of the recommendations made cannot be accomplished, especially on public works. For instance, contrary to Principle (5),* the engineer cannot relieve surety companies on public works from giving a bond to pay for the labor and materials expended, because the statute laws of the United States and of some States require that such a bond must be given.

Principle (4)* is a sound one. According to law, an owner may not specify the manner and method of doing the work and, at the same time, stipulate the materials to be used and then exact a warranty of results from the contractor. It is dangerous and productive of litigation to specify both how and of what materials the work shall be done and then say that the contractor "guarantees" or warrants it. The Court says that, if the structure leaks, although the contractor "warrants" it, he may not be held to the warranty. The speaker distinguishes between the words "guarantee"—which is a promise to pay the debt of another man—and "warranty"—which is a warrant of the work done and materials used.

The engineer usually obtains the "Information for Bidders".† He makes soundings or borings; he determines quantities; the elevation of water surfaces or sub-strata; the character of excavations; the presence or absence of materials of construction, etc.; at great expense to the owner or municipality. Then what is done?

The contractor is told that nothing is definitely known about these quantities and conditions; that the owner will not be held liable; and that the contractor must be responsible for them. In a week or ten days he must get information that the engineers may have taken nine months to collect at a cost of thousands of dollars. That is a universal practice in advertising for bids, on either public or private work.

It is done to protect the owner from the possibility of adverse Court decisions if the judgment and knowledge of the engineer should prove to be inexact. As the author states,‡ "nobody can see very far under water, or into the earth"; but the engineer can judge better than the contractor. Owners may be sure, therefore, that the contractor will add to his price because of the repudiation of the engineer's estimate. (See Principle (3).)‡

The speaker suggests writing to 100 or 1 000 contractors and asking how much they would decrease their bids to a municipality if they had a positive

* *Proceedings*, Am. Soc. C. E., November, 1928, Papers and Discussions, p. 2440.
† *Loc. cit.*, p. 2447.
‡ *Loc. cit.*, p. 2439.

declaration as to quantities and conditions existing on the proposed work and if they were not required to guess or take information from engineers only to have it later repudiated in the "Notice to Bidders".

To deny, or refuse, to support the engineers' report, is an act that deprives the owner or municipality of the money spent for engineering and investigation. The contractor must then be responsible for engineering data obtained by the owner and he adds a liberal amount to his bid or contract price accordingly. In municipalities the practice has been promulgated by the legal departments in an effort to forestall or prevent successful litigation by the contractor.

The speaker has sometimes believed that the engineer quite approved the practice because it shifted the responsibility for errors in engineering work. The practice is justified by attorneys because when a contractor has obtained a large judgment against the owner, through engineering errors or misjudgment, it is a definite lump sum that is apparent and is felt by the defeated attorney; whereas the untold sums added by contractors to their bids are not so apparent. They are, nevertheless, met, in even larger quantities, by the owner or municipality.

The trouble with engineers is that they allow the lawyers to dictate to them what shall be included in the contract. Of what value is a report of investigations to contractors when the owner's lawyer (who prepares the contract) states that, in all events, the contractor shall be held responsible for the accuracy of the work of some one else?

The speaker believes that engineers do not realize how important the surety is to companies in the letting of contracts. The present tendency of corporations and municipalities is to make of the contractor a casulty insurance company by forcing him to indemnify the owner, not only for any negligence of himself or anybody that may act for him but, for anything at all that may happen—even under the direction of the engineer.

That is current practice in the City of New York. Where is there any justification for it? Again and again, the speaker has tried to have the legality of these clauses passed upon by the Court. State laws forbid anybody from writing insurance—casualty or otherwise—unless they are duly and properly authorized by the State and unless they accumulate and hold large bonuses. A contractor is not authorized to write insurance for the public or for municipalities but the ordinary construction contract requires him to do just that very thing.

When confronted with this issue, judges of this State have avoided it by deciding the case on other grounds. Some day it will be decided definitely and, the speaker believes, that will stop the practice of requiring the contractor to act as an insurance company.

EDWARD W. STEARNS,* M. AM. SOC. C. E. (by letter).†—There is so much that is good in this paper that it deserves careful reading by those whose duties include the preparation of plans and specifications for contract work. There

* Asst. to Bridge Engr., The Port of New York Authority, New York, N. Y.
† Received by the Secretary, November 28, 1928.

are, however, a number of points included which, in the writer's opinion, are quite controversial and there is, furthermore, one very broad principle which is quite axiomatic and which deserves greater emphasis and amplification than was given it by Mr. Bush. The method of treating the subject adopted by the author is quite ideal and lends itself admirably to reading, discussion, and reference.

*Principle (1).*—Fundamentally, Principle (1)* is correct, but its application should be followed with certain discretion. In the preparation of contracts and specifications it frequently happens that a broad general requirement, clearly and fully stated in its proper place, should be emphasized later by mention in special items subject to the requirement. When this is done, of course, it is of the utmost importance that each subsequent statement does not disagree in the slightest manner from the broad requirement first stated. No doubt a number of writers, and more particularly those of the legal profession, will take exception to this practice on the theory that by mention or emphasis of any particular application, the points not mentioned are, by inference, weakened if not entirely invalidated. In order to overcome this objection it is the writer's practice to include a clause of the following nature in the contract:

"The enumeration of particular instances in which the opinion, judgment, discretion, or determination of the Engineer shall control or in which work shall be performed to his satisfaction or to his inspection or of particular items to be performed without additional compensation, shall not imply that only the matters of a nature similar to those enumerated shall be so governed and performed, but without exception all work shall be so governed and so performed."

In the case of contract documents for a building, the broad requirement is laid down that the contractor shall comply with all Federal, State, and municipal laws and regulations and shall secure all necessary permits. These documents are prepared with the intention in mind that there will be a single general contractor on the work and that it is his obligation to comply with this provision. There will, however, be many sub-contractors engaged in the construction of the building, and in all likelihood the general contractor will expect each sub-contractor to comply with the laws prescribed and to secure such permits as are particularly required for his trade. It seems entirely proper, therefore, to indicate in the sections of the specifications pertaining to each trade, that the owner expects the contractor, meaning the general contractor, to see to it that all laws and regulations are complied with and that all necessary permits are secured, so that the sub-contractor shall be aware of the requirement and that he may determine his obligations from the general contractor.

*Principle (2).*—The writer desires to add to the discussion regarding Principle (2)* a consideration of "cost plus" work. Many people engaged in construction work seem to have little conception of the fact that a very important item in a contractor's cost is the question of overhead and that there is no

* *Proceedings*, Am. Soc. C. E., November, 1928, Papers and Discussions, p. 2439.

fixed method among all contractors for the apportionment of overhead expenses. When the contractor is called upon to do work on the basis of cost plus percentage a definite agreement should be reached before any work is done, as to just how properly to compensate him for his overhead expenses and just how the item of cost will be measured.

It should be borne in mind that usually "extra work" is not undertaken because of the desire of the contractor for it, but because the owner finds it the most convenient way of handling certain minor matters. With this thought in mind it is only equitable that the contractor should receive not only his actual direct outlay in cash for labor and materials, but also both his overhead expense and a fair profit. By clearly understanding at the start just how the word, "cost", is to be defined, much controversy and ill feeling can be avoided and, as a result thereof, better work can be obtained due to a more sympathetic feeling between the contractor and the owner.

*Principle* (*6*).*—Probably the greatest uncertainties which a contractor assumes in any construction work are those which are incident to foundation conditions. Those are the uncertainties which cannot be seen and must be felt. The writer thoroughly believes in sub-surface explorations, commensurate with the importance of the work involved, and believes further in furnishing to the bidder the complete information which he obtains from such explorations exactly as he receives it. It is obvious that his plans represent his interpretation of the results of such explorations. The writer does not believe in guaranteeing to the contractor either the correctness of the sub-surface explorations or that the conditions indicated by them will be found actually to exist; or, in other words, that the interpretation as indicated by the plans is correct. Those risks must and should be taken by the contractor.

As an example, the writer recalls a job on which extensive borings indicated a thick stratum of soft clay overlying a bed of gravel or hardpan. The plans required that the piers be supported on piles. After studying the samples from the borings, the contractor ordered piles of sufficient length to be driven through the clay and into the harder strata below, but in driving, it was found that the piles would penetrate only partly into the clay. The cost of the pile cut-off was placed upon the contractor, the contract being a unit price job with a price per foot for piles driven. This ruling may seem to have been a hardship on the contractor. It should be borne in mind, however, that the full information furnished from the borings was in his possession at the time he prepared his bid, and further, that he did not elect, before ordering his piles, to drive a few test piles by which he could have determined the probable penetration. Consequently, he assumed the full risk when he ordered the long piles and should not have been compensated for the loss incurred in the pile cut-offs.

The foregoing, however, does not, and in Court cannot be made to, hold if the sub-surface explorations are obviously insufficient or grossly incorrect. The writer recalls one job on which borings indicated rock at such an elevation below water level that the excavation could presumably be done by the open

* *Proceedings*, Am. Soc. C. E., November, 1928, Papers and Discussions, p. 2440.

coffer-dam method, and the contractor so calculated his bid. When the so-called rock was exposed, however, it developed that it was not bed-rock, but a rather unusual layer of large boulders. Subsequent exploration then revealed the fact that bed-rock lay at a depth so much greater than that orginally contemplated that pneumatic caisson work became necessary. It is obvious that, to force the contractor to perform the pneumatic work at the price bid for coffer-dam work would have been inequitable, and could not have held in Court.

The foregoing should not apply to sub-surface man-made structures. The engineer should endeavor to determine to his best ability the existence, location, and character of all such structures and furnish such information to the contractor, thereby making him solely responsible. Should structures not shown on the plans be encountered, the risk thereof and any work involved in properly caring for them should be solely at the cost of the owner. As an example, on one job in the writer's experience there was encountered a 24-in., high-pressure water main which was not shown on the plans and which had to be moved. One entire section of the city was dependent almost wholly on this main for domestic use and fire protection. The removal of the main from the vicinity of the work was not an expensive operation, but the risk entailed was tremendous if the main had been broken or if a fire had occurred at the time necessary to shut off the water. Obviously, here the equity lay only in the owner assuming both the cost and the risk.

*Principle* (7).—The surety premium represents the charge for carrying certain risks. Obviously, then, if the risks are reduced the premium will be reduced and Principle (7)* is only a statement of this fact in regard to particular risks. The writer believes, however, that the State lien laws are based on sound thought and, therefore, that the State and Government should provide the same protection as is given to the laborer or material man on private work. The risk for the payment of such claims is clearly one to be covered by insurance and, therefore, the surety should be required to guarantee the cost of unpaid labor and material bills.

*Principle* (*13*).†—Except when there is no surety bond—and it is the writer's opinion that every contract should include a surety bond—the "retained percentage" is not withheld by the owner as a safeguard in case of default. On the contrary it is withheld for other purposes, as follows: (1) To cover incidental work of minor importance usually deferred until the end of the contract; (2) to serve as a liquid fund in case the contractor defaults; and (3) to act as a counter fund to offset any possible over-payment due to the approximate nature of the monthly estimates. Other reasons of lesser importance might be mentioned.

The writer believes that in the early stages of the contract the percentage which the "retained percentage" bears to the cost of work done, should be relatively high in order to build up a considerable cash fund, but that, as the work progresses and nears completion, both this percentage and the amount of the "retained percentage" should decrease until, when the work is entirely complete, they both should be relatively small. In one contract recently writ-

* *Proceedings*, Am. Soc. C. E., November, 1928, Papers and Discussions, p. 2441.
† *Loc. cit.*, p. 2443.

ten, the writer specified that 10% of the value of the work performed be withheld until such a time as the retained amount equaled 5% of the total estimated contract price, after which time no further amount should be withheld. In another large contract he specified that until 25% of the work had been done, 10% should be withheld, after which the percentage was reduced on a sliding scale until, at the completion of the work, only 2½% was retained.

There is a feeling among many engineers and architects that a contract should be worded so as to force the contractor to finance himself very extensively. To this opinion the writer takes strong exception. He believes that monthly payments to the contractor should be arranged so as to compensate the contractor as far as possible for the cost of the work as it proceeds. Usually this cannot be done in a simple way. In the early part of the work the contractor necessarily has a heavy investment and a relatively small amount of work performed so that his estimates do not and cannot fully compensate him. It is for this reason that his financial ability is important. After the work has progressed, however, if payments are properly arranged and if the retained percentage is not exorbitant, the time should quickly come when the payments should carry the contractor's cost.

Usually, the owner is in a much better position to secure the use of funds than the contractor. By compensating the contractor, fairly and reasonably, as the work progresses and reducing his necessity of obtaining loans, sometimes at quite high rates of interest, the owner establishes that feeling of good-will between himself and the contractor which is conducive not only to greater effort on the part of the contractor to get his work done well and expeditiously, but also to cause him to reflect such feeling in his bid price. This is particularly true when the owner is in the market at more or less frequent intervals for construction work; but it is nevertheless also true when the owner is contemplating and planning only one piece of construction work, providing the contract and specifications are so worded as to show clearly to the bidder that he can expect fair treatment.

*Principle* (*17*).*—It is with considerable humiliation as an engineer that the writer discusses this obvious principle. Unfortunately, the competition in engineering work has become so acute that the fees for it have become proportionately meager. Commercial organizations frequently seem to be so short-sighted as to consider the cost of the preparation of their drawings and contract documents as a whole as unproductive costs. This is fundamentally wrong. Care, thought, and study cost money, but when they are applied so as to produce drawings and contract documents that reduce to a minimum the points in which the contractor must use his imagination, the money so invested is saved many times over.

The engineer or the architect, whether he be employed as an outside agency or whether he be a regular employee of the owner on salary, has imposed upon him a great trust, namely, to construct for the owner, at the lowest possible cost, a structure which can best serve the owner's needs. Such results can only be obtained by the careful and thorough preparation of the drawings and contract documents. When he lends himself to any other practice on the

* *Proceedings*, Am. Soc. C. E., November, 1928, Papers and Discussions, p. 2444.

grounds that he does not receive sufficient funds to justify such careful and thorough preparation, the engineer or architect is breaking his trust most disgracefully and lowering the standard of the profession as a whole.

*Principle (21).**—The writer believes that it is best not to state the estimated cost of the work, even with limits as suggested by the author. The particular limits to be adopted would be subject to considerable variation in practice at different offices, and if made broad enough to provide the possible range of price as found in bids for the majority of work, they would be of little value in indicating the size of the work. A general statement of the approximate nature and dimensions of the work contemplated would seem to be a better basis for indicating the magnitude and the character of the work to the bidder.

*Principles (34), (35), and (36).†*—It is essential for the writer to consider these three principles together. He takes issue with the author on four points, namely: (1) That the cash deposit required of the bidder is primarily required as a "bidding security"; (2) that it should be as much as 10 or 15% of the "probable contract price"; (3) that bid bonds are not desirable; and (4) that bid letters are not desirable.

The writer believes that the best bidding security is the bid bond or the agreement of proposed surety which is called a bid letter by the author. Since the responsibility of the surety, who must be acceptable to the owner, is apt to be greater than the responsibility of the bidder himself, in the writer's opinion the cash deposit at the time of submitting the bid has the chief purpose of giving a measure of the contractor's financial ability. Consequently, the amount of the bid deposit should be only sufficient to indicate that the bidder, if awarded the contract, has sufficient liquid capital or credit to proceed with his contract without embarrassment to the owner. Before awarding a contract the owner should require the contractor to submit an up-to-date financial statement. This, together with his bid deposit, should furnish the owner with sufficient information of the bidder's financial resources. It is difficult, if not impossible, to fix a formula for determining the amount of the bid deposit. It has been the writer's practice to fix it somewhere between 3 and 8% of the estimated cost of the work, depending on the class of work, the cost, and the length of time to be involved in completing the work.

In conclusion, there is one broad principle applicable to the preparation of plans and contract documents which is axiomatic, but which nevertheless seems only too often to be entirely overlooked. Mr. Bush touched on it a number of times, but failed to emphasize it by setting it among his other principles. Briefly it may be stated as follows:

"Clarity and consistency are of prime importance".

Close attention should be paid to this principle, particularly in the preparation of the contract documents. Although, due to certain tradition and practice, certain legal phrases more or less difficult for the layman to read seem, by custom, to be unavoidable, the preparation of such documents never-

* *Proceedings*, Am. Soc. C. E., November, 1928, Papers and Discussions, p. 2447.
† *Loc. cit.*, p. 2452.

theless is only the practicable application of the English language. It is obvious of course that the individual preparing the contract documents must know what he wants and, to a certain extent, how to get it. Unfortunately, however, the documents are often so worded as to leave the contractor completely ignorant of the real intent. The point cannot be too strongly emphasized that every statement in such documents should be carefully weighed with a view to determining whether or not they correctly carry the thought which the writer desires to transmit. As a rule, the more brief and concise the documents are made, the more intelligible they are to those using them; but this consideration should not be carried to such an extreme that much is left to inference or interpretation.

Only too often, the writer fears, is it true that contract documents previously prepared—possibly even with careful analysis and study—are used as a guide in the preparation of subsequent documents. This practice often results in inconsistencies which are sometimes serious and many times quite ludicrous. Recently, the writer had to review a specification prepared by another engineer, in which an entire chapter was devoted to a description of a type of work with which he was unfamiliar. Throughout the specifications reference was made to this work, thereby giving the impression that it was of considerable importance. At the first opportunity he questioned the man who prepared the specifications as to the details involved in this particular type of work and after getting a reasonably clear conception of what was intended he inquired where it was to be used in the particular construction contemplated. Much to his surprise he discovered that none of it was contemplated, and the entire chapter and the references to it had merely been copied from a specification for an entirely different piece of work. Such practice is in much the same category as that of the calculator who claimed that he made only a very minor error in his calculation when he misplaced the decimal point. To some, the foregoing criticism may seem ridiculously obvious, but when one has encountered such conditions not occasionally but frequently over a period of years it becomes a very serious matter.

The preparation of drawings and contracts is only part of the story. After having attempted to emphasize the importance of fairness, clarity, and consistency in the wording and treatment of such documents a word to those whose duty it is to execute them and carry out their provisions is not remiss. In all the work that the writer has ever done he has made it an unvarying practice to carry to the contractor in one way or another the feeling that the contractor and the engineer are both working to the same end, with the same purposes in mind, and with the same obligations, namely, to complete the construction, whatever it may be, in the best possible manner, in the shortest space of time, and at the least possible expense both to the contractor and the owner with the intent of the plans and specifications and the purposes for which the structure is to be used. All the contract documents, therefore, become only a portrayal of the broad general principles under which the work is to be performed. It behooves the engineer in charge to establish with the contractor, by fair dealing, fair interpretation of the contract provision, and sympathetic consideration of the contractor's problems that spirit of close

co-operation and good will which alone will result to the best interests of the owner.

Too many engineers, from the lowliest inspector to the chief in charge of the work, seem to have the impression that the provisions on the contract documents are immutable; that they must be followed to the very letter rather than to their intent; and that their prime obligation in carrying out the work is, if not completely to break the contractor, at least to see to it that he makes no profit. The contractor is entitled to a profit; to all the profit he can make, consistent with thorough and first-class workmanship. To deny him that profit by means of unfairly rigid and restrictive rules and by narrow-minded and short-sighted interpretations is the best way to create an unhappy condition in the execution of the work; a feeling of deep distrust and, in cases, even dishonesty on the part of the contractor, and finally, expensive and uneconomical work.

RICHARD S. KIRBY,* M. AM. SOC. C. E. (by letter).†—The writer is in thorough accord with almost the entire paper. He has been struggling for years to inculcate just such principles into successive generations of engineering students. In his own mind he has classified Mr. Bush's "thirty-nine articles" into five groups, as follows:

(*a*).—More orderly arrangement of the material forming the contract documents: The writer suspects that much of the confusion to which the author properly calls attention has resulted either from leaning too confidently on precedent or from a division of responsibility in the preparation of the documents. Neither condition is typical of the engineer at his best.

(*b*).—A great degree of explicitness in the wording of the documents and in the data shown on the drawings, so as to remove uncertainties: To the writer's mind this group needs all possible emphasis, and Principle (3)‡ is one of the most important of the entire thirty-nine, and the most difficult to put into practice. The writer would phrase Principle (9)§ differently—"Prepare each contract so carefully that nothing will remain for the Court to interpret." Courts and lawyers flourish on agreements entered into loosely. Construction work contracts ought to be so complete that if contractor, owner, and engineer should die the day after a contract is signed, their successors would nevertheless find themselves in a position to carry out an agreement which did not lack precision at any point. The writer agrees especially with the suggestion that the graphic language be used in greater variety on plans. (See Principle (17)).‖ Isometric drawings are so easily made and, at times, are so effective that it seems a shame that they are not used more often to supplement orthographic views.

(*c*).—More thorough and extensive engineering investigation of the project in advance: The writer agrees heartily with this general principle. He differs

* Associate Prof. of Eng. Drawing, Sheffield Scientific School, Yale Univ., New Haven, Conn.

† Received by the Secretary, January 7, 1929.

‡ *Proceedings*, Am. Soc. C. E., November, 1928, Papers and Discussions, p. 2439.

§ *Loc. cit.*, p. 2441.

‖ *Loc. cit.*, p. 2444.

with Mr. Bush on Principle (27)* only, as he can see no serious objection to percentage-unit bidding, provided the engineer gives proper thought to the preparation of his estimate.

(*d*).—Uniform treatment of bidders; improvements or changes in letting procedure: Doubtless the author had private, rather than public, work in mind in some of his suggestions; and it may be that architects are greater offenders than engineers in these respects. The writer questions the practicability of Principle (24)† only.

(*e*).—Reduction of sureties' duties, responsibilities, or liabilities: Some of this group of suggestions raises doubts in the writer's mind; especially Principle (39).‡ It is plainly within the engineer's province to study carefully any matters concerning physical conditions relating to a project and to inquire into a contractor's reputation for good work, judged by past performances. How far an engineer should pose as a financial expert is another matter. Is not this plainly within the scope of the service which a surety company is expected to render?

* *Proceedings*, Am. Soc. C. E., November, 1928, Papers and Discussions, p. 2449.
† *Loc. cit.*, p. 2448.
‡ *Loc. cit.*, p. 2455.

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# PAPERS AND DISCUSSIONS

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## PUMPED-STORAGE HYDRO-ELECTRIC PLANTS

### Discussion*

BY MESSRS. JOHN R. FREEMAN, AND H. ALDEN FOSTER.

JOHN R. FREEMAN,† PAST-PRESIDENT, AM. SOC. C. E. (by letter).‡—The use of pumped storage on a very large scale was suggested about 1900 by the late Julius M. Howells, M. Am. Soc. C. E., as an important part of a development of 50 000 to 100 000 h.p. from the Feather River over a transmission line about 200 miles in length, of the Western Power Company of California.

Mr. Howells was an engineer of unusual creative imagination, as was proved by his pioneer work in the construction of large earth dams by the hydraulic-fill method. It has long been a matter of deep regret to the writer that Mr. Howells did not receive the credit due him for his inventive genius in being the first to apply the methods developed in the hydraulic mines of California to the building of large earth dams. This method has now become so well established under such widespread application that it has ceased to be thought of as an invention which required great courage, as well as technical skill in its methods, by an engineer who made use of it in important structures. The writer firmly believes that Mr. Howells was entitled to the credit for that invention.

If his associates or financial backers had possessed equal confidence and courage, Mr. Howells, long ago, would have demonstrated this method of pumped storage for electric developments on a grand scale. At the beginning of this century the United States was in the early stages of large scale hydro-electric development, and it was not then so easy to find either capital for building great plants, or customers to absorb their product.

In brief, Mr. Howells' project was to transmit the electric current at a constant 24-hour, 7-day rate to a point about 15 miles from San Francisco,

* This discussion (of the paper by W. W. K. Freeman, Assoc. M. Am. Soc. C. E., published in November, 1928, *Proceedings*, but not presented at any meeting of the Society), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Consulting Hydraulic Engr., Providence, R. I.

‡ Received by the Secretary, November 10, 1928.

Calif., thereby obtaining the greatest possible economy in the use of a transmission line nearly 200 miles in length and to utilize all surplus power in off-peak hours for pumping the water into a reservoir about 750 ft. above sea level and ½ sq. mile in area, in the valley on the northeasterly slope of Mt. Tamalpais.

Mr. Howells' preliminary estimates assumed that the combined efficiencies of motor and pump would be about 70% during the storage period and that in the period of return the combined efficiency of the Pelton wheel and generator would also be about 70%, or that, as a whole, about 50% of the power expended in pumping would be returned.

This pumped storage station also was to be largely relied on to take the place of the steam reserve station near the terminus, which, in those days, was considered an essential part of any long-distance transmission through rough country.

It was a fascinating proposition, but 20 or 25 years ago the time was not ripe for it. Moreover, at that time, the efficiency of multi-stage centrifugal pumps, and of impulse wheels of the Pelton type, had not become so highly developed as in recent years.

H. Alden Foster,* Assoc. M. Am. Soc. C. E. (by letter).†—An interesting example of the use of pumped storage was involved in a hydro-electric project on which the writer recently made some extended studies. Since the problems treated were rather unique, it is believed that the methods used in solving them may be of interest. The writer does not claim any credit for originating the general scheme of development described.

Fig. 6 is a map and profile of the project. The river flows around a large bend, *A C E*, in a distance of about 14 miles. The total fall from *A* to *E* is 420 ft., as indicated on the profile. Several plans were proposed for the development of this head, one of which consisted of building a dam across the river at *C*, to raise the water level to Elevation 1 300. At this elevation, the water would overflow the main river valley into the water-shed of the creek flowing from *D* to *E*. A second dam, therefore, was located at *D*. The resulting reservoir had a flow line at Elevation 1 300 (Fig. 6 (*a*)). From *D* to *E* it was proposed to build a tunnel, which would deliver the water to a power house at *E*, where the full head of 420 ft. could be utilized.

The reservoir itself was to be used to regulate the flow of the river down stream from *E*. The use of the reservoir, however, was complicated by the fact that, when the water was drawn down below Elevation 1 265, the lake would divide into two parts, as indicated by the flow line shown in heavy broken lines on the map. The water flowing into the reservoir from above *A* was diverted into that part which was directly tributary to the tunnel, through the canal, *H*, by a weir constructed across the river at *B*, the crest of this weir being at Elevation 1 265. The only way in which the water in that part of the reservoir between *B* and *C* could be diverted to the tunnel (when the reservoir was drawn down below Elevation 1 265) was by pumping. This was arranged by locating a pumping plant at *G*, which received the water through the pipe line, *G C*, and discharged into the other part of the reservoir at *F*. The avail-

* With Parsons, Klapp, Brinckerhoff & Douglas, New York, N. Y.

† Received by the Secretary, December 8, 1928.

able storage was thus divided into three parts, as shown diagrammatically in Fig. 7 (*a*):

*Section I.*—The storage capacity above Elevation 1 265: This part of the total capacity can be drawn upon whenever the reservoir surface is above Elevation 1 265.

*Section II.*—The part of the reservoir capacity below Elevation 1 265 and directly tributary to the tunnel: This volume of stored water can be drawn upon whenever the reservoir surface is below Elevation 1 265. The minimum drawdown is to Elevation 1 190 (see Fig. 6 (*b*)).

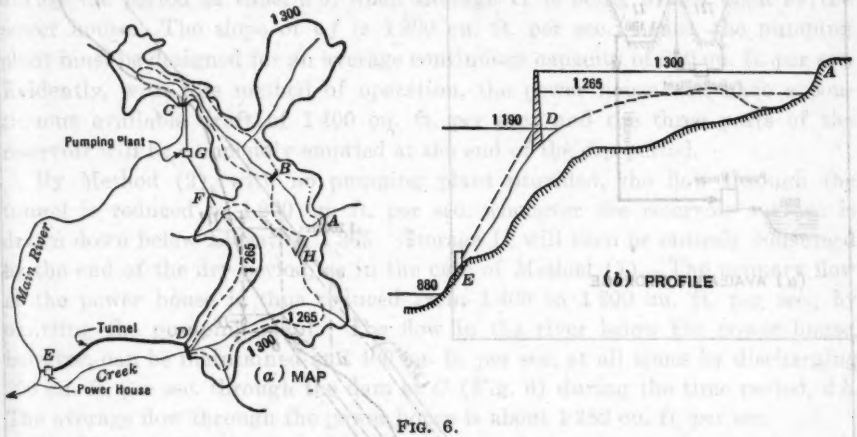


FIG. 6.

*Section III.*—The part of the reservoir below Elevation 1 265, and lying between the weir, *B*, and the dam at *C*: This volume of storage is available when the reservoir surface is below Elevation 1 265; but it cannot be used by the power house unless it is pumped into Section II (Fig. 7 (*a*)).

The pumping plant would operate against a static head varying from zero to 65 ft., plus a friction head based on the size of pipe line adopted and varying with the actual rate of discharge, with a maximum total head of about 80 ft. The water so pumped was available for power under a static head of between 310 and 385 ft., depending on the actual level of the water surface in Section II.

In order to determine the capacity for which the pumping plant should be designed, a mass curve (Fig. 7 (*b*)) of the unregulated stream flow was prepared, covering the dryest period in the available records. At the left of this diagram, a vertical line, *c a*, shows the three parts of the total storage, as already explained. A straight line, *a b*, is drawn tangent to the lowest hollow of the mass curve, the slope of this line being 1 400 cu. ft. per sec. This represents the maximum rate of flow that can be maintained in the river below the power house if the total capacity of the reservoir can be utilized.

There are four methods of operation illustrated by Fig. 7 (*b*): (1) pump water from Section III to Section II after Section I has been discharged; (2) without pumping, draw water from Section I at the rate of 1 400 cu. ft. per sec. and then from Section II at 1 200 cu. ft. per sec.; (3) without pumping, draw

water from Sections I and II at the rate of 1 400 cu. ft. per sec.; and (4) without pumping, draw water from Sections I and II at the rate of 1 372 cu. ft. per sec.

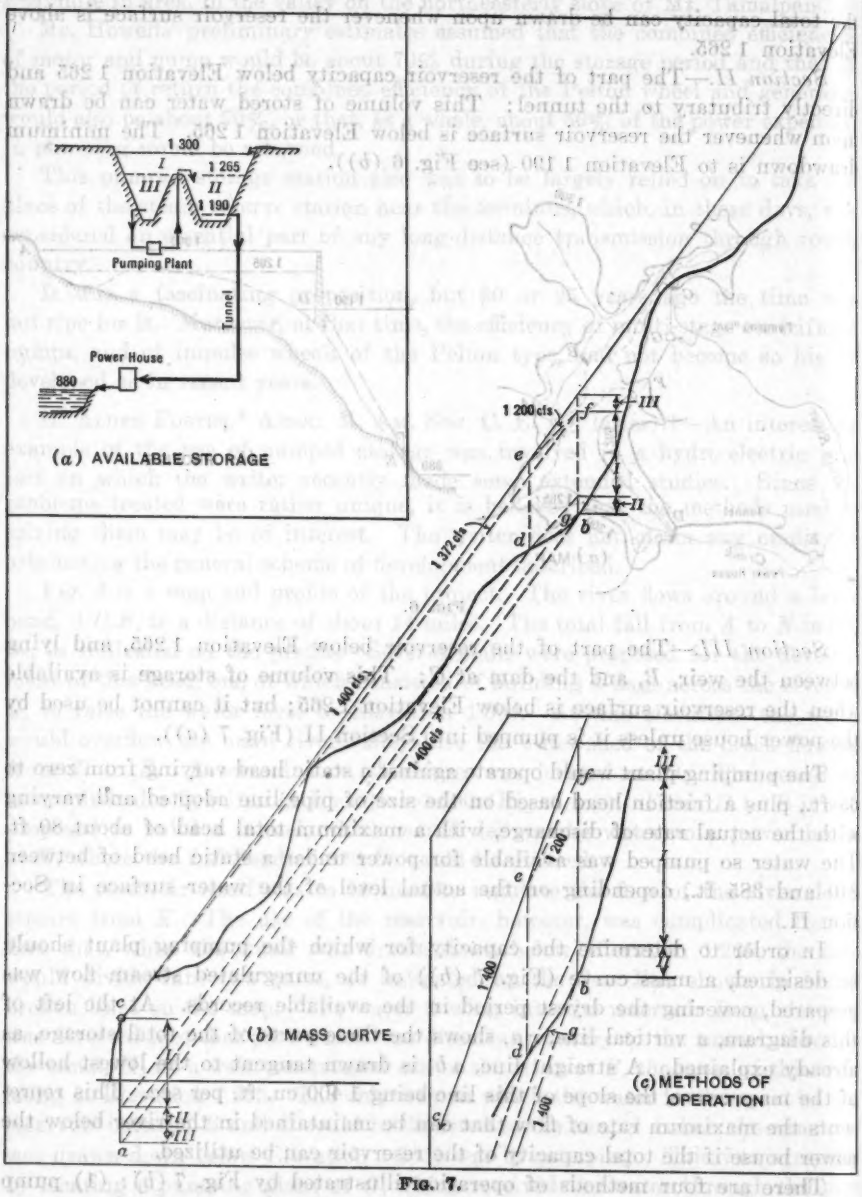


FIG. 7.

By Method (1) the pumps would be operated at their full capacity at all times when Storage II is below Elevation 1 265, in order to secure the greatest possible benefit from the pumps by increasing the operating head at the power

house. The sequence of operation as shown by the mass curve is to use Storage I from $c$ to $d$, and Storages II and III together, from $d$ to $b$. This section of the mass curve is shown at an enlarged scale in Fig. 7 ($c$). At $b$, lay off Storages II and I vertically upward to the point, $f$. From $e$, at a distance, $I$, above $d$, draw a straight line to the point, $f$. Then, the slope of $e\,f$ will give the maximum rate at which water can be drawn through the tunnel during low-water stages without using Storage III. The difference between the slopes of $e\,f$ and $c\,e$ will give the minimum discharge at which the pumping plant must operate in order to transfer all of Storage III into Storage II during the period of time, $d\,b$, when Storage II is being drawn upon by the power house. The slope of $e\,f$ is 1 200 cu. ft. per sec.; hence, the pumping plant must be designed for an average continuous capacity of 200 cu. ft. per sec. Evidently, with this method of operation, the power house will have a continuous available draft of 1 400 cu. ft. per sec., and the three parts of the reservoir will be completely emptied at the end of the dry period.

By Method (2), with no pumping plant provided, the flow through the tunnel is reduced to 1 200 cu. ft. per sec. whenever the reservoir surface is drawn down below Elevation 1 265. Storage II will then be entirely consumed by the end of the dry period, as in the case of Method (1). The primary flow at the power house is thus reduced from 1 400 to 1 200 cu. ft. per sec., by omitting the pumping plant. The flow in the river below the power house, however, can be maintained at 1 400 cu. ft. per sec. at all times by discharging 200 cu. ft. per sec. through the dam at $C$ (Fig. 6) during the time period, $d\,b$. The average flow through the power house is about 1 382 cu. ft. per sec.

By Method (3), the flow through the tunnel is maintained at 1 400 cu. ft. per sec. during the dry period until Storages I and II are all consumed (Point $g$ on the mass curve). From $g$ to $b$, the tunnel flow will be reduced to the natural stream flow entering the reservoir. The flow in the river below the power house can be maintained at 1 400 cu. ft. per sec. during this period by discharging sufficient water through the dam at $C$.

By Method (4) the discharge through the tunnel is maintained at 1 372 cu. ft. per sec. at all times (slope of line, $c\,f$, Fig. 7 ($b$)), and no water is discharged through the dam at $C$; hence, the regulated flow in the river below the power house is reduced from 1 400 to 1 372 cu. ft. per sec.

Under the first three methods, Storage III would be used during the dry period; but under Method (4), Storage III would never be used.

The relative advantages of Methods (2), (3), and (4), are not dependent on the pumping plant; they are mentioned chiefly to illustrate an interesting use of the mass diagram. Having determined which of these methods of operation is the most desirable, the economic justification of the pumping plant may be examined by the following procedure:

($a$) Compute the annual fixed charges and average annual operating expense for the pumping plant.

($b$) Compute the increase in firm capacity and average annual output, at the power house, $E$, and at any other power houses farther down the river, which may be obtained by the use of the pumping plant.

(*c*) Compute the kilowatt capacity required for the pumping-plant motors, and the average annual kilowatt-hours consumed in pumping.

(*d*) The difference between the capacity and energy increases of Procedure (*b*) and those required in Procedure (*c*) will represent the net increase of firm capacity and the kilowatt-hour output obtained by using the pumping plant. If the annual value of these increases is greater than the total annual cost of the pumping plant, the construction of the plant will be justified.

It will be seen that this problem is a special case of the "part-head" type of plant discussed by the author.*

* *Proceedings*, Am. Soc. C. E., November, 1928, Papers and Discussions, p. 2466.

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# PAPERS AND DISCUSSIONS

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## ECONOMIC COMPARISONS OF VARIOUS TYPES OF ROAD SURFACING

## A SYMPOSIUM

### Discussion*

By Arthur R. Hirst, M. Am. Soc. C. E.

Arthur R. Hirst,† M. Am. Soc. C. E.—The paper by Mr. Kuelling‡ is probably not just as the speaker might have written it. However, he is quite sure that the figures and conclusions in it are not as radical as they may appear at first reading.

The speaker has been familiar with the American highway problem since the spring of 1903 when he first served with the Maryland Highway Department. It is one of the most astounding problems that engineers have ever faced. It is extremely unfortunate, and, in fact, it has been almost fatal, that it has had to be prosecuted in an atmosphere which was not conducive to the close engineering reasoning that Mr. Kuelling's discussion shows. To the speaker it seems to be the most interesting of all problems in the history of the world. That statement is made advisedly.

In 1928 probably 23 000 000 motor vehicles will be licensed in America. Undoubtedly, those vehicles will travel at the average rate of 3 000 miles per vehicle, or a total of 69 000 000 000 miles. As compared to these figures, both in ton-miles and passenger-miles, the total traffic on American railroads is inconsiderable.

If a real consideration of this problem and some real highway service could save this traffic 1 cent per mile, the annual saving in 1928 would be

* This discussion (of the Symposium on Economic Comparisons of Various Types of Road Surfacing, presented at the meeting of the Highway Division, New York, N. Y., January 18, 1928, and published in November, 1928, *Proceedings*), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Cons. Engr., Madison, Wis.

‡ *Proceedings*, Am. Soc. C. E., November, 1928, Papers and Discussions, p. 2482.

$690 000 000, which, capitalized at 5%, represents a principal sum that the speaker hesitates to mention. If engineers could save this traffic even 0.1 cent per mile, the annual saving would be $69 000 000, which capitalized at 5% is an imposing amount. The average investment in these vehicles is probably $600 each, or $13 000 000 000. The annual cost of operating them is probably more than one-half the total investment in them.

The trouble in solving the problem has been, and apparently always will be, that two factors are involved: First, the public cost of the construction and maintenance of highway structures, which is quite well known and is considered appalling by the tax-paying public; and, second, the total cost of motor vehicles to individuals, which is not generally recognized at all.

This second cost is that which Mr. Kuelling dwells on. It is the speaker's experience that the average American citizen prefers to pay $10, when, and as he pleases, rather than have the Government tell him that he must give it 10 cents. He has not believed—although many have tried to impress the fact on him—that the public tax is the smallest part of the cost of operating his vehicle.

Mr. Kuelling states* that "users of highways have advanced and are now ahead of the builders". The speaker believes that the builders have never "caught up"; that they never have built any roads that were adequate for the traffic ten years later. The highway engineer is not at fault. The trouble is that the American people have not at all realized the astounding problems of this phase of transportation, and they have not been willing to pay the price for adequate highway service.

One of the most striking developments in the history of highways is the present tendency of the American public to turn, in an unprecedented degree, to the construction of the larger bridges and tunnels rendered imperative by the demands of traffic. In order to pay for them they are turning back 100 years and more to the toll system because it has been found impossible to sell these works directly to the public even when economically justified. They can be sold to bankers but not to the public. The speaker ventures the prediction that Americans are about to enter on the most tremendous era of toll-road construction in the history of this country, simply because the economics of the situation demand them.

The public is unwilling and unable to pay in mass for what they will gladly pay on the installment plan, as they go, in the form of tolls. In most cases, they have proved quite willing to pay in the form of tolls (gasoline taxes, now in effect in forty-five States) for highway improvements. Gasoline taxes are nothing else in the world but a return to the old toll-gate in a different form. He who uses the highway pays in proportion to that use.

America built the science of automotive engineering, and the story of its building is one of the most wonderful of all times. There are 23 000 000 motor vehicles on American highways and with very few exceptions the free and ready use of those vehicles is everywhere strangulated by traffic congestion—especially in the East. When a car, capable of going safely at 70 miles per

* *Proceedings*, Am. Soc. C. E., November, 1928, Papers and Discussions, p. 2482.

hour, requires 15 hours to travel from New York to Washington, D. C. (240 miles ±), the machine has been strangulated, despite the fact that the States of New York, New Jersey, Pennsylvania, Delaware, and Maryland, and the District of Columbia have spent millions in the construction of their highways. The traffic cannot move economically or swiftly, because at almost every block there is a traffic officer or a red light.

Before highway engineers really solve this problem, especially in the congested areas near and in the principal centers of population, they are going to see (within the next decade) a return to the toll system in order to build highways which are really adequate for traffic demands.

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# PAPERS AND DISCUSSIONS

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## RELATION OF THE LANDSCAPE ARCHITECT TO THE ALLIED PROFESSIONS ENGAGED IN CITY PLANNING

### Discussion*

BY MESSRS. HAROLD M. LEWIS, AND S. HERBERT HARE.

HAROLD M. LEWIS,† M. AM. SOC. C. E.—No better example of the interrelationship between professions in City Planning can be found than the work of the New York Regional Plan, which was organized on the assumption that the highest kind of co-operative effort would be required. In its search for fundamental data, four lines of investigation were started: (1) A physical or engineering survey; (2) an economic and industrial survey; (3) a social and living condition's survey; and (4) a legal survey.

As soon as these varied groups of investigators were put to work, Advisory Committees were created on which architects, landscape architects, and the leaders in the city planning field—irrespective of the particular profession to which they belonged—were represented. The work has been carried on through the medium of a staff made up of the heads of each of the different Survey and Planning Groups, and under the guidance of a General Director of Plans and Surveys.

The modern city is so complex that it cannot be planned simply by the application of engineering, architectural, landscape architectural, and other professional principles of design and adornment. Many modern communities are the victims of serious ailments which must be cured before they can grow in a healthy manner. This requires a diagnosis based on careful research which should be instituted by the professional city planners, but which may require the assistance of many other branches of endeavor. The speaker calls attention to just two of such ailments: (*a*) The so-called "vicious circle" of

* This discussion (of the paper by Arthur A. Shurtleff, Esq., presented at the meeting of the City Planning Division, New York, N. Y., January 18, 1928, and published in December, 1928, *Proceedings*), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Executive Engr., Regional Plan of New York and Its Environs, New York, N. Y.

rapid transit and congestion, new rapid transit and increased congestion, etc., *ad infinitum;* and (*b*) the effect of rapidly increasing land values on the form and character of urban growth. In the first case, how can new lines be built according to the best city planning principles when the people demand them to be built through crowded areas and parallel to lines already congested? In the second case, how can planners do away with blighted districts and the false values created by excess speculation?

It is obvious that such problems cannot be solved without the aid of many groups of both professional and business men.

S. HERBERT HARE,* ESQ. (by letter).†—It would probably be difficult to cite a more apt illustration of the value of specialization and co-operation than has been given by Mr. Shurtleff. This seems to be an age of specialization and, therefore, must be an age of co-operation, because these two must ever go hand in hand. Every profession has become more or less specialized within its own boundaries. This is particularly true of the field of engineering and is true to a certain extent already in the somewhat younger profession of landscape architecture. Therefore, co-operation must be practiced within the various professions as well as between professions.

The author has traced in an interesting way the evolution which has taken place in the field of landscape architecture from the problems involved in designing parks and private estates through the era of the development of parkways and connected park systems to the much broader problems of city planning which require much the same basic training as the simpler problems.

The profession of Landscape Architecture, while utilizing knowledge from the fields of engineering, architecture, design, and horticulture, cannot be said to be evolved from any one of these fields; it stands out as a distinct calling, from which the public has come to expect certain definite results. Has the work of city planning come to be recognized as a distinct profession? The answer is probably "No". It is a name given to a field of endeavor which normally implies co-operation of several professions, the line of demarcation between the work of each being somewhat less fixed by the titles under which the various practitioners operate, than by their individual training, experience, and natural ability. One thing is certain, that in nearly all city planning problems, the public is demanding more and more that proper attention be given to the beauty as well as the utility of the result, and some one in this scheme of co-operation must assume that task.

If specialization implies co-operation, then co-operation, in turn, implies a sympathetic understanding of the aims and ideals, if not of the technique, of the various fields involved. An engineer, an architect, or a landscape architect who understands the most about the other professions will not only co-operate to the best advantage in a problem, but will usually be the most ready to do so.

* Landscape Archt. and City Planner (Hare & Hare), Kansas City, Mo.

† Received by the Secretary, January 19, 1928.

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# PAPERS AND DISCUSSIONS

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## ENGINEERING APPLIED TO NATIONAL PARKS

### Discussion*

BY H. K. BISHOP, M. AM. SOC. C. E.

H. K. BISHOP,† M. AM. SOC. C. E. (by letter).‡—The author has covered this subject so thoroughly that little has been left to discuss. The speaker agrees entirely with Mr. Mather.

The U. S. Bureau of Public Roads has benefited greatly by its co-operation with the Park Service. In the beginning the personnel of the Bureau was perhaps a bit inclined to think too much of economy in road construction and too little of scenic effects and the preservation of the beauty of the country through which the roads ran. Through the National Park Service the Bureau has been brought into contact with landscape engineers and consequently a new appreciation of scenic factors will be evident in the character of the roads built hereafter in the National Parks and National Forests of the West.

The speaker has a great admiration for the work that the Park Service is doing. Those who have visited the parks in the West will appreciate it. The Service is opening trails and roads to scenes of beauty that were hitherto inaccessible.

Mr. Mather mentioned two projects:§ the Transmountain Road in Glacier National Park, and the road in Zion National Park, each of which will cost more than $1 000 000. The State of Arizona is now building a bridge across the Grand Canyon at Lee's Ferry which will change entirely the situation in that part of the United States. The impassable canyon that heretofore has cut off highway travel between the north and south and, to some extent, between the east and west, will be bridged at Lee's Ferry, and a road will

* This discussion (of the paper by Stephen T. Mather, Esq., presented at the meeting of the City Planning Division, New York, N. Y., January 18, 1928, and published in December, 1928, *Proceedings*), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Chf., Div. of Constr., U. S. Bureau of Public Roads, Washington, D. C.

‡ Received by the Secretary, December 17, 1928.

§ *Proceedings*, Am. Soc. C. E., December, 1928, Papers and Discussions, p. 2676.

probably be constructed from Flagstaff, Ariz., to Zion National Park, Bryce Canyon, and Salt Lake City, Utah. The speaker predicts that this road, in time, will be one of the heavy traffic roads of the West.

Mr. Mather describes* the snow-removal problem. There has been a great deal of trouble in opening some of the passes through the Rocky Mountains in the West in time for the traffic in the spring. Great strides have been made since 1925, but snow removal work is still a new problem in the parks.

In the Western States the best road surfacing is obtained from fine crushed rock or gravel; but experience has taught the Bureau of Public Roads that it must do something to eliminate the dust which becomes a serious nuisance when the traffic increases. For that purpose a method has been developed of mixing a road oil in place, which is believed to be something new in road building.† After the crushed rock road has been built and traffic-bound, it is scarified to a depth of 2 or 3 in. Then it is treated with just an ordinary fuel oil in two or three applications, the harrows being run over it after each time. A blade machine is used to move the material back and forth on the road until it is thoroughly mixed, as indicated by the color of the mass. After mixing in this manner the loose material is spread uniformly over the width to be surfaced and the road is opened to traffic, by which it is consolidated.

This kind of surfacing costs from $800 to $1 500 per mile for the oil and mixing process, and it is very effective. The speaker has seen some sections that appeared as if they were paved with bituminous concrete.

* *Proceedings*, Am. Soc. C. E., December, 1928, Papers and Discussions, p. 2681.
† *Loc. cit.*, p. 2682.

# MEMOIRS OF DECEASED MEMBERS

## JOHN CHARLES CHURCHILL, M. Am. Soc. C. E.*

DIED APRIL 19, 1928.

John Charles Churchill was born at Oswego, N. Y., on September 20, 1856, the second son of Judge John C. and Kathleen Sprague Churchill. His father was one of Oswego's leading citizens, having been a Representative in Congress and also a Justice of the Supreme Court. He received his preparatory education in the public schools of Oswego, and was graduated, in 1878, from Lafayette College, Easton, Pa., with the degree of Mining Engineer.

From March, 1879, to July, 1881, Mr. Churchill was Surveyor and Draftsman for the Federal Government on harbor improvement work on Lake Erie. During the remainder of 1881, and until the end of 1884, he was Assistant Engineer for the Oregon Railway and Navigation Company, for two years in charge of the construction of bridges and trestles, and of tracklaying on the Baker City Branch. He returned to New York State in 1885, and from May to October was in charge of harbor improvement work at Ogdensburg, N. Y. From October, 1885, to October 1886, he was Assistant Engineer on the Rome, Watertown and Ogdensburg Railroad, in charge of the construction of a ferry slip at Ogdensburg, and of construction work on the branch from Norwood to Massena, N. Y. From October, 1886, to August, 1887, he was Assistant Engineer for the Federal Government on various surveys for harbor improvements in Northern New York and Vermont. In August, 1887, he resigned from the Government service to enter the contracting business, continuing in this work until July, 1889.

From July, 1889, to January, 1891, Mr. Churchill was Engineer and Assistant to the Building Superintendent on the construction of the St. Lawrence State Hospital, at Ogdensburg. In 1892, he acted as City Engineer of Ogdensburg, during which service he designed and constructed the South Street main sewer, and rebuilt the sea wall in the Oswegatchie River.

In 1892, he returned to the Federal Government Service in charge of harbor improvement work at Ogdensburg, continuing until August, 1895, when he was made Assistant Engineer in charge of the St. Lawrence River improvement. From January, 1896, to July, 1898, he was in charge of examination and surveys on Lake Champlain, the St. Lawrence River, and Lake Ontario, having been also from January to May, 1898, in charge of the U. S. Engineer Office at Oswego, N. Y., during the absence of the District Engineer. In July, 1898, the Oswego District was combined with the Buffalo, N. Y., District, and Mr. Churchill was given charge of the Oswego Branch Office, in which position he continued until it was again separated from the Buffalo District in August, 1909. When the Oswego Dis-

* Memoir prepared by Lynn L. Davis, M. Am. Soc. C. E.

trict Office was re-established in August, 1909, he became Principal Assistant to the District Engineer, the District including all harbors on the south shore of Lake Ontario and the St. Lawrence River. In May, 1913, the Oswego District was again combined with the Buffalo District, and Mr. Churchill for the second time was made Assistant Engineer in charge of the Oswego Sub-Office, in which capacity he continued to serve until his death.

In addition to this regular service he was employed in a consulting capacity as follows: By the State of New York as Consulting Engineer and Expert Witness in matters pertaining to water power on the Oswego River; and for a time as Chairman of the Water Commission of the City of Oswego. He also served as Fuel Administrator of the City of Oswego during the World War. He had applied for a commission in the Corps of Engineers during the war, but his request was refused on account of his age. All the members of his family were engaged in some capacity during the war, his three sons having been in active service.

Mr. Churchill served the Federal Government on harbor improvement work for more than thirty-eight years, on Lakes Erie, Ontario, and Champlain, and on the St. Lawrence River. He designed and built many of the breakwaters and piers on Lake Ontario and the St. Lawrence River. His knowledge regarding the needs of commerce on these lakes and of the character and condition of all the structures in the former Oswego U. S. Engineer District was very valuable, and his advice was sought on all projects for improvement or maintenance.

He died at his home in Oswego, on April 19, 1928, and is survived by his widow, formerly Mary Beekman Westbrook, three sons, John W. Churchill, of Washington, D. C., Maj. Lawrence S. Churchill, Air Corps, U. S. A., Langley Field, Va., and Walter A. Churchill, of Salem, Mass.; one brother, Lawrence W. Churchill, of White Plains, N. Y., and St. Petersburg, Fla.; and one sister, Mrs. R. R. Fisher, of Oswego.

Mr. Churchill was one of God's noblemen. His kind and genial disposition and splendid upright character won for him the highest respect of all who were privileged to know him. In his business and professional relations he was held in the highest regard by all who were associated with him. His City will miss his wise and judicial counsel and the Federal Government will feel the loss of a valued and able servant whose wisdom was always sought in all matters of policy and details of Federal work in his District.

Mr. Churchill was elected a Member of the American Society of Civil Engineers on February 1, 1899.

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## JOHN BYERS HOLBROOK, M. Am. Soc. C. E.*

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DIED AUGUST 20, 1928.

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John Byers Holbrook was born in New York, N. Y., on July 12, 1872. He was the son of Edwin W. and Fanny Chick Holbrook, both parents being descendants of families prominently identified with the early history of Man-

* Memoir prepared by W. W. Ricker, Esq., New York, N. Y.

hattan. His father was engaged in commercial banking for many years, and at the time of his death was the oldest living member of the Union League Club. His son became associated with this Club on attaining his majority.

On completing his academic schooling in New York, Mr. Holbrook entered the Pennsylvania Military College, at Chester, Pa., from which he was graduated in 1892 with the degree of Civil Engineer. In 1893, he entered Cornell University, graduating in 1896 with the degree of Mechanical Engineer. At Cornell, he joined the Delta Kappa Epsilon Fraternity, with which, during his life, he maintained an active connection. He held the office of President of the Alumni Association of his Chapter and was diligent in his efforts to influence the young students in making the most of their college life.

Upon leaving college, Mr. Holbrook spent five or six years in perfecting himself in steam power plant design and operation, by employment with various companies, notably the E. W. Bliss Company, Brooklyn, N. Y., and the New York Edison Company and the American Blower Company of New York City. In 1901, he began private practice as Consulting Engineer, specializing in steam power plants and heating and ventilating. He designed and supervised the installation of the power plants in many of the earlier skycrapers of downtown New York, among which the Trinity Building, City Investing Building, and Commercial Cable Building are of importance. Later, his work extended throughout the United States and was marked by a thoroughness and conscientious attention to detail that resulted in very successful installations.

Mr. Holbrook served an enlistment of five years in Squadron A, New York National Guard. He always maintained his interest in the Veteran Organization and, in 1916, returned to active service for the duration of the World War.

His personality was characterized by a rare cheerfulness and geniality which drew to him, and held continuously, an unusual number of loyal friends.

He was married on October 3, 1903, to Blanche V. Meehan, of New York, who survives him.

Mr. Holbrook was elected a Member of the American Society of Civil Engineers on April 6, 1909.

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## DAVID JANNEY HOWELL, M. Am. Soc. C. E.*

DIED AUGUST 28, 1928.

David Janney Howell was born in Charlestown, W. Va., on September 8, 1863, the son of Samuel and Adele (LaFitte) Howell. His father was of Scotch ancestry, while his mother was of French descent.

Mr. Howell received his early education in the Charlestown Academy, Charlestown, and afterward attended Washington and Lee University, at Lexington, Va.

His early experience—starting as Rodman and afterward being promoted to Instrumentman and Topographer on railroad surveys in Virginia—was followed by his Civil Service appointment in 1884 in the United States

* Memoir prepared by Beaudric L. Howell, Assoc. M. Am. Soc. C. E.

Geological Survey as Assistant Topographer. This position he held until his resignation in 1887 to enter private practice in civil engineering in Washington, D. C.

From 1890 to 1897 Mr. Howell's practice embraced the planning, surveys for, and superintendence of, the improvements of various suburban properties within the District of Columbia and the adjoining States. He was also engaged on surveys for and superintendence of the construction of the walks, roads, lake, and bridge in the National Zoological Park; municipal improvements in Alexandria, Va. as City Engineer; surveys, construction, and superintendence of electric railways in Washington, D. C.; an 80-ft. iron span bridge and 50-ft. short-span bridge over Great Hunting Creek in Fairfax County, Virginia; and engineering work and superintendence on drainage canals in North Carolina. In addition, he acted as Chief Engineer for the Washington, Alexandria, and Mount Vernon Railway Company between Washington and Alexandria, exclusive of electrical equipment.

From 1897 to 1910 his work embraced the following positions: Assistant Engineer under the United States Board of Engineers on Deep Waterways in charge of surveys, investigations, and report of the Oswego Route, Eastern Division, from Albany to Herkimer, N. Y., about 90 miles for the ship canal from the Great Lakes to the Atlantic Ocean; Engineer to the New York State Canal Advisory Committee (appointed by President Roosevelt, then Governor of New York State), making estimates of cost and report on the enlarged Erie Canal, from Troy to Buffalo, N. Y.; Consulting Engineer to the New York State Engineer and Surveyor, Edward A. Bond, M. Am. Soc. C. E., in charge of surveys, plans, estimates, and printed report to the New York State Legislature on the enlargement of the canals of the State known as the Barge Canal, the estimated cost of which was about $100 000 000; surveys, plans, and specifications for the water-works system, at Fultonville, N. Y., and surveys and plans for the development of water power on the Shenandoah River above Harpers Ferry, W. Va., for the Shenandoah Electric Light and Power Company; Engineer to the late F. W. Hodgdon, M. Am. Soc. C. E.

Mr. Howell also served as Engineer of the Massachusetts River and Harbor Commission, on plans and estimates of cost of locks on the Taunton River, Boston Ship Canal; report on improvements to water-works system at Frostburg, Md.; electric railway surveys and report, Canandaigua, N. Y., to Lake Ontario, 30 miles; the planning and construction, as part owner, under franchise of the water system and electric plant at Welch, W. Va., and the direction of its operation until it was sold in 1911; the investigation and report on additional water supply for Bristol, Va., and the preparation of plans for, and the superintendence of the construction of, a spring supply and 4 000 000-gal. reservoir; plans and report on the new distributing reservoir, Alexandria Water Company, Alexandria, Va.; railroad surveys from Cherry Run to Cumberland, Md.; investigation and full report with estimates of cost for a dam and large storage reservoir and 5½ miles of pipe line, and the purchase of land and water rights for the same, for the Alexandria Water Company, known as Barcroft Dam and Reservoir; plans for and the general supervision of the

new reservoir, at Warrenton, Va.; report on improvements for the water-works system, Washington County Water Company, Hagerstown, Md.; general report and appraisal of plant, Alexandria Water Company; surveys and plans for a new dam and storage reservoir and pipe line, Charlottesville, Va.; surveys and reports on the sewerage system, Laurel, Md.; and consultation and report on an additional water supply and about 7 miles of pipe line in Bristol, Va.-Tenn.

From 1910 until 1918 Mr. Howell was engaged in the following work: Surveys for the planning of and superintendence of the construction of surface and underground improvements for the development of numerous suburban properties in the District of Columbia, Maryland, and Virginia; all engineering work and superintendence of construction of the Barcroft Dam and Reservoir for the Alexandria Water Company, the dam being 500 ft. long, with a maximum height of 70 ft., and the reservoir storage being about 600 000 000 gal.; and the investigation and report on a proposed storage dam for electric power in Virginia and various valuations and reports on water-works properties and sewerage systems.

Mr. Howell was made Manager of the Alexandria Water Company, of Alexandria, and acted as such for a period of about two years. His work with this Company embraced plans for and superintendence of its filtration plant. He was also engaged on complete surveys, plans, and superintendence for a separate sewerage system for the Town of Potomac, Va.; and a complete report, with estimates of cost, for a water supply and sewerage system for Arlington County, Virginia, embracing about 30 sq. miles.

During 1918 and 1919, Mr. Howell served as Supervising Engineer in the United States Quartermaster Department, in charge of surveys, plans, and the construction of surface and underground improvements at the Enlisted Men's Barracks, South Potomac Park, Washington.

From the opening of his private offices in 1887 to the time of his death, he acted as Engineer for the larger real estate operators of Washington, and in adjoining States, and had full charge of planning and developing many beautiful subdivisions in and around Washington.

On August 25, 1928, he was appointed, by Governor Byrd of Virginia, a member of the Commission to confer with a similar Commission of the State of Maryland for the purpose of settling a dispute which had arisen between the two States.

In 1910, he organized, with his son, Beaudric L. Howell, Assoc. M. Am. Soc. C. E., as Junior Member, the present firm of David J. Howell and Son, of Washington. He continued as an active member of this engineering firm until his death. Mr. Howell died suddenly of heart failure while engaged in an inspection of construction work on August 28, 1928.

Mr. Howell was a member of the New England Water Works Association, the American Water Works Association, and the Washington Society of Civil Engineers, and was a Certified Professional Engineer of the State of Virginia.

In 1887, he was married to Mary M. Gibson, who, with one son, survives him.

Mr. Howell was elected a Member of the American Society of Civil Engineers on September 5, 1900.

## HOWARD GEORGE KELLEY, M. Am. Soc. C. E.*

DIED MAY 15, 1928.

Howard George Kelley was born in Philadelphia, Pa., on January 12, 1858. He was educated in the public schools of Philadelphia and was a student in civil engineering at the Polytechnic College of Pennsylvania.

On leaving school Mr. Kelley desired to follow the lure of the sea which inclination was derived from the experiences of his grandfather and his father, both sea-faring men, although his father in middle age had given up his ship and become a merchant in Philadelphia. Mr. Kelley, however, engaged to sail on a small ship of less than 400 tons from San Francisco, Calif. He became well versed in the study of navigation which he found a pleasing diversion. His grandmother, who had taken care of him in his youth, prevailed upon him to give up his sea-faring proclivities and he became a Land Surveyor in California. His later work, fortunately, gave him numerous opportunities to indulge in his love of and pleasure in ocean travel. He crossed the Atlantic a dozen times in the line of duty and during the winter of 1924 and 1925 he and Mrs. Kelley enjoyed a trip around the world.

The record of his services is most interesting in its diversity and its steady accumulations of success. It demonstrates his close attention to his chosen profession and shows the result of the intensity with which he carried on every duty in the line of his work.

In April, 1881, Mr. Kelley entered the service of the Northern Pacific Railway Company at Tacoma, Washington Territory, on the original preliminary and location surveys from Kalama, Washington Territory, to Portland, Ore., running lines on both sides of the Columbia and Willamette Rivers. He then joined the crew on the preliminary and location surveys from Sand Point, Idaho, on Lake Pend'Oreille, around the north and east sides of the lake, and thence up the Valley of Clarks Fork of the Columbia River, through the Cœur d'Alene and Cabinet Mountains, to its junction with the Missoula River in Montana, to the point now known as Paradise, at that time a totally uninhabited country.

On the completion of the surveys, he was appointed Transitman for the Division Engineer in charge of bridges and, on the retirement of the Division Engineer on account of illness, Mr. Kelley was placed in charge of the party and completed the work to the Missoula River, including the bridge over Clarks Fork at that point, then known as the "Third Crossing". This work included the design and construction of bridges, trestles, viaducts, piers, and abutments, all built of timber, and performed by company forces. In July, 1883, he was transferred to the Kalama-Portland Division as Resident Engineer in charge of bridge construction. Later in the autumn, he was put in charge of the work on the Upper Yakima River for about two months.

Mr. Kelley was then engaged for several months in making a survey of the water-front at Seattle, Wash., on which the later improvements by the Northern Pacific Railway Company were based. Subsequently, he was placed in charge

* Memoir prepared by Carl H. Gerber, Esq., Omaha, Nebr.

of the field work on the survey and soundings of Tacoma Harbor and the Lower Puyallup River. He was engaged on reconnaissance in the Cœur d'Alene Mountains, in Idaho; on the western slopes of the Cascade Range, in Washington Territory; and in the Olympic Mountains south of the Straits of San Juan de Fuca, also in Washington Territory. His work with the Northern Pacific Railway Company and allied service covered a period of four years, terminating in the spring of 1885.

During the remainder of 1885, and for the succeeding year, Mr. Kelley was engaged in mining, his work including nearly every job pertaining to a mine, incidentally, also, learning the blacksmith trade, which resulted in his being placed in charge of the development of the Mapleton and Champion Mines of the Summit Mountain Mining District.

In December, 1886, he was appointed Resident Engineer of Maintenance of Way on the St. Louis, Arkansas and Texas Railway, and, later, on the St. Louis Southwestern Railway, which extended from Birds Point, Mo., opposite Cairo, Ill., southward through Missouri, Arkansas and Texas, 395 miles of this railway having been changed from narrow to standard gauge at that time. The position of Superintendent of Bridges and Buildings was later added to his duties as Resident Engineer.

The entire original railway was rebuilt, including all the bridges over the rivers which constituted the principal drainage lines of that section of the country; one item was the sinking of two pneumatic piers with a penetration of 88 ft. below the surface of the water. This work was started under contract, but completed by company forces under Mr. Kelley's direction. During this period the railway was being extended in Missouri, Arkansas, Louisiana, and Texas, to a total of 1 228 miles, and on the completion of the grading and track laying of these extensions, he took charge of the construction of the permanent structures and was appointed Chief Engineer.

As Chief Engineer Mr. Kelley continued in direct charge of bridges and buildings; he located and built the line from Delta to Grays Point, Mo., together with the yard and incline at Grays Point, and also located the first line to the Mississippi River for a bridge to Thebes, Ill.; he made the reconnaissance personally and reported on the terminal property required for the route to Dallas on which the line was built; he also made several reconnaissances in Texas and reports on other smaller properties.

The valuation of the St. Louis Southwestern Railway Company's properties in Texas, aggregating between 500 and 600 miles, made under a General Order of the State Railroad Commission, was carried out under his personal direction and supervision. His services with this Company extended throughout a period of eleven years.

On March 1, 1897, Mr. Kelley resigned his position to become Chief Engineer of the Minneapolis and St. Louis Railroad Company, being retained for another year as Consulting Engineer for the St. Louis Southwestern Railway Company. As Chief Engineer of the Minneapolis and St. Louis he had charge of maintenance, including bridges and buildings, real estate, and new construction.

While in the service of the Minneapolis and St. Louis Railroad Company, Mr. Kelley made an examination and report of the Iowa Central Railway, about 500 miles, and of the Des Moines and Fort Dodge Railway, about 139 miles, and on the basis of his report these lines were acquired and became part of the Minneapolis and St. Louis System. A number of important changes and extensions were made, including reconstruction for improvement in grades and alignment on the main line of this System, as well as on the Iowa Central Railway, all of which was heavy work, covering a distance of 296 miles. This reconstruction was performed by company forces with the exception of 15 miles on the Minneapolis and St. Louis Railroad between Hopkins Junction and Chaska, Minn. and 7 miles on the Iowa Central Railway which was done under contract. In addition, there was also the location and construction of 363 miles of new lines for the Minneapolis and St. Louis Railroad Company in Minnesota, Iowa, and South Dakota, and a number of reconnaissance examinations for other projected lines. The valuation of these lines, made by order of the Minnesota Railroad and Warehouse Commission, was under Mr. Kelley's direct supervision. Throughout the work he had charge of the operation of the lines under construction. His services with the Minneapolis and St. Louis Railway Company continued for a period of nine years.

In July, 1907, Mr. Kelley was appointed Chief Engineer of the Grand Trunk Railway System, for both the American and Canadian lines, aggregating 4 785 miles. After four years in that position, in October, 1911, he was made Vice-President in charge of operation, maintenance, and construction. Six years later, in October, 1917, he was elected President of the entire Grand Trunk System, the Grand Trunk Pacific Railway, and its steamship lines, in all more than 8 000 miles of railway. He also served as Chairman of the Board of the Central Vermont Railway Company, President of the Toronto Terminal Company, and Director of the Chicago and Western Indiana Railroad Company, and the Belt Railway Company of Chicago, which positions he retained until the Grand Trunk Railway System was taken over and made part of the Dominion of Canada Government Railways in 1922.

Important among the noteworthy items accomplished during this period of fifteen years was a substantial improvement of the physical condition of the property and a very heavy increase in business; also the reconstruction of bridges over the St. Lawrence and Niagara Rivers; the improvement of grade line and elimination of grade crossings from York through Toronto to Mimico, Ont., Canada; the construction of the Union Station building, at Toronto; the elimination of grade crossings, covering several miles, at Detroit, Mich.; the preliminary plans for the abolition of grade crossings in the City of Montreal, Que., Canada, covering more than six miles and including three terminal yards and the preliminary plans for a new terminal station; the plans and estimates for an entirely new terminal for the Chicago and Western Indiana Railroad Company in Chicago, Ill.; the electrification of the St. Clair River Tunnel from Sarnia, Ont., Canada to Port Huron, Mich.; the construction of the Montreal and Southern Counties Railway, an electric freight and passenger line of 44 miles; the construction of 4.5 miles of new line

between Birch and Wybridge, Ont., Canada, and 18.8 miles of new line from Cass City to Bad Axe, Mich.

The valuation of the American lines under orders from the Interstate Commerce Commission, including the lines of the Central Vermont Railroad Company, was started and subsequently carried on by the Valuation Department of the Grand Trunk Railway System. The valuation of the entire Grand Trunk System, together with a report on its physical condition, was made in 1920 and 1921, in connection with the arbitration proceedings under which the property was taken by the Dominion Government. This included all the owned and partly owned properties of the System; the Grand Trunk Railway, American and Canadian Lines; the Grand Trunk Pacific Railroad and Steamship Lines; the Central Vermont Railroad; the Detroit Terminal Company; the Toledo Terminal Company; the Detroit and Toledo Shore Line Railroad; the Chicago and Western Indiana Railroad and Belt Railway of Chicago; the Montreal and Southern Counties Railroad (Electric); the Oshawa Railway (Electric); the car ferries on Lakes Michigan and Ontario; the Central Vermont Transportation Company Steamships; the Grain Elevator Companies; the Hotel Companies, and a number of other properties.

A noteworthy accomplishment, demonstrating Mr. Kelley's ability, energy, and tireless attention to meet emergencies, was his success in directing the movement of men, munitions, and supplies, occasioned by the World War. The record shows efficient and prompt movement of all traffic and, further, that no ship assigned to transportation overseas for Canada's participation in the war was delayed more than 24 hours in the whole 4-year period.

At the close of his services with the Grand Trunk System, he had planned to take a well-earned vacation and intended to go on a purely pleasure trip to Europe, but before he could put that plan into effect he was called to Boston, Mass., for a conference with the members of the Joint New England Railroad Committee, and in October, 1922, was selected as Technical Adviser to that Committee which he served until November, 1923, when the work was completed.

The Joint New England Railroad Committee was composed of the Chairmen of six State Committees appointed in the six New England States by their respective Governors. The members had been selected from among the leading bankers and business men for the purpose of studying the transportation and industrial situation and to report on such study with recommendations for the future advancement of New England and for the consolidation of the railways.

The work required a study of the manufacturing industrial situation of New England, the present and possible future markets, the transportation facilities to reach such markets, the various routes of distribution by rail, ocean, and lake, and the possible development of the principal ocean ports. It also included an inspection, study, and analysis of each of the several railroads of New England, their financial structures, operating conditions, methods and results, maintenance, facilities, requirements, and the probable effect on them and on the New England industries of the several methods of possible consolidation.

It involved a series of public hearings held in Boston, Hartford, Conn., Providence, R. I., and Montpelier, Vt., the States of New Hampshire and Maine being heard in Boston. The published report of the Committee required a book of 334 pages, supplemented by maps and charts, but the actual data assembled filled several thousand pages.*

From June until late in November, 1924, Mr. Kelley was engaged upon an analysis and report covering the location, construction, operation, value, and earning power of the Virginian Railway Company, together with its branches and terminal properties. This study included a complete review of this property from its inception to date (1902 to 1924) and covered the original location, operating grades secured, character of country traversed, type and character of construction, conditions under which construction had been conducted, the motive power and equipment, the operating results throughout the period of its existence, the business, past, present, and prospective, available for traffic and, finally, the earning power and value of the railway.

This was supplemented during the months of October and November, 1925, by a re-arrangement of the data contained in the report of 1924 for direct testimony presented in hearing before the Bureau of Valuation of the Interstate Commerce Commission in connection with the valuation case of the Virginian Railway Company.

In November, 1925, within a week after the completion of the Virginian hearing, Mr. Kelley left New York City to go to Russia to make an examination of two railways in Southern Russia, aggregating more than 3 000 miles.

The work involved a detailed inspection, study and analysis of the roadway, engine terminals, shop facilities, station and other buildings, and all other physical elements; an analysis of operating performances and statistics covering the period of years after the World War and for several years prior to the war; an estimate for the completion of the rehabilitation work undertaken by the Russian Government and the requirements for increasing the capacity and extending the lines of these two railways to properly serve the coal, iron ore, and manganese mines and the several steel plants in the Ukraine Province. It also included the examination of the seaports on the Black Sea. Nearly three months were required to complete the inspection and to obtain the desired data in Russia. A report on the project was compiled after Mr. Kelley's return to New York City and completed in June, 1926.

During August and September, 1926, an analysis and report were made of the Buffalo, Rochester and Pittsburgh Railroad Company, which report was used in the testimony presented before the Interstate Commerce Commission in connection with the hearing relating to the proposed lease of the Buffalo, Rochester and Pittsburgh by the Delaware and Hudson Company to form a part of the Loree project for an additional trunk line from New York City westward.

On May 20, 1927, Mr. Kelley again sailed from New York City, this time to make a study and an analysis of the State Railways of the Kingdom of Roumania, including their location, physical condition, equipment, operating

* Part of Interstate Commerce Comm., Docket, 12964.

methods and results, and earning power. These lines covered a total of 11 085 km., or 6 888 miles, including 150 miles of second track. The work involved a detailed inspection of the physical properties, the obtaining of data on operating performances and results, and an estimate for the rehabilitation and extensions to serve properly the traffic requirements of Roumania, including all commodities produced or transported within and throughout that country.

Mr. Kelley returned to New York on August 26, 1927, and for the next three months was occupied in compiling a report on his findings which he completed in December, 1927, at Omaha, Nebr. Subsequently, he was called to New York (January, 1928) for discussion and conferences in connection with both the Russian and Roumanian work. Concluding his stay in New York, he went to his home in Dallas, Tex., to attend to an accumulation of matters relating to his personal affairs and in April, 1928, went to San Diego, Calif., for rest and recreation.

His plans at that time were made in expectation of further service in connection with the two foreign projects, and he anticipated returning to Europe some time during June or July, 1928, to continue the investigations and conferences incident to the negotiations with the two Governments; but fate willed otherwise, and he passed on to that bourne from which no man returns on May 15, 1928, at San Diego.

Mr. Kelley was affiliated with numerous religious, fraternal, social, and business organizations, including the Protestant Episcopal Church and the Church of England; the Masonic Order; the Mount Royal Club; the Forest and Stream and Mt. Bruno Country Clubs (Montreal); the New York Yacht Club (New York City), the Congressional Country Club (Washington, D. C.), and many others.

He was a member of the Executive Committee of the American Railway Association from June, 1919, to January, 1923; the Standing Committee of the Association of Railway Executives from its inception until he retired from the Grand Trunk System; the Committee on Relations (now Section V) of the American Railway Association from 1911 to 1921, inclusive; the Per Diem Rules Arbitration Committee (one of five members); the American Railway Engineering Association, of which he was President for two years; the Institution of Civil Engineers, of London, England; the Canadian Institute of Civil Engineers; and a Director of the Royal Trust Company of Canada, from 1917 until his death.

Mr. Kelley was married in 1897 to Cora Lingo, of Denison, Tex., who survives him. He is also survived by two brothers, Walter S. Kelley, of Wheeling, W. Va., and Henry Kelley, of Big Spring, Tex.; and one sister, Mrs. Edith Homan, of Oklahoma City, Okla.

The passing of Howard G. Kelley is a grievous loss and he is mourned by many. His character and personality were of the highest type, his ability recognized by all, and his energy and thoroughness known to those with whom he came in contact. He was honored and respected as was his due.

Kindly and considerate, appreciative and generous, with that unusual understanding of conditions and personal temperament affecting those who worked with him and for him; always ready to listen and to give sound counsel to

every one; charitable to all; and the most delightful companion in every phase of life—these are only a few of the excellent attributes which he possessed.

His power of analysis and his sound judgment were accepted in determining methods and rules for guidance in the successful direction and advancement of transportation affairs by the owners and executives of the railways of the United States and Canada. His capacity for work seemed almost unlimited and his success in the railroad and business world are proof of his motto, "Without work, nothing".

Mr. Kelley was elected a Member of the American Society of Civil Engineers on May 1, 1889.

## EMILE LOW, M. Am. Soc. C. E.*

DIED MAY 25, 1927.

Emile Low, the eldest son of Sigismund and Anna (Hussell) Low, was born on April 2, 1854, in Pittsburgh, Pa. His father was a graduate of the Karlsruhe Polytechnicum, Karlsruhe, Germany, and was one of the founders of the *Pittsburgh Volksblatt*. Emile Low's education was obtained in a private school and, later, in the public schools of Pittsburgh.

His first professional work was done in 1869 in connection with the surveys of the Monongahela Valley Railroad. In 1870, he was engaged on the surveys for the Western Extension of the Pittsburgh and Connellsville Railroad (now a part of the Baltimore and Ohio Railroad), of which his father was Engineer in Charge; and during 1870 and 1871, he was employed on construction work for the Cumberland, Md., extension of the same line.

Subsequently, Mr. Low became Assistant on the Hiland Avenue Reservoir of the Pittsburgh Water-Works, with which he was connected until 1875. In 1876, he served as Assistant City Engineer. Early in 1877, he was connected with the survey of the Pittsburgh and Lake Erie Railroad Bridge across the Ohio River. Later, he entered the service of the Pittsburgh, Castle Shannon, and Washington Railroad Company as Principal Assistant Engineer; this railroad subsequently was absorbed by the Baltimore and Ohio Railroad Company. In 1879, he was engaged on work for the Shousetown Railroad and also for the Pittsburgh and Atlantic Railroad Companies.

In the fall of 1879, Mr. Low was appointed Engineer of Construction and of Maintenance of Way for the Pittsburgh and Western Railroad Company and, later, became Chief Engineer. This railroad, also, became a part of the Baltimore and Ohio Railroad. He remained with the Company until March, 1881, when his father was appointed Engineer in Charge of the Guanajuato Branch of the Mexican Central Railway, and Mr. Low accompanied him to Mexico as his Assistant. This work terminated in June, 1882. During the pursuit of his engineering career, Mr. Low encountered many dangerous situations, particularly that experienced in a silver mine in Mexico where he barely

* Memoir prepared by H. J. March, L. L. Davis, and S. T. M. B. Kielland, Members, Am. Soc. C. E.

escaped with his life because of a dynamite blast discharged by careless workers.

Upon his return to the United States, he was appointed Engineer in charge of location for the Pittsburgh, McKeesport and Youghiogheny Railroad Company with which he remained until the latter part of 1884. Early in 1885 he served as Assistant Engineer with the Monongahela Railroad Company of Pittsburgh, which position he retained until March, 1885, when he was made Locating Engineer of the Pennsylvania (Schuylkill Division) Railroad. In December, 1886, he made the remarkable location of a mountain line, 20 miles long, in 10 days during winter weather, between Delano and Hazelton, Pa.

In 1887, Mr. Low was appointed Locating Engineer of the Norfolk and Western Railroad. Subsequently, he became Division Engineer and Engineer in charge of coal operations of the Ohio Extension. In 1892, he was made Engineer of the Mathieson Alkali Works, of Saltville, Va. The late John F. Wallace, Past-President, Am. Soc. C. E., who was Vice-President of the Works, took occasion at a speech-making event to commend most highly the accuracy and thoroughness of Mr. Low's work. He stressed the thought that a great many engineers like Mr. Low, spend their lives in obscure sections of the country and in consequence nothing praiseworthy is heard of them; therefore, when opportunity presents itself for commendation, it should be given freely. Mr. Low's connection with the Mathieson Alkali Works terminated in 1896, after which he became associated with Maj. Thomas W. Symons, Corps of Engineers, U. S. Army, in the preparation of a report on the Erie Canal. He also was associated with Major Symons in the preparation of the well-known report for a canal from the Great Lakes to the Hudson River.*

From 1897 to 1903, Mr. Low was Resident United States Assistant Engineer, in charge of the building of the Buffalo Breakwater under the direction of Colonel Symons. He was awarded the Norman Medal by the Society in 1904 for the presentation of a paper describing this work.†

Mr. Low continued as U. S. Assistant Engineer until June 1, 1904, after which he was in the service of the State of New York as Resident Engineer in charge of the Barge Canal work in Oneida County, having been granted a three-year furlough from the U. S. Engineer Department. While he was engaged in this work which involved some difficult engineering problems, he published a number of interesting papers, one of which was entitled "Wash-drill Borings in the New York State Barge Canal",‡ and another, "Hydraulic Dredging on New York State Barge Canal".§ On April 1, 1907, Mr. Low re-entered the Federal Service as Assistant Engineer at Buffalo, N. Y., where he remained until his retirement in April, 1924, having served under the following officers of the Corps of Engineers, U. S. Army: Col. H. M. Adams, Col. W. L. Fisk, Col. J. G. Warren, M. Am. Soc. C. E., Maj. L. V. Frazier, Col.

* The respective reports upon these matters may be found in the Annual Report of the Chief of Engineers, U. S. Army, for 1897.

† "The Breakwater at Buffalo, New York," *Transactions*, Am. Soc. C. E., Vol. LII (1904), p. 73.

‡ *Engineering News*, January 17, 1907.

§ *Loc. cit.*, April 10, 1913.

Clarke S. Smith, M. Am. Soc. C. E., Maj. William E. R. Covill, and Maj. P. S. Reinecke, M. Am. Soc. C. E.

Mr. Low was married on January 23, 1883, to Estelle Reed, of Zelienople, Pa.; there were three children, Estelle Low, Emile Reed Low, and Mary Virginia Low.

Mr. Low was esteemed most highly by his fellow engineers of Buffalo for his remarkable professional attainments. He was an indefatigable worker in every engineering pursuit which he followed. In this respect he was an exemplary character for so many engineers who take joy in service for their fellow men and their country and who contribute of their energy of mind, body, and soul, as it were, with little or no personal reward.

Mr. Low was elected a Member of the American Society of Civil Engineers on May 2, 1888.

## ADDISON MOFFAT SCOTT, M. Am. Soc. C. E.*

DIED APRIL 27, 1927.

Addison Moffat Scott, the son of Chauncey L. and Margaret L. (Moffat) Scott, was born in Dryden, N. Y., on February 20, 1843. He was of mixed Scotch-Irish and English ancestry.

When he was about sixteen, and for several years thereafter, he attended the Ithaca Academy, at Ithaca, N. Y., directing his studies in large part to mathematics and surveying. While attending the Academy, he usually taught school during the winters.

In the spring of 1866 Mr. Scott made a trip to the Northwest and at St. Paul, Minn., joined a party then organizing under Gen. G. K. Warren, U. S. Corps of Engineers, for a survey of the Upper Mississippi River. On the disbanding of this party at St. Louis, Mo., late in 1866, Mr. Scott returned home and renewed his studies at the Ithaca Academy. While he was attending school in the fall of 1867, General Warren offered him a position as Assistant Engineer on the improvement of the Upper Mississippi, which was then in progress. He accepted this position with much hesitation, as it interfered with his plan of study, which included a course at college. He remained on this work for several years, having been employed the latter three years of the time, as Assistant Engineer on the construction of a large railroad and highway bridge built by the United States Government across the Mississippi River, between Rock Island, Ill., and Davenport, Iowa. He was active in this work from the first survey for the location of the bridge until its completion in the spring of 1873. On the completion of the bridge, General Warren, who had been transferred to the East with headquarters at Newport, R. I., tendered Mr. Scott a position under him there. This would have been accepted but for another, and what appeared a more promising, offer made about the same time.

In that year, 1873, the United States Government undertook the improvement of the Great Kanawha River, which was in the Ohio River Department,

* Memoir prepared by C. P. Fortney, M. Am. Soc. C. E.

then in charge of the late Col. William E. Merrill, U. S. Corps of Engineers, M. Am. Soc. C. E., with headquarters at Cincinnati, Ohio. Mr. Scott was placed in local charge of this work, residing at Charleston, W. Va. Throughout the several changes that took place in the organization of the river improvement force, Mr. Scott was continuously retained in his position as Chief Assistant or Resident Engineer. During his long service there, he devoted himself with persevering zeal and energy to the duties of his professional position, mastering thoroughly the theory and practical details of the Chanoine movable dams which the Government adopted in the improvement of the Great Kanawha River. The first movable dams in America were built in this river.

Mr. Scott remained in local charge of the Great Kanawha River Improvement until the fall of 1901, when he resigned, establishing a record of continuous service of more than 28 years on that work and of 34 years as Civil Engineer in the United States Corps of Engineers. A reference to his professional service should not omit the following tribute paid to him by a distinguished Engineer Officer, the late Gen. William P. Craighill, Past-President, Am. Soc. C. E., under whom he served for 21 years, a period embracing the beginning and nearly the entire completion of the lock and dam system of the Great Kanawha. General Craighill resigned from command of the Baltimore Division of River and Harbor Works, which included the Great Kanawha Improvement, in 1895, when promoted to the highest office in his Corps, the Chief of Engineers of the Army. In an article on the Kanawha Improvement,* General Craighill stated:

"I desire also to call attention in the most emphatic way to another member of the American Society of Civil Engineers, who has for more than twenty-five years been connected with this important improvement which has been most successful from a commercial as well as an engineering standpoint. I refer to Mr. A. M. Scott, who was principal Engineer on this work when I took charge of it for the United States in 1874; he has remained on it up to the present time when it is completed. The conduct of the work has been most economical, wise and excellent in every way. Mr. Scott has exhibited an unusual degree of skill as a designing and constructing engineer in the management of the improvement and in dealing with the many perplexing problems which have presented themselves for solution in its progress. To him is due, more than to any other person, the success of this work. I say what I know, and it gives sincere pleasure thus to bear record to the merit of a most faithful and deserving man."

To all his subordinates Mr. Scott was always courteous and kind, winning their respect and esteem, while he himself possessed the unbounded confidence and kind regards of superior officers. In his profession he achieved an enviable reputation for capacity, skill, and executive ability. In his general intercourse with the public Mr. Scott was a modest, unassuming, courteous gentleman. He was accurate, prompt, and thoroughly reliable in all business engagements and transactions.

Mr. Scott continued to make Charleston his home until the time of his death. At one time he was Vice-President of the Charleston Chamber of Commerce; he was a Mason of long standing—Past High Priest of Royal Arch Masons and Past Eminent Commander of Knights Templar of the Charleston Commandery. He was long identified with the Kanawha Presby-

* *Engineering Record*, June 17, 1899.

terian Church of Charleston and was a member of its Board of Trustees. Although never prominent in politics, Mr. Scott took a real interest in all public matters. He was a life-long Democrat.

He was married in 1907 to Florence L. Jeroloman, a daughter of Judge John Jeroloman, of New York City, who survives him.

Mr. Scott was elected a Member of the American Society of Civil Engineers on June 2, 1886.

## HIRAM ELLSWORTH BALDWIN, Assoc. M. Am. Soc. C. E.*

DIED APRIL 16, 1927.

Hiram Ellsworth Baldwin was born in Youngstown, Ohio, on May 2, 1862, the son of George Simeon Baldwin and Hannah (Grey) Baldwin. Both parents were of sturdy American pioneer stock and spent their lives in Youngstown and the neighboring town of Niles, Ohio.

Mr. Baldwin's early education was obtained in the public schools of Niles. Following his steadfast boyhood desire for an engineering training, he entered the Massachusetts Institute of Technology, graduating therefrom in the Class of 1890 with the degree of Bachelor of Science.

Immediately following his graduation, he secured employment with The Brown Hoisting Machinery Company, of Cleveland, Ohio. His choice of this Company was a deliberate one, as he was greatly impressed with the developments then being carried on in machinery for the handling of ore and coal by Mr. Alexander E. Brown, the inventor. Mr. Brown had drawn around him a brilliant staff of young engineers, many of whom were to make names for themselves in later years. Mr. Baldwin was placed at once in the Engineering Office and his advance was rapid. While with this Company he not only achieved splendid success in the office, but also was sent on important engineering and erection work in Europe as well as in the United States. An especially notable accomplishment was the installation, under his direction, of the 100-ton capacity, balanced crane on the *Hercules*, of the U. S. Navy, the largest floating crane at that time.

He was appointed Chief Engineer of Special Work by The Brown Hoisting Machinery Company, and under his supervision came many developments in machinery for the rapid and economical handling of ore, coal, limestone, and other materials. Furnace hoists, bins, larries, locomotive cranes, overhead cranes, trollies, etc., were also designed and developed.

Mr. Baldwin left The Brown Hoisting and Machinery Company in 1908, and became identified as Chief Engineer with The Variety Iron Works Company, of Cleveland, Ohio. In this capacity, his work was of a most varied character, including the design of furnace hoists, open-hearth furnaces, ore and coal bins, as well as general structural steel work.

During the World War period he was instrumental in designing and building much important work for the United States Army and Navy, on which he labored indefatigably to the detriment of his health.

* Memoir prepared by E. C. Pierce, Esq., Cleveland, Ohio.

He resigned his position with The Variety Iron Works Company in 1922, to become Chief Engineer of The Brown Hoisting Machinery Company, with which he remained until ill health compelled him to resign. His death occurred on April 16, 1927; interment was at his boyhood home in Niles.

Mr. Baldwin was married on September 19, 1893; he had three children, one girl and two boys. One son was killed in action in the Argonne during the World War. Mr. Baldwin's widow, two children, and a sister, Lida F. Baldwin, survive him.

He was a member of the Cleveland Engineering Society, the National Aeronautic Association, the Cleveland Chamber of Commerce, and the Chagrin Valley Country Club.

He had a strong and forceful character, and was beloved by his friends and respected by all with whom he came in contact. His was a keen and analytical mind, and his work in his chosen profession was always well done and frequently most brilliant. His loss is a distinct one.

Mr. Baldwin was elected an Associate Member of the American Society of Civil Engineers on January 2, 1895.

## ROBERT WAITE DAVIS, Assoc. M. Am. Soc. C. E.*

DIED NOVEMBER 18, 1927.

Robert Waite Davis, the son of Thomas Burt and Alice Elizabeth (Finch) Davis, was born in Wartrace, Tenn., on January 16, 1890, of Welsh ancestry. He was educated in the public schools and later attended and was graduated from the Branham-Hughes Military Academy, at Spring Hill, Tenn., in 1906. He afterward worked for several years before entering the University of Tennessee, from which he was graduated in 1918.

Mr. Davis began his engineering career in 1907 as Rodman, Chainman, Mapper, etc., on land surveys. From March, 1909, to January, 1910, he was employed as Surveyor, Rodman, Concrete Inspector and Tester on lock and dam construction for the War Department, U. S. Engineer Office. From January to September, 1910, he was Assistant to the Construction Foreman, Instrumentman, and Material Chaser on plant construction for the Tennessee Coal, Iron and Railroad Company. In 1915, he returned to that Company as Instrumentman and Temporary Chief of Party on mineral land surveys.

In December, 1915, Mr. Davis accepted a position with the Nashville, Chattanooga and St. Louis Railroad Company as Office Assistant to the Location Engineer, also acting as Draftsman, Computer, etc. He held this position until September, 1916. From June to September, 1917, he was engaged as Draftsman in the office of the Louisville and Nashville Railroad Company in Louisville, Ky.

In the fall of 1917 and spring of 1918 Mr. Davis finished his education at the University of Tennessee and immediately upon his graduation in June, 1918, he joined the U. S. Coast Guard and, later, was transferred to the Engineer Officers' Training Camp, at Camp Humphreys, Virginia, where he re-

* Memoir prepared by N. E. Lant, M. Am. Soc. C. E.

mained until the Armistice was signed. He was honorably discharged from the Service in December, 1918.

In April, 1919, Mr. Davis accepted a position as Resident Engineer on railroad construction with the Cuyamel Fruit Company, in Honduras, Central America. He remained in this position until October, 1919, at which time he returned to the United States. In January, 1920, he again went to Honduras and until October of that year, acted as Resident Engineer and Locating Engineer on railroad construction and location for the Cuyamel Fruit Company.

After Mr. Davis returned to the United States in October, 1920, he accepted a position as Resident Engineer on highway construction with the Louisiana Highway Commission. His first assignment was on the construction of the Federal Aid Highway between Covington and Slidell, La. This project was completed in 1921, and he was then sent to Mansfield, La., as Resident Engineer on the Evelyn-Mansfield Highway, which was completed in 1922. Each of these two projects involved grading, drainage, structures, and gravel surfacing on approximately twenty-five miles of highway. After the disastrous floods in the Mississippi Valley in the spring of 1922, Mr. Davis was sent to Poydras, La., a short distance below New Orleans, where the levees and highway had been destroyed by the Poydras Crevasse. He superintended the rebuilding of the highway at this location, the work being done on a day-labor basis and under great difficultty, due to the wet weather and heavy traffic.

Due to exposure at Poydras, Mr. Davis suffered a recurrence of tropical fever which he had contracted in Honduras. As a result of this he spent several months in the fall of 1922 at Hot Springs, Ark., trying to regain his health. In November, 1922, after recovering his strength somewhat, he returned to the Louisiana Highway Commission, at Baton Rouge, and worked in the office as Checker and Computer during the winter. In the spring of 1923, he asked to be placed on construction work again and was sent to Slidell, as Resident Engineer on the Federal Aid Highway project, extending from Rigolets, La., to Slidell and from Pearl River Station, La., to the Mississippi State line. Each of these sections was approximately six miles long. The work at Slidell required Mr. Davis to be in the swamps and marshes practically every day, both sections of this road being through a virgin country without a trail of any kind over which to drive. Considerable relocation was necessary on the northern section, from Pearl River to the Mississippi State line, and this was all carried on with dispatch and accuracy. In the spring of 1925, contracts were let for the bridges over the East Pearl and West Pearl Rivers, and as the highway construction which Mr. Davis had in charge was rapidly approaching completion, he was given charge of the construction of these two bridges, both of which were steel truss bridges on concrete piers.

Mr. Davis continued in charge of the highway and bridge construction at Slidell until August, 1925, when due to ill health he was unable to continue this work. He was given a furlough and lived at Slidell several months before moving to Birmingham, Ala., where he died of a combination of tropical diseases and inflammatory rheumatism on November 18, 1927.

Mr. Davis was a careful and accurate engineer, fully investigating all details of each problem which presented itself to him. On highway location and construction, his notes and records were always complete and reliable. His sickness and death proved a great loss to the Louisiana Highway Commission and to his fellow employees.

He was a member of the Slidell Lodge of Masons, and of the American Geographical Society. Although he was not affiliated with any church, Mr. Davis had a strong Christian character and high moral standards.

He was married in 1924, to Constance Osborne, who, with one child, a daughter, is now living in Louisville.

Mr. Davis was elected an Associate Member of the American Society of Civil Engineers on November 21, 1921.

## WILLIAM PENDLETON PALMER, Affiliate, Am. Soc. C. E.*

DIED DECEMBER 17, 1927.

William Pendleton Palmer, the son of James Stewart and Eleanor Pendleton (Mason) Palmer, was born in Pittsburgh, Pa., on June 17, 1861. His father was a Captain in the 155th Regiment of Virginia and died during the Civil War. His mother was a descendant of an old Colonial family.

After completing a four years' course which included Mechanical Drafting, he was graduated from the Pittsburgh Central High School in 1878. Subsequently, he entered the employ of the Lewis, Oliver and Phillips Company, with which he remained until May, 1881, occupying various minor positions, as well as that of Traveling Salesman.

On May 27, 1881, Mr. Palmer began his service with Wilson, Walker, and Company, which, on January 1, 1886, was absorbed by Carnegie, Phipps, and Company, Limited. In July, 1892, the latter Company was united with the Carnegie Steel Company, Limited, and, with these various Carnegie interests, Mr. Palmer occupied, successively, the positions of Draftsman, Traveling Salesman, Secretary, General Sales Agent, and Assistant to the President.

In 1896, he resigned from the latter position to accept an appointment as Second Vice-President of the Illinois Steel Company, in Chicago, Ill. When the American Steel and Wire Company of New Jersey was organized, on January 13, 1899, he became a Director and General Manager of that Company. On May 9, 1900, Mr. Palmer was elected President of the American Steel and Wire Company and held this position until his death.

Mr. Palmer was also President of the American Mining Company. He was made a Director of the United States Steel Corporation in 1922, and was re-elected in 1924 and 1927. He likewise acted as a Director of the H. C. Frick Coke Company and of the Cleveland Trust Company, and was a Trustee of the Lakeside Hospital, Case Library, and Oberlin College. He was a member of the Protestant Episcopal Church.

* Memoir prepared by George H. Tinker, M. Am. Soc. C. E.

Mr. Palmer had a predilection for historical literature. He gathered from various sources all that is known of the wire industry. This information, in the form of written data, together with exhibits of machinery used in the industry, forms a notable collection known as the Industrial Museum of the American Steel and Wire Company, located at Worcester, Mass.

It was also Mr. Palmer's pleasure to collect historical data and Americana, particularly books, maps, manuscripts, photographs, etc., both Union and Confederate, relating to the Civil War, Abraham Lincoln, and slavery. This collection, believed to be the largest in the country outside the Library of Congress, he presented to the Western Reserve Historical Society of Cleveland.

Mr. Palmer was married on July 24, 1898, to Mary Boleyn Adams, of Chicago, who, with a daughter, Mrs. William Chisholm II, and a son, William Pendleton, Jr., survives him.

He had served as President of the Western Reserve Historical Society since 1913 and was a member of the following societies and clubs: The Engineers Society of Western Pennsylvania; the Cleveland Chamber of Commerce; The Chamber of Commerce of the United States; the Society of Colonial Wars; the American Antiquarian Society; the Military Order of the Loyal Legion; the Union Club; the Rowfant Club; the Cleveland Country Club; the Tavern Club; the Mayfield Club; the Kirtland Club; and the Chagrin Valley Hunt Club of Cleveland; the Duquesne Club and the Pittsburgh Club, of Pittsburgh, Pa.; the Chicago Club, of Chicago, Ill.; and the Engineers' Club and India House, of New York, N. Y.; also the Worcester Club and the Rufus Putnam Society, of Worcester, Mass.; and the Megantic Fish and Game Corporation of Belmont, Mass.

Mr. Palmer was elected an Affiliate of the American Society of Civil Engineers on March 3, 1896.